NAME: __________________________________________

General Instructions

- No collaboration of any kind is permitted on this examination.
- You may bring 2 double sided pages of reference notes. No other material may be consulted.
- Write all your solutions in the space provided. No sheets should be added to the exam.
- Make diagrams and sketches as clear as possible, and show all your derivations clearly.
  Incomplete solutions will receive only partial credit, even if the answer is correct.
- If you find you are unable to complete part of a question, proceed to the next part.

Please initial the statement below to show that you have read it

`By affixing my name to this paper, I affirm that I have executed the examination in accordance with the Academic Honor Code of Brown University. PLEASE WRITE YOUR NAME ABOVE ALSO!`


1 (7 points) __________

2 (14 points) __________

3. (9 points) __________

4. (5 points) __________

5. (5 points) __________

TOTAL (40 points) __________
1. The figure shows the ‘tablecloth’ trick demonstrated in class. The bottle has diameter \( d \) at the base, and its center of mass is a height \( h \) above the table. The coefficient of friction between cloth and bottle is \( \mu \). The cloth is pulled horizontally with an acceleration \( a > \mu g \) so the cloth slips under the bottle.

1.1 Draw the forces acting on the bottle on the figure below.

1.2 Assuming that the bottle does not tip, calculate its horizontal acceleration.

1.3 Show that the bottle will tip over if \( h/d \) exceeds a critical value, and give an expression for the maximum allowable value of \( h/d \) for the trick to work.
2. An unbalanced rotor that is spun at constant speed by a motor attached to its hub can be idealized as a particle with mass $m$ located at the center of mass of the rotor, which is a distance $L$ from the hub as shown in the figure.

2.1 Write down the position vector of the particle (i.e. center of mass) in terms of $L$ and the angle $\theta$. Hence, derive expressions for its acceleration in terms of $\theta$, $\omega = \frac{d\theta}{dt}$, and $L$. Use the basis shown, and assume that $\omega$ is constant.

2.2 Draw the forces and moments acting on the rotor on the figure provided. Gravity should be included.

2.3 Hence, calculate expressions for the horizontal and vertical reaction forces acting at the rotor hub as functions of time.
2.4 The graphs show reaction forces measured experimentally. Determine the mass of the rotor, and the distance of the center of mass from the axis of rotation. Use SI units.

[5 POINTS]
3. A gymnast swinging on a high horizontal bar can be idealized as a pendulum shown in the figure, with a point mass at B and pin joint at A. The length of member AB varies with time according to the equation \( L(t) = L_0 + \Delta L \sin \Omega t \), were, \( L_0 \) and \( \Delta L \) are constants, and \( \Omega \) is the (constant) frequency at which the athlete ‘pumps’ to start swinging.

3.1 Find the acceleration vector of the mass at B, in terms of the angle \( \theta \) and its time derivatives, \( L_0 \), \( \Delta L \), and \( \Omega \). Express your answer using the polar coordinate basis vectors shown in the figure.

3.2 Draw a free body diagram showing the force acting on the mass at B.

3.3 Using Newton’s law, show that the equation of motion for the angle \( \theta \) is

\[
\left( L_0 + \Delta L \sin \Omega t \right) \frac{d^2 \theta}{dt^2} + 2\Delta L \Omega \cos \Omega t \frac{d\theta}{dt} + g \sin \theta = 0
\]

3.5 Rearrange the equation of motion into a form that MATLAB could solve.
4. The figure shows an apparatus to measure the impulse exerted by a sub-surface explosive device. It consists of a piston with mass $m$ supported by a frame. The system is initially at rest. The explosion then propels the piston vertically, and its maximum height $h$ is measured. Derive an expression that relates the piston mass $m$ and the height $h$ to the impulse $I$ exerted on the piston by the explosion. Friction can be neglected.

(Figure from Ehrigott, et al, *Experimental Techniques*, doi: 10.1111/j.1747-1567.2009.00604.x)
5. The figure shows a collision between two spheres with identical mass. Before the collision, sphere A moves at 45° to the i direction while sphere B is at rest. The coefficient of restitution between the spheres is $e=0$. One of figures (a)-(e) shows correctly the position of the spheres a short time after the collision (the dashed circles show the positions of the spheres at the instant the collision occurs, for reference). By answering the true/false questions below, identify the correct figure.

(a) Momentum is conserved  T  F
   The restitution coefficient formula is satisfied  T  F

(b) Momentum is conserved  T  F
   The restitution coefficient formula is satisfied  T  F

(c) Momentum is conserved  T  F
   The restitution coefficient formula is satisfied  T  F

(d) Momentum is conserved  T  F
   The restitution coefficient formula is satisfied  T  F

(e) Momentum is conserved  T  F
   The restitution coefficient formula is satisfied  T  F

[5 POINTS]