NAME: ____________________________

General Instructions

- No collaboration of any kind is permitted on this examination.
- You may bring 2 double sided pages of reference notes. No other material may be consulted.
- Write all your solutions in the space provided. No sheets should be added to the exam.
- Make diagrams and sketches as clear as possible, and show all your derivations clearly.
  Incomplete solutions will receive only partial credit, even if the answer is correct.
- If you find you are unable to complete part of a question, proceed to the next part.

Please initial the statement below to show that you have read it

`By affixing my name to this paper, I affirm that I have executed the examination in accordance with
the Academic Honor Code of Brown University. PLEASE WRITE YOUR NAME ABOVE ALSO!

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1 (5 points) ____________
2. (10 points) ____________
3. (5 points) ____________
4. (10 points) ____________

TOTAL (30 points) ____________
1. The projectile in a gas gun is subjected to a propulsive force

\[ F = F_0 \left(1 - \frac{v}{3c}\right)^5 \]

where \( F_0, c \) are constants and \( v \) is the projectile’s speed. The projectile has mass \( m \). It starts at rest at time \( t=0 \) at position \( x=0 \).

1.1 Use Newton’s law to determine the acceleration of the projectile and hence determine an expression for its speed as a function of time and other parameters. Air resistance and friction may be neglected.

[3 POINTS]
1.2 Find a formula for the distance traveled by the projectile as a function of time.
2. A ‘prey’ particle with mass \( m \) is subjected to a propulsive force with magnitude \( F_{\text{max}} \) that is applied at a constant angle \( \alpha \) to its direction of motion. It also experiences a drag force (not shown in the figure) with magnitude \( cV \) (where \( V \) is the magnitude of the velocity and \( c \) is a constant) that acts opposite to its direction of motion. The random force is zero. As a result, it travels at constant speed \( V \) around a circular path with radius \( R \).

2.1 Write down expressions for the propulsive force vector and drag force vector acting on the particle, in terms of \( F_{\text{max}}, \alpha, c \), and \( V \), expressing your answer in polar coordinates \( \{ \mathbf{e}, \mathbf{e}_\theta \} \).

2.2 Write down an expression for the acceleration vector in terms of \( V \) and \( R \), and hence use Newton’s law to show that

\[
V = \frac{F_{\text{max}}}{c} \cos \alpha \\
R = \frac{mF_{\text{max}}}{c^2} \frac{\cos^2 \alpha}{\sin \alpha}
\]
2.3 Find a formula for the time required for the particle to complete a full circle.

[1 POINT]

2.4 In the MATLAB code controlling the prey, the propulsive force is calculated from the formula 
\[ \mathbf{F} = F_{\text{max}} \cos \omega t \mathbf{i} + F_{\text{max}} \sin \omega t \mathbf{j}, \]
where \( \omega \) is a constant. Use the solutions to 2.2 and 2.3 to find a formula relating the radius of the path \( R \) to \( \omega, F_{\text{max}}, m \) and \( c \). The formula \( \cos \theta = \frac{1}{\sqrt{1 + \tan^2 \theta}} \) might be helpful.

[2 POINTS]
2.5 The prey starts with a total energy supply $E$. Find a formula for the maximum time that the particle can continue traveling around the circular path without exhausting the energy supply, in terms of $E, F_{\text{max}}, m, \omega, c$. 

[2 POINTS]
3. Two spheres with identical mass and restitution coefficient \( e=0 \) have initial positions shown in the figure below. Before impact sphere B is stationary and sphere A has velocity \( V \). The collision is frictionless. By answering the true/false questions below, identify which of the figures (a-d) shows the correct position of the spheres after collision.

**CIRCLE ONE RESPONSE TO EACH STATEMENT BELOW**

(a) Total Momentum is conserved in the j direction T F
Momentum of B is conserved in the t direction T F
The restitution formula is satisfied in the n direction T F

(b) Total Momentum is conserved in the j direction T F
Momentum of B is conserved in the t direction T F
The restitution formula is satisfied in the n direction T F

(c) Total Momentum is conserved in the j direction T F
Momentum of B is conserved in the t direction T F
The restitution formula is satisfied in the n direction T F

(d) Total Momentum is conserved in the j direction T F
Momentum of B is conserved in the t direction T F
The restitution formula is satisfied in the n direction T F

Correct figure: a b c d

[5 POINTS]
4. The figure shows a schematic diagram of a pendulum apparatus designed to measure the stiffness of an intervertebral disk. The disk is idealized as a spring with stiffness $k$ and unstretched length $L_0$ (the slider at A allows the spring to remain vertical at all times). The goal of this problem is to find the relationship between $k$ and the measured period of oscillation of the pendulum.

4.1 Write down a formula for the speed of the mass $m$ in terms of $h$ and $d\theta/dt$.

4.2 Hence, write down the total potential and kinetic energy of the system, in terms of $k, d, h, m, g$ and $\theta$ and its time derivatives.
4.3 Hence, show that $\theta$ satisfies the equation of motion

$$mh^2 \frac{d^2 \theta}{dt^2} + kd^2 \sin \theta \cos \theta + mgh \sin \theta = 0$$

[2 POINTS]

4.4 Linearize the equation of motion for small $\theta$ and hence find a formula for the natural frequency of vibration of the pendulum, in terms of $k, d, h, m, g$.

[3 POINTS]
4.5 Rearrange the equation in 4.3 into a form that MATLAB could solve using the ode45 function.