Kaldor–Hicks compensation. The need to judge one situation better than another motivates much of economics, and almost all of welfare economics. There are questions on the level of the individual: When is $y$ better than $x$ for one particular person? And if $y$ is better, by how much? How much does the person gain by going from $x$ to $y$? There are similar-looking questions on the level of society: When is $y$ better than $x$ for society? And if it is better, by how much? Economists have had a fair amount of success in analysing and answering the individual-level questions. They have had less success with the societal-level questions.

In the view of nineteenth-century utilitarian philosophers and economists, a person's degree of happiness, or satisfaction, or utility, could in theory be measured much like a physical attribute such as weight. Measured utility would have the obvious characteristic of reflecting preferences — so $u(y) > u(x)$ if the individual prefers $y$ to $x$, and $u(y) = u(x)$ if he likes them equally well. But it would also reflect strength of preference, so, for example, $u(y) - u(x) > u(w) - u(z)$ if the individual finds the utility advantage of $y$ over $x$ exceeds the utility advantage of $w$ over $z$, and, for example, $u(y) = 5u(x)$ if the individual finds $y$ five times as good as $x$. Moreover, according to classical utilitarians, utility, like weight, is comparable between individuals. This last means that, if $u_a = Adam’s$ utility and $u_e = Eve’s$ utility, it makes sense to add $u_a(x)$ and $u_e(x)$ together, and it makes sense to say $u_a(y) + u_e(y) > u_a(x) + u_e(x)$ when, and only when, situation $y$ is better for society than situation $x$.

In short, for nineteenth-century utilitarians, utility is an interpersonally comparable, cardinal metric. It can be applied to both individual-level questions and societal-level questions, and it can be relied upon to answer questions about when $y$ is better than $x$, and by how much. However, although there are still some utilitarians among us, most twentieth-century economists have turned away from this approach, mainly because of the impossibility of measurement. No one has derived a generally accepted way to measure one person’s utility function. No one knows how to test the statement that $u(y) = 5u(x)$. No one is sure about how to scale Adam’s and Eve’s utilities so that the sum $u_a(x) + u_e(x)$ makes sense.

Modern economic analysis leans on two axioms, the first being that utility is ordinal. That is, writing $u(y) > u(x)$ when the individual prefers $y$ to $x$ (and $u(y) = u(x)$ when he is indifferent) makes sense, but a utility function has no meaning beyond this. It shows only preference. It does not show strength of preference, and it is not comparable between Adam and Eve (see, e.g., Morey 1984). The second axiom is that money is observable, measurable and comparable. That is, for example, $2$ is more than $1$ (ordinality); but, moreover, $2$ is twice as much as $1$ (cardinality); if Adam has $1$ under $x$ and $3$ under $y$, he gains $2$ in the switch from $x$ to $y$ (cardinality); and if Eve has $2$ under $x$ and $1$ under $y$, then society has a net gain of $1$ in the switch from $x$ to $y$ (cardinality plus interpersonal comparability). In short, money is an interpersonally comparable, cardinal metric. Unfortunately, as we shall see, these two axioms of economics do not mesh well.

Hicks, Kaldor, Scitovsky and Samuelson. If utility is ordinal, and no utility comparisons between individuals are permitted, it seems almost impossible to make judgments about whether situation $y$ is better for society than situation $x$. This is because, in most real policy choices, the switch from $x$ to $y$ produces some gainers (those for whom $u(y) > u(x)$), and some losers (those for whom $u(y) < u(x)$), as well as some who are indifferent. If interpersonal utility comparisons are prohibited, how can one judge the gains of the gainers against the losses of the losers?

To facilitate such judgments, Kaldor (1939) and Hicks (1939) developed the compensation criterion that bears their names. Consider a move from policy $x$ to policy $y$. In Kaldor the switch is the repeal of the English Corn Laws in 1846, which harmed landowners but helped consumers of bread. If those who gain from the switch could in theory compensate those who have been harmed, and remain better off, then the move is desirable, and $y$ is better for society than $x$. Note that the compensation is not necessarily paid; it is a theoretical possibility, not a fact.

For the economist to make his case for a reform, according to Kaldor:

... it is quite sufficient for him to show that even if all those who suffer as a result of the reform are fully compensated for their loss, the rest of the community will still be better off than before. Whether the landlords, in the free-trade case, should in fact be given compensation or not, is a political question on which the economist, qua economist, could hardly pronounce an opinion (1939: 550).

Hicks proposes the same criterion: ‘... the reforms we have studied are marked out by the characteristic that they will allow of compensation to balance that loss, and they will still show a net advantage’ (1939: 711). Hicks does not recommend that compensation of the losers always be made, although he does urge economists/reformers to be more explicit about what compensation policies (e.g., adjustments of taxes) would be appropriate.

So the Kaldor–Hicks compensation criterion involves the
Kaldor–Hicks compensation

Theoretical possibility of compensation payments. The payments, if made, would leave everyone as well off after the reform as they were before, and leave some better off. A reform which in fact (not in theory) makes everyone as well off and some people better off (and which would therefore get unanimous support) is called a Pareto improvement, and consequently the Kaldor–Hicks criterion is sometimes called the potential Pareto improvement test. Also, note that those theoretical compensation payments are clearly not transfers of utility. They are potential transfers of goods, or of money. And so there are two types of analysis that can be done here, analysis in terms of goods transfers, or of money transfers.

Shortly after Hicks and Kaldor proposed compensation tests, Scitovsky (1941) used a careful goods-transfer model to reveal a troubling problem with the Kaldor–Hicks test. Scitovsky’s model has two individuals, say Adam and Eve, and two goods, say A and B. There are two alternatives, say x and y, between which society must choose. Each alternative is an allocation of the two goods between the two people, and each allocation involves certain totals of the two goods. In a move from x to y, the total quantity of one good rises, and the total quantity of the other good falls. Given y (or x), society could in theory redistribute the totals of goods A and B between Adam and Eve, so as to achieve any redistribution with the same respective totals. In Scitovsky’s example, the distributions x and y are both Pareto optimal or efficient given the respective totals. (That is, it is impossible to redistribute those totals so as to make one individual better off, without hurting the other.) Now Scitovsky shows (with his graphical example) that the move from x to y is a Kaldor–Hicks move. In other words, the gainer, say Adam, could potentially transfer some of one or both goods to the loser, say Eve, so as to fully compensate her, and remain better off than he was at x. However, at the same time, the reverse move from y to x is also a Kaldor–Hicks move. That is, in a move from y to x the gainer, Eve, could potentially transfer some of both goods to the loser, Adam, so as to fully compensate him, and remain better off than she was at y.

In other words, when the total quantity of one good is increasing but the total quantity of another good is decreasing, and when the theoretical compensation payments are mapped out in terms of goods transfers, the attractive Kaldor–Hicks criterion may be logically inconsistent. It may say y is better than x, and, at the same time, x is better than y, which Scitovsky rightly calls an ‘absurd result’. His remedy is a two-edged test: ‘We must first see whether it is possible in the new situation to redistribute income as to make everybody better off than he was in the initial situation; secondly, we must see whether starting from the initial situation it is not possible by a mere redistribution of income to reach a position superior to the new situation, again from everybody’s point of view. If the first is possible and the second impossible, we shall say that the new situation is better than the old was’ (1941: 86–7). In other words, the move from x to y satisfies the Scitovsky compensation criterion if it is a Kaldor–Hicks move, but the move from y to x is not.

Samuelson (1950) uses a utility frontier diagram to analyse the Scitovsky criterion, similar to Figure 1 below. For the purpose of this diagram, any ordinal utility representation of Adam’s preferences may be used, as well as any ordinal utility representation of Eve’s preferences. (So this is not an exercise in (cardinal) utilitarian analysis, no meaning being attached to terms like \( u_a(x) + u_e(y) \); all that matters are inequalities like \( u_a(y) > u_a(x) \).)

In Figure 1, Adam’s utility is shown on the horizontal axis and Eve’s on the vertical. Each point shows both a utility level for Adam and for Eve; for instance, \( u(x) = (u_a(x), u_e(x)) \) shows their utility levels given the initial situation x. Now, if situation x is chosen, there are many possible compensation transfers that could in theory be made, and they give rise to a large set of resulting utility combinations for Adam and Eve; the outer boundary of this set is the utility frontier \( U(x) \). The fact that \( u(x) \) is on the frontier \( U(x) \) means that, given the goods totals or whatever other fundamental constraints are produced by the choice of situation x, the distribution of goods between Adam and Eve is Pareto optimal or efficient. Figure 1 illustrates a pair of alternatives, x and y, with Scitovsky reversal. The move from x to y is a Kaldor–Hicks move because the point \( u(x) \) lies well inside the frontier \( U(y) \). But for a completely analogous reason \( u(y) \) is inside the frontier \( U(x) \), the move from y to x is also a Kaldor–Hicks move. On the other hand, situation z (which is one of the possibilities that could be reached from y) is better than situation x according to the two-edged Scitovsky test. That is, the move from x to z is a Kaldor–Hicks move, but the move from z to x is not. Not only does Figure 1 illustrate the kind of reversal that makes the Kaldor–Hicks compensation criterion unsatisfactory; it also reveals the potential weakness of the Scitovsky compensation criterion: the Scitovsky test finds z is better than y. But Figure 1 shows the set of utility possibilities based on z, which is the same as the set based on y, i.e. \( U(y) \), is not at all more attractive than the set based on x, i.e. \( U(x) \).

Samuelson writes that what Scitovsky should have done is make ‘the comparison depend on the totality of all possible positions in each situation’ (1950: 11). That is, for y to be declared better than x, the utility frontier \( U(y) \) should lie entirely outside the utility frontier \( U(x) \). Call this the Samuelson compensation criterion.
The main purpose of Samuelson’s paper was to discover what is the connection, if any, between an increase in real national income – roughly speaking, the money value of the goods and services consumed by society – on the one hand, and an increase in the welfare of society on the other. (This connection had been explored earlier by many economists, including Hicks (1940).) Suppose, for example, that in situation x the list of prices of goods and services is \( p_x = (p_{1x}, p_{2x}, \ldots) \), and that in situation y the list of prices is \( p_y = (p_{1y}, p_{2y}, \ldots) \). Suppose, further, that there is a shift in relative prices as society goes from x to y; that is, the \( p \) vector is not proportional to the \( x \) vector. Finally, let \( x \) be the bundle of goods and services consumed by Adam under situation x, with similar definitions for \( y \), \( x \), and \( y \).

Now, everyone knows that it would be wrong to infer anything about Adam’s welfare from a comparison of the money he spends under x with the money he spends under y, or \( p_x \cdot x = p_{1x} \cdot x_1 + p_{2x} \cdot x_2 + \ldots \) with \( p_y \cdot y \). There might, for example, be general price inflation or deflation between x and y, which would render such a comparison meaningless. So the alternative bundles x and y must be evaluated at one set of prices only.

If the evaluation is done at the prices corresponding to y, the test for Adam would be to see if \( p_y \cdot y > p_x \cdot x \). If this inequality holds when society moves from x to y (and relative prices as well as consumption bundles change), then Adam is buying a new bundle of goods \( y \) when he could more than afford his old bundle \( x \), and he must therefore be better off. This raises two questions: (1) Is \( p_y \cdot y - p_x \cdot x \) a correct dollar measure of Adam’s increase in welfare? (2) If \( p_y \cdot y > p_x \cdot x \), can we conclude that society is better off because of the move from x to y? Samuelson’s answer to the latter question is negative: the price–quantity inequality says nothing at all about whether the utility frontier \( U(y) \) lies outside the utility frontier \( U(x) \). The former question is related to a concept called consumer’s surplus.

**Consumer’s Surplus.** Starting with the work of Dupuit (1844) and Marshall (1890), economists and others have attempted to measure the money gains from desirable public projects like roads or bridges, or from the existence of markets for given goods, and the money losses from undesirable things like monopolies, or taxes, or tariffs. This kind of measurement is essential for intelligent judgment about alternative macroeconomic policies. The Kaldor–Hicks compensation criterion judges y better than x if the gainers in the move from x to y could in theory compensate the losers. To measure those gains and losses, according to Hicks (1941), the apparatus to use is consumer’s surplus.

Marshall’s definition of consumer’s surplus seems straightforward. Suppose a consumer is purchasing something. He pays some price for the quantity he buys, but if he were given a choice between paying a higher price for that quantity, or going entirely without, there is some maximum amount he would be willing to pay. The difference between the maximum he would be willing to pay, and what he actually does pay, is his consumer’s surplus. This is a money measure of the value of his opportunity to buy the commodity at the given price.

Hicks formalized the Marshallian notion of consumer’s surplus in two ways (see especially Hicks 1942). Suppose a change from x to y is proposed. As before, \( p \) and \( x \) represent the pre- and post-change lists of prices, and \( x \) and \( y \), represent the pre- and post-change bundles of goods and services consumed by Adam. The compensating variation measure of Adam’s gain from the x to y move is the answer to the following question: Starting at y, and based on the \( p \) prices, what is the maximum amount of money Adam could give up, and remain as well off as he was at his original position \( x \)? This measure is illustrated in Figure 2.

Figure 2 shows two budget lines for Adam, a pre-change budget \( B \), and a post-change budget \( B' \). As well, Adam’s chosen bundles \( x \), pre-change and \( y \), post-change. Note that the change from x to y makes Adam better off – in fact, the figure represents a change that simply involves a drop in the price of good A, with Adam’s income and the price of good B remaining constant. Assume without loss of generality that the price of good B is normalized at 1 – so $1 of income is equivalent to one unit of good B. In the figure, \( HB \) and \( HB' \), are hypothetical budget lines; \( HB \) has a slope determined by the \( p \) prices but is located so that it would make Adam exactly as well off as he is at \( x \). \( HB' \), has a slope determined by the \( p \) prices but is located so that it would make Adam exactly as well off as he is at \( y \). Under the assumption that the good B price equals 1, the vertical intercept of any budget line gives the income level corresponding to that budget.

With all this apparatus in place, we can now note that the compensating variation measure of Adam’s gain must be equal to the difference, on the vertical axis, between the intercept of the B’ budget – that is, his expenditure level, contingent on prices \( p' \), when he is at the post-change point – and the intercept of the HB’ budget – that is, his expenditure level, also contingent on prices \( p' \), if his income is reduced so much that he is exactly as well off as he is at \( x \). This difference is labelled c.v. in the figure.

Note that c.v. is close to, but not exactly equal to, the measure \( p_y \cdot y - p_x \cdot x \), mentioned in the last section. It was
asked there whether \( p \cdot y - p \cdot x \) is a correct dollar measure of Adam's increase in welfare, and we can now see that the answer is 'not exactly'. (To find \( p \cdot y - p \cdot x \), draw another line, through \( x \), and with the same slope as \( B_2 \), find its intercept on the vertical axis, and take the difference between that intercept and the \( B_2 \) intercept.)

Note that the compensating variation measure meshes perfectly with what is required for the Kaldor–Hicks test: one can imagine taking away a sum of money (or good 2) from Adam, up to the amount c.v., and transferring that money to the loser Eve so as to compensate her for her loss.

The second formalization of consumer's surplus that Hicks provides is the equivalent variation measure. It is the answer to this question: Starting at \( x \), and based on the \( p \), prices, what is the minimum amount of money that Adam would require to become as well off as he would be at \( y \)? This is identified as c.v. in the figure. Note that, as Figure 2 shows, c.v. will generally differ somewhat from e.v., at least if relative prices change when going from \( x \) to \( y \).

Intuitively, compensating variation represents the money gain that Adam could bribe Eve with, if the change from \( x \) to \( y \) were made; equivalent variation represents the bribe that Adam would require to forego the change. Careful examination of Figure 2 should convince the reader that c.v. and e.v. are related in this way: Adam's c.v. associated with the move from \( x \) to \( y \) must be the negative of Adam's compensating variation associated with the reverse move, from \( y \) to \( x \). Therefore, if compensating variation is the money metric used, and if we calculate the effect on Adam of the round trip, first from \( x \) to \( y \), and then from \( y \) to \( x \), his gain on the \( x \) to \( y \) leg is c.v. of Figure 2, and his loss on the \( y \) to \( x \) leg is e.v. (Note that the loss from the \( y \) to \( x \) leg seems greater than the gain from the \( x \) to \( y \) leg; this should puzzle and disturb the reader.)

At this point we have an embarrassment of riches for measuring Adam's gain when society moves from \( x \) to \( y \), a collection of more or less intelligently devised money metrics. These include \( p \cdot (y - x) \), already introduced, which is close to what is wanted but not exact; compensating variation; equivalent variation; and various related demand-curve-based measures connected to e.v. and c.v. It turns out that both c.v. and e.v. are in fact exact represenations of Adam's preferences, in the limited sense that they must be positive when Adam prefers \( y \) to \( x \), zero when he is indifferent, and negative when he prefers \( x \) to \( y \).

A great deal has been written about how the e.v. and c.v. money metrics are close to each other and close to observable areas under Marshallian demand curves (Willig 1976); about how the classical Marshallian 'deadweight loss' triangles are approximately equal to the properly calculated e.v. or c.v. welfare change measures; about how classical formulae for welfare change, such as \( 1/2 (p_1 + p_2)(y_1 - x_1) \), are appropriate or close to appropriate (Weitzman 1988; Diewert 1992). We can safely say at this stage that, from the standpoint of theory and empirical work, there are good money measures for Adam's gain from a move from \( x \) to \( y \).

**Consumers' surplus.** Note well the position of the apostrophe in this section's heading. Can we now combine, in a logically consistent way, consumer's surplus for Adam and for Eve, so as to determine with a consumers' surplus-money metric whether or not society gains in the move from \( x \) to \( y \)?

Under some circumstances we can. If there is only one good, there is no logical problem with aggregating gains and losses. This is the underlying assumption of much law and economics theorizing, wherein a legal rule is chosen to maximize aggregate wealth. The money metric for Adam or Eve becomes the quantity of the one good he or she possesses. Society's gain in going from \( x \) to \( y \) becomes \((y + x) - (x + x)\), which may be easily measurable, and, although perhaps morally unattractive to some, this approach creates no internal contradictions.

Or, if there are two or more goods but relative prices do not change when society moves from one alternative to another, there is no logical problem with aggregating the money metric gains and losses. This case is essentially equivalent to the one-good case. This invariance of relative prices is the standard underlying assumption of economic cost–benefit studies.

Or, if relative prices do change when society moves from one alternative to another, but Adam's and Eve's preferences are such that they do not substitute one good for another as the first becomes relatively cheaper, there is again no logical problem with aggregating the money metrics. Or, if Adam's and Eve's preferences are (i) homothetic – meaning that as income changes, if relative prices are held constant, the proportions of various goods in their consumption bundles will not change – and (ii) identical – at least close to the alternative points – then there is no logical problem with aggregating the money metrics. Many of these results and the negative results below are surveyed in Blackorby and Donaldson (1990).

However, in the general case, where relative prices do change, where Adam and Eve do substitute cheaper for dearer goods, where their preferences do differ, the consumer's surplus money metric is logically inconsistent, just as the Kaldor–Hicks compensation test is logically inconsistent.

The fatal problem was discovered by Broadway (1974). In the spirit of Scitovsky (1941), Broadway constructs an example in which society (that is, Adam and Eve) move from \( x \) to \( y \), where both \( x \) and \( y \) are Pareto optimal or efficient allocations. In Broadway's example, unlike Scitovsky's, \( x \) and \( y \) are efficient points in the same Edgeworth–Pareto box diagram. (Because both are efficient and the goods totals are constant, neither can be superior to the other in terms of the Kaldor–Hicks test.) However, \( x \) and \( y \) do differ in the sense that (i) the move from \( x \) to \( y \) makes Adam better off and Eve worse off, and (ii) relative prices are different at \( x \) and \( y \). Broadway shows that, in the move from \( x \) to \( y \), the sum of Adam's and Eve's compensating variations must be positive. But if society moves back, from \( y \) to \( x \), the sum of the compensating variations is again positive. Since the move out is a social improvement, and the move back is also, the c.v. money metric is inconsistent, a Scitovsky-style 'absurd result'.

Nor is Broadway's example unique. Blackorby and Donaldson (1990) demonstrate that, in general, for an exchange economy model or an exchange and production model, when society moves from an efficient \( x \) to an efficient \( y \), the
sum of compensating variations will always be non-negative and will generally be positive (the latter if there are relative price changes and substitution). Hence, by the criterion of positive summed compensation variations, the move from $x$ to $y$ will almost always seem to make society better off, as will the move from $y$ to $x$, and Bowday inconsistency is the rule, not the exception. In fact, the compensating variation money metric is a worse (i.e. more logically inconsistent) test than the Kaldor–Hicks compensation test discussed above: whenever the Kaldor–Hicks test is inconsistent, the compensating variation money metric test will also be inconsistent, and, as the Bowday example shows, the compensating variation test will sometimes be inconsistent even when the Kaldor–Hicks criterion is not.

In conclusion, from the perspective of the economic theorist who seeks a consistent method which allows judgments about when the move from $x$ to $y$ is a social improvement, a method that does not require constant relative prices or very similar consumers with very special preferences, all of the compensation criteria discussed in this essay are fundamentally disappointing.

However, for the applied economist, the policy-maker, or the law-maker interested in economic efficiency, it is necessary to hope that relative price changes are not too large, to make a leap of faith, and to weigh together Eve’s losses and Adam’s gains. The applied economist uses cost-benefit analysis, consumers’ surplus measures and the Kaldor–Hicks test to boldly go where the theorist fears to tread.

ALLAN M. FELDMAN

See also compensation for regulatory takings; contingent valuation; eminent domain and just compensation; law-and-economics from the perspective of critical legal studies; Pareto optimality; takings; value maximization; wealth maximization.

Subject classification: 2a(ii); 4e(ii); 6e(iv).

BIBLIOGRAPHY


Karl Llewellyn and the early law and economics of contract. Karl Llewellyn (1893–1962) taught law at the Yale, Columbia and Chicago Law Schools from the early 1920s to the 1950s (the standard biography is Twining 1973). Llewellyn is best known to lawyers for three major contributions: he wrote the first casebook on the law of sales (Llewellyn 1930); he was the leading drafter of the Uniform Commercial Code, America’s most important commercial statute; and he was perhaps the most important scholar in the legal realist movement. Llewellyn is also known to the wider world for his original contributions to legal anthropology.

It is less well known that Llewellyn developed a normative theory of contracts to help decisionmakers regulate commercial transactions. Llewellyn’s theory drew heavily on economics, and was an important precursor of the modern law and economics approach to contracts and commercial law. This essay sketches Llewellyn’s theoretical contribution and argues that while his general approach remains relevant to modern, his solutions to particular commercial problems are not. This is because Llewellyn lacked the economic tools, such as game theory and finance, that the contract theorists of today employ to resolve these problems. The essay that follows is drawn from the thirteen Llewellyn works cited in the bibliography below.

The decisionmakers that Llewellyn’s theory was meant to help were courts and private law reform organizations. Legislatures played a minor role. The substantive part of the theory told decisionmakers what to do. The institutional part matched decisionmakers to the problems they could best solve and also specified the appropriate level of abstraction that particular rules should assume.

Regarding substance, Llewellyn explicitly rejected distributional norms because he thought they could not be effectively pursued in contracting contexts. The commercial parties in the theory were commonly both buyers and sellers. As a consequence, a rule that sought to shift wealth to sellers would hurt actual commercial sellers, because they were also buyers, and similarly for pro-buyer roles. The regnant norm in Llewellyn’s contract theory was thus efficiency, as an intelligent law professor then would understand it.

Llewellyn’s particular applications of the efficiency norm followed from three premises: