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RECONTRACTING STABILITY

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This paper contains a relatively simple proof for the stability of Edgeworthian recontracting. It also applies that proof to a random recontracting process in an economy with a finite number of utility vectors, in which the recontracting process is a Markov chain and all non-core utility vectors correspond to transient states.

1. INTRODUCTION

THE BASIC economic act is exchange. Since exchange need not be constrained by prices, barter exchange is in a sense more fundamental than price exchange. A typical barter economy has a set of distributions of goods or utilities which cannot be upset by further bartering. This equilibrium set is called the core.

In this paper we will be concerned with the stability of the bartering, or recontracting, process; we want to know whether it actually carries the economy to a core distribution. Our point of view is analogous to that of economists such as Negishi [**4**] who have investigated the stability of the price adjustment, or tâtonnement, process. Jerry Green has treated the stability of recontracting in several papers [**2** and **3**]; his first treatment has been extended by Neuefeind in [**5**]. We will elaborate on the similarities and differences between these approaches and ours below. Other work (see, e.g., [**1**]) has been done on the stability of solution searching procedures for n-person games. Scarf's [**6**] proof of existence of the core is constructive, and its algorithm for finding the core could be called "stable," but, of course, that algorithm is not recontracting.

Recontracting is a process of proposal, challenge, and counterproposal, in which no exchanges are actually made outside the core. At each stage of the process an allocation of goods or utilities is put before the economy. If some group of traders finds that it can do better on its own, it challenges or blocks the proposal. It then proposes an alternative allocation, which it prefers, and which it can somehow achieve by itself, but which may make its non-members worse off than they were before. Two types of recontracting stability can be defined. First, if for any allocation which can be blocked by some group, there exists a sequence of recontracts which leads to an allocation which cannot be blocked by any group (or is in the core), the recontracting process is stable in a potential sense. Second, if we specify a random mechanism for choosing blocking groups and blocking proposals, and if that choice mechanism leads from any allocation to the core with probability one, then the process is stable in a probabilistic sense. We will prove two theorems which provide relatively simple sufficient conditions for each sort of recontracting stability.

2. THE MODEL

It is possible to discuss recontracting and the core from two points of view. The first considers simultaneously a set of allocations of goods and a corresponding set of utility vectors. The second abstracts from the allocations and considers only the utility vectors. The latter approach is notationally much simpler, and we will adopt it in this paper.

We will suppose that there are n individuals in the economy, and we will let N be defined as the set of all members of the economy. A *coalition* is a non-empty subset of N. If S is a coalition, we will let $V(S)$ denote the set of all utility vectors attainable by S. Whenever S and T are coalitions, $S \subset T$, and $u \in V(T)$, we will write u^S for the projection of u onto R^S, the Euclidean subspace whose coordinates are indexed by the members of S. If, for example, $S = \{1, 2\}$, $T = \{1, 2, 3\}$, and $u = (8, 0, 4)$, then $u^S = (8, 0)$. We will also let I denote the set of integers: $I \equiv \{\ldots -2, -1, 0, 1, 2, \ldots\}$; I^N will denote the n-fold Cartesian product of I, whose coordinates are indexed by the members of N, and I^S will denote the subspace of I^N whose coordinates are indexed by the members of S.

If $u \in V(S)$, and there is no $v \in V(S)$ such that $u \leqslant v$, we will say that u is *Pareto optimal for* S.[1] If $u \in V(N)$, and there exists a $v \in V(S)$ such that $u^S \leqslant v$, then we will say that S *blocks* u, or S *blocks* u *with* v. The *core* is the set of all utility vectors in $V(N)$ which are blocked by no coalitions.

We will make the following assumptions about the attainable utility sets.

ASSUMPTION 1 (Finiteness): *For every* $S \subset N$, $V(S)$ *is a finite set. Without loss of generality, we will assume that* $V(S) \subset I^S$.

ASSUMPTION 2: *For* $i = 1, 2, \ldots, n$, $V(\{i\}) = 0$.

ASSUMPTION 3 (Superadditivity): *If* S_1 *and* S_2 *are disjoint coalitions, and if* $v_1 \in V(S_1)$ *and* $v_2 \in V(S_2)$, *then* $(v_1, v_2) \in V(S_1 \cup S_2)$.

ASSUMPTION 4 (Very strong superadditivity for pairs): *If* S *has two members, there exists a vector* $(a, b) \in V(S)$ *such that* $(0, 0) < (a, b)$.

ASSUMPTION 5 (Free disposal of utility): *If* $v \in V(S)$, $0 \leqq u \leqq v$, *and* $u \in I^S$, *then* $u \in V(S)$.

Assumption 1 is somewhat unusual in economic and game theoretic work because it typically makes analysis rather difficult. The assumption of finiteness, however, makes our task much more tractable, and so it is perhaps appropriate to justify it. There are two possible justifications. First, in the real world, most goods are traded in indivisible units, and there can be only a finite number of allocations of indivisible goods in finite supply. Second, utility itself is probably not a psychological continuum; individuals may only have a finite number of stages of happiness, even if there are an infinity of allocations. In either case, it is

[1] If $x \in R^k$ and $y \in R^k$, we will write $x = y$ whenever $x_i = y_i$ for $i = 1, \ldots, k$; $x \leqq y$ whenever $x_i \leqq y_i$ for $i = 1, \ldots, k$; $x \leqslant y$ whenever $x \leqq y$ and $x \neq y$; and $x < y$ whenever $x_i < y_i$ for $i = 1, \ldots, k$.

reasonable to assume that $V(S)$ is finite. There is no loss of generality in the assumption that $V(S) \subset I^S$, because any finite set of real valued utility vectors can be transformed into a set of integer valued utility vectors without changing the *relative* sizes of any coordinates, and the relative sizes are all that interest us.

Assumptions 2 and 3 are innocuous; Assumption 2 is assumed for notational convenience, and Assumption 3 holds in any exchange economy in which there are no externalities of consumption. Assumption 4, which requires that every pair of traders be able to make some mutually profitable trade, is rather strong in the context of a finite utility space model. Assumption 5 is one variant of a common free disposal assumption; we have restricted it to preserve the finite character of the model. This is again a rather strong assumption in a finite model since it requires that individuals be able to dispose of units of utility. It is clear that if the real world is "very" finite (or indivisibilities are very important), Assumptions 4 and 5 are unrealistic. However, if the real world is full of large indivisibilities, the standard continuous utility space and continuous free disposal assumptions are also unrealistic.

3. THE PARTIALLY SPECIFIED RECONTRACTING PROCESS

We will suppose that there are R vectors, $v_1, v_2, \ldots, v_R$ in $V(N)$. By Assumption 1, $R < \infty$, and by repeated applications of Assumptions 2 and 3, $(0, 0, \ldots, 0) \in V(N)$, so $1 \leqslant R$.

We will let $Q \equiv 2^n - 1$ be the number of coalitions in the economy, and we will label a typical coalition S_q, where $q \in \{1, 2, \ldots, Q\}$. When the context makes our meaning clear, we will drop the subscript q, and we will often use u or v without a subscript to denote an element of $V(N)$.

If $v \in V(N)$ and S is a coalition, we will define

$$B_S^*(v) = \{u^* \in V(S) : u^* \geqslant v^S\},$$

and[2]

$$B_S(v) \equiv \left\{u : u = \begin{cases} u^* & \text{on } S, \\ 0 & \text{on } N - S, \end{cases} \quad \text{and } u^* \in B_S^*(v)\right\}.$$

By Assumptions 2 and 3, $B_S(v) \subset V(N)$. The set of blocking vectors for the coalition S is $B_S^*(v)$ and the set of extended blocking vectors is $B_S(v)$.

Recontracting is a process of movement from proposal to proposal. Starting from a particular utility vector, a possible blocking coalition is chosen. If in fact that coalition can block, a blocking vector is chosen from among those vectors that are feasible for it, and extended by the assignment of zeros to non-members of the coalition. If the coalition cannot block, no movement is made. This completes one iteration. More formally, we will make the following assumption.

[2] The symbol 0 is used for the zero vector of the appropriate dimension, and $N - S$ is a set-theoretic difference: $N - S \equiv \{i : i \in N \text{ and } i \notin S\}$.

ASSUMPTION 6: *The recontracting process works as follows. Suppose we are at v in $V(N)$. An S is chosen. Then a u in $B_S(v)$ is chosen if $B_S(v)$ is not empty. If $B_S(v)$ is empty, we stay at v. This completes one iteration, and the process continues ad infinitum.*

We will call the process described by Assumption 6 a *partially specified recontracting process* (p.s.r.p.). It is "partially specified" because we have not said how S is chosen, or how a vector in $B_S(v)$ is chosen. We will leave that specification to the next section.

Let us remark that $v \in V(N)$ is in the core if and only if the partially specified recontracting process cannot move away from it. This is clearly the case, for if v is in the core, then no matter what S we choose, $B_S(v)$ is empty, and, conversely, if $B_S(v)$ is empty for all S, v cannot be blocked. The truth of this remark depends only on Assumption 6, which in turn depends on Assumptions 2 and 3. It does not depend on Assumptions 1, 4, and 5, and it is vacuously true if the core is empty. Since recontracting cannot leave the core once it has arrived, the question of recontracting stability reduces to the question of whether or not the process will in fact lead to the core.

We will say that the *partially specified recontracting process is stable* if it is possible to recontract from any non-core utility vector in $V(N)$ to a core vector, in a finite number of steps. The process is *not* stable if it can get trapped in a set of non-core allocations; in the final section of this paper we provide an example of an unstable p.s.r.p. in an economy with a non-empty core. It is clear from the definition of stability that the p.s.r.p. cannot be stable if the core is empty. We can now proceed to prove that if Assumptions 1–6 hold, non-emptiness of the core implies stability of the p.s.r.p.

THEOREM 1: *Under Assumptions 1–6, the partially specified recontracting process is stable if and only if the core is non-empty.*

PROOF: The proof of Theorem 1 will depend on the following lemma.

LEMMA: *Under Assumptions 1–6, if $v \in V(N)$ is not in the core, then it is possible to recontract from v to a core utility vector or to a utility vector with 2 ones and $n - 2$ zeros.*

PROOF OF THE LEMMA: (For notational convenience we will write "S_1" and "v_1" for "S_{q_1}" and "v_{i_1}" respectively, and so on. Also, we will call a blocking coalition *minimal* if none of its proper subsets can block a proposed utility vector.)

By assumption, v is not in the core. Choose a minimal blocking coalition, say S_1, to block it. If $S_1 = N$, then S_1 can block v with a $u \in V(N)$ which is Pareto optimal for N and, therefore, clearly in the core, and we are finished. Suppose, then, that $S_1 \neq N$.

Now choose $v_1^* \in V(S_1)$ so that v_1^* is Pareto optimal for S_1, and so that $v^{S_1} \leqslant v_1^*$.

Let

$$v_1 = \begin{cases} v_1^* \text{ on } S_1, \\ 0 \quad \text{on } N - S_1. \end{cases}$$

By Assumptions 2 and 3, $v_1 \in V(N)$. If v_1 is in the core we are finished. Assume, therefore, that it is not in the core.

Case 1: S_1 has $n - 2$ members or fewer. Now let S_2 be any pair of traders in $N - S_1$. Therefore, $v_1^{S_2} = (0, 0)$. By Assumptions 1, 4, and 5, S_2 can block v_1 with (1, 1). Extend (1, 1) to a utility vector for the whole economy by assigning zeros to nonmembers of S_2. The result is a vector with 2 ones and $n - 2$ zeros.

Case 2: S_1 has $n - 1$ members. Suppose individual i is the single trader who is not in S_1. Since v_1 is not in the core, it can be blocked by a minimal coalition S_3. Because S_1 was chosen so that none of its proper subsets could block v, because $v^{S_1} \leqslant v_1^{S_1}$, and because $v_1^* = v_1^{S_1}$ was chosen Pareto optimal for S_1, $S_3 \subset S_1$ is impossible. Therefore, $i \in S_3$.

Case 2a: $S_3 = N$. S_3 can block v_1 with v_2 which is Pareto optimal for N. Now v_2 must be in the core, since S_3 is a minimal blocking coalition for v_1 and $v_1 \leqslant v_2$.

Case 2b: S_3 has $n - 1$ or fewer members. Choose a vector to block v_1 which gives individual i not more than one unit of utility. This can be done by Assumptions 1 and 5. Now extend the blocking vector by assigning zeros to non-members of S_3. Call the result v_3. Suppose $j \in N - S_3$. By construction, $v_3^{\{i,j\}} \leqslant (1, 0)$. By Assumptions 1, 4, and 5, $\{i, j\}$ can block v_3 with (1,1). Extend (1, 1) to a utility vector for the whole economy by assigning zeros to non-members of $\{i, j\}$. The result is a vector with 2 ones and $n - 2$ zeros, which completes the proof of the lemma.

PROOF OF THEOREM 1: The "only if" part of the theorem is obvious. Suppose then that the core is non-empty.

By the lemma, if $v \in V(N)$ is not in the core, it is possible to recontract from v to a core utility vector or to a utility vector with 2 ones and $n - 2$ zeros. We now claim that if v has 2 ones and $n - 2$ zeros and v is not in the core, it is possible to recontract from v to a core utility vector. If $n = 2$, this is obvious. If $n \geqslant 3$ and if u is any core vector, then $0 \leqslant u$ by Assumption 2, and by Assumption 4, u has no more than one zero. Therefore, u has two positive, integer valued, coordinates; without loss of generality we will assume its first two coordinates are positive. Therefore, $u \geqslant (1, 1, 0, \dots, 0)$.

Now if $v = (1, 1, 0, \dots, 0)$, we can recontract from v to u via a block by N. If $v \neq (1, 1, 0, \dots, 0)$, then $v^{\{1,2\}} = (0, 0)$, or $(1, 0)$, or $(0, 1)$. Therefore, by Assumptions 1, 4, and 5, coalition $\{1, 2\}$ can block v with (1, 1); the extended blocking vector $(1, 1, 0, \dots, 0)$ is in $V(N)$ by Assumptions 2 and 3, and we can recontract, via a block by N, from $(1, 1, 0, \dots, 0)$ to u. *Q.E.D.*

At this point we will make an observation about our use of Assumption 1 in Theorem 1. What we have used is not the fundamental notion that $V(S)$ is finite, but the secondary assumption that utility vectors are integer valued. The theorem could easily be generalized by changing Assumption 1 to 1′: *$V(S)$ is closed and bounded for every $S \subset N$*; and 5 to 5′: *If $v \in V(S)$, and $0 \leqslant u \leqslant v$, then $u \in V(S)$.* Now we could define $\varepsilon \equiv \min_{\{i,j\} \subset N} \varepsilon_{i,j}$, where $\varepsilon_{i,j} \equiv \min\{a, b\} > 0$ comes from Assumption 4, and we could replace the "units" of utility used throughout the proof of the theorem by ε's.

The assumption of finiteness is useful, however, when a random choice mechanism is overlaid on the partially specified recontracting process.

4. THE RANDOM RECONTRACTING PROCESS

We will now suppose that a positive probability $\hat{p}_r$ is associated with each utility vector $v_r \in V(N)$. We will assume that $\Sigma_{r=1}^{R} \hat{p}_r = 1$. We will also suppose that a positive probability p_q is associated with each coalition S_q, and, of course, that $\Sigma_{q=1}^{Q} p_q = 1$.

Let us specify a random mechanism for choosing blocking coalitions and blocking vectors: Suppose the recontracting process is at $v_i \in V(N)$. Choose a coalition S_q according to the probability assigned to it, p_q.

If S_q can block v_i, choose $v_j \in B_{S_q}(v_i)$ according to the conditional probability

$$\hat{p}_j \Big/ \sum_{\{m: v_m \in B_{S_q}(v_i)\}} \hat{p}_m .$$

If S_q cannot block, stay at v_i. This completes one iteration.

If we define

$$p_{qij} \equiv \begin{cases} \dfrac{\hat{p}_j}{\sum_{\{m: v_m \in B_{S_q}(v_i)\}} \hat{p}_m} & \text{if } v_j \in B_{S_q}(v_i) \text{ and } B_{S_q}(v_i) \neq \varnothing, \\ 0 & \text{if } v_j \notin B_{S_q}(v_i) \text{ and } B_{S_q}(v_i) \neq \varnothing, \\ 1 & \text{if } i = j \text{ and } B_{S_q}(v_i) = \varnothing, \\ 0 & \text{if } i \neq j \text{ and } B_{S_q}(v_i) = \varnothing, \end{cases}$$

and let $p_{ij} \equiv \Sigma_q p_q p_{qij}$, the random recontracting process as we have specified it is a finite Markov chain, where the transition probability for moving from state i (that is, v_i) to state j (that is, v_j) is given by p_{ij}. Moreover, v_i is an element of core if and only if $p_{ii} = 1$, or state i is absorbing.

We can now give a probabilistic definition for recontracting stability: We will say that a *random recontracting process is stable* if, from any $v_i \in V(N)$, it carries the economy to a point in the core, in a finite number of steps, with probability one. Theorem 2 is a straightforward consequence of Theorem 1.

THEOREM 2: *Under Assumptions 1–6 and our positivity assumptions on the $\hat{p}_r$'s and p_q's, the random recontracting process is stable if and only if the core is non-empty.*

PROOF: "Only if" is again obvious. Suppose then that the core is non-empty. Suppose v_i is not in the core. By Theorem 1, given any $v_i \in V(N)$, it is possible to recontract from v_i to a point in the core; moreover, it requires no more than five iterations to do so. By our specification of the random recontracting process, there is a *positive probability* attached to that *possibility*. But core points are absorbing. Therefore, state i of the Markov chain is transient. Since the probability is zero that a finite Markov chain stays forever in its set of transient states, the random recontracting process carries the economy from v_i to a point in the core, after a finite number of steps, with probability one. Therefore, it is stable, as claimed. *Q.E.D.*

5. ALTERNATIVE APPROACHES AND EXAMPLES

In [**2** and **3**] Jerry Green analyzes random recontracting stability in economies with closed utility possibility sets, and in [**5**] Neuefeind clarifies and extends the analysis of [**2**]. In several respects all of our approaches are similar. Any proof of global recontracting stability must rule out the possibility of a cycle of non-core allocations which recontracting cannot escape. This is in a sense the barter analog of the exclusion, in a tâtonnement stability analysis, of non-damped price oscillations. We all show that it is always possible to escape cycles and to recontract to a core allocation. Any proof of *random* recontracting stability must bound the probabilities of reaching the core, or some neighborhood of the core, away from zero; Green does this with his "strong-superadditivity" assumption [**3**, Assumption 4] and his assumptions on the probability measure which dictates the choice of blocking utility vectors. We do the same thing by requiring that each element of $V(N)$ and each coalition be assigned a positive $\hat{p}_r$ or p_q. In Green's and Neuefeind's papers and our own, the core is assumed non-empty.

The single most striking difference between the Green-Neuefeind approaches on one hand and ours on the other is our assumption of finite utility possibility sets. This greatly simplifies the technical analysis, because finite Markov chains are transparent compared to infinite state-space Markov processes. There are, however, a number of other differences between Green's analysis and our own. First, in [**3**] (but not in [**2**]) it is assumed that the blocking coalition is assigned a utility vector which is Pareto optimal for it and the complement of the blocking coalition is also assigned a Pareto optimal utility vector. We assume that the blocking coalition is assigned a blocking utility vector which need not be Pareto optimal, and its complement is assigned a "no-trade" utility vector. Second, Green assumes [**3**, Assumption 1] that individuals can (with positive probability) be assigned non-individually-rational utilities; in fact, the singleton coalition is a key blocking unit in the proof of stability in [**3**]. Our recontracting process, on the other hand, can be entirely restricted to individually rational (i.e., non-negative) utility vectors, since the key blocking unit in our proof is the pair. Third, Green assumes [**3**, Assumption 2] that any coalition of size 2 or larger can achieve a utility vector which makes all of its members better off than they are without trade; we make this assumption for coalitions of size 2 only. Fourth, a monotonicity type of assumption is made in [**3**, Assumption 3]; such an assumption is unnecessary for our purposes. Fifth and finally, the continuous probability structure of his model makes it necessary for Green to assume [**3**, Assumption 4] that there is a core utility vector which is "large" in the sense that its projections cannot be achieved by any proper subset of N; the discrete probability structure of our model makes such an assumption unnecessary for us.

It is useful to consider an example to highlight the differences between the Green assumptions on the utility possibility sets and our own. Suppose there are three traders in the economy, and assume that

$$V(\{i\}) = 0 \quad \text{for} \quad i = 1, 2, 3,$$

$$V(\{i,j\}) = \left\{u : \begin{pmatrix}0\\0\end{pmatrix} \leqslant u \leqslant \begin{pmatrix}1\\1\end{pmatrix}, u \in I^2\right\}$$

for any pair $\{i, j\}$, and

$$V(\{1,2,3\}) = \left\{u : u \in I^3;\ \begin{pmatrix}0\\0\\0\end{pmatrix} \leqslant u; \text{ and } u \leqslant \begin{pmatrix}2\\2\\0\end{pmatrix} \text{ or } u \leqslant \begin{pmatrix}1\\0\\1\end{pmatrix} \text{ or } u \leqslant \begin{pmatrix}0\\1\\1\end{pmatrix}\right\}.$$

These sets meet all our assumptions, and partially specified recontracting is stable in this economy since the core is non-empty;

$$\begin{pmatrix}2\\2\\0\end{pmatrix}$$

cannot be blocked. But the economy does not meet Green's Assumptions 1–4. (Assumption 3 fails because of finiteness, but the others are meaningful for discrete utility economies.)

In [**3**] (but not in [**2**], or in [**5**]), it is assumed that a blocking coalition receives a utility vector that is Pareto optimal for it. The distinction between such Pareto-optimal blocking and non-Pareto-optimal blocking is not insubstantial; in fact, Pareto-optimal blocking can be destabilizing. A final example (in a non-finite utility space economy) will throw some light on this difference between the approach of [**3**] and the approach of this paper.

Let $V(\{i\}) = 0$ for $i = 1, 2, 3$,

$$V(\{1,2\}) = \left\{u : \begin{pmatrix}0\\0\end{pmatrix} \leqslant u \leqslant \begin{pmatrix}3\\1\end{pmatrix}\right\},$$

$$V(\{1,3\}) = \left\{u : \begin{pmatrix}0\\0\end{pmatrix} \leqslant u \leqslant \begin{pmatrix}1\\3\end{pmatrix}\right\},$$

$$V(\{2,3\}) = \left\{u : \begin{pmatrix}0\\0\end{pmatrix} \leqslant u \leqslant \begin{pmatrix}3\\1\end{pmatrix}\right\},$$

and

$$V(N) = \left\{ u : \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \leqslant u \leqslant v, \text{ where } v \in \left\{ \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} \right\} \right\}.$$

Note that

$$\begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$$

cannot be blocked. These utility possibility sets satisfy our assumptions 1′, 2, 3, 4, and 5′, and a partially specified non-Pareto-optimal blocking recontracting process is stable in this economy. Assumptions 1 and 3 of [**3**] are not met, however, and it is easy to see that Pareto-optimal blocking recontracting will *not* be stable. Suppose, for example, that we start at

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

and coalition $\{1,2\}$ blocks. We must then move to

$$\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}.$$

But

$$\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$$

can only be blocked by coalition $\{2, 3\}$ and the result is

$$\begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix}.$$

When this vector is blocked, we go to

$$\begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix},$$

and then, completing the cycle, to

$$\begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}.$$

In short, the process never reaches the core. Now this example is really perfectly intuitive. The fundamental barrier to recontracting stability is the cycle of noncore utility vectors, and the essence of a recontracting stability proof is the step that allows the economy to slip out of a possible cycle. It is clearly easiest to slip out if the economy is not required to move along the highest ridges in utility space.

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