Buying Time: A Model of the Dollar Value of Extra Years of Life

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I. Introduction

The purpose of this paper is to develop a simple value of life model in which a rational individual decides how much to spend to increase his lifespan. This paper is about "buying time." The model is partly based on that of Ehrlich and Chuma (1990). In contrast to the standard value of life model of Schelling (1987), Jones-Lee (1974, 1976, 1980, 1984, 1992), Mishan (1982a,b), Viscusi (1992, 1993) and many others, the model of this paper has no uncertainty. The story here is not about *risk of death,* it is about *length of life.* The standard willingness-to-pay-to-avoid-risk model may relate well to real world quandries about how much to spend to reduce the likelihood of, say, automobile accidents, which are highly uncertain events. The model of this paper relates well to real world quandries about how much to spend on medical care to prolong the lives of patients (all of us, ultimately) who can be kept alive, but who cannot be cured. Death is certain, but if we spend more it will be delayed. We can buy time.

Some of the paradoxes of the standard willingness-to-pay-to-avoid-risk model may arise because of the probabilistic nature of that model. In the standard model, value-of-life (*VOL*) is usually defined as willingness-to-pay for an increment in survival probability; per unit of additional survival probability. One paradox that arises under fairly general conditions is that *VOL* depends on the survival probability. Given a survival probability of .95, my life is worth \$X, but given a survival probability of .05, my life is worth \$Y, with X and Y vastly different. Another paradox, related to the first, and again very general, is what I call the Broome paradox (Broome (1978). Also see Buchanen and Faith (1979), Jones-Lee (1979), and Williams (1979)). This is the observation that, with small death probabilties, individuals may act as if they value their (statistical) lives at modest and finite amounts. Based on these valuations, a government may use cost-benefit analysis to decide to undertake a project, like a tunnel, that will, on average, result in the loss of some lives. But if the government knew *whose* lives were to be lost, and queried those individuals as to what money compensation they would need to accept death, they would likely answer with very large, possibly infinite amounts. Were these valuations inserted in the cost-benefit calculation, the project would be soundly rejected. The Broome paradox itself may be viewed as a special case of a general paradoxical inconsistency between social welfare as calculated *ex ante* (before the fact), and social welfare as calculated *ex post* (after the fact). Ulph (1982) shows this nicely, and points out that only utilitarianism is *ex ante* and *ex post* consistent. Thus the standard willingness-to-pay-money-to-reduce-risk model is inescapably inconsistent. (See also Blackorby and Donaldson (1986) for devastating criticism of the consistency of the willingness-to-pay model of *VOL*.)

So the model of this paper is not about reducing risk in a world of uncertainty; rather, it is about buying time in a world of certainty. Since we will all certainly die, and since we can often buy some extra days, months, or years by, for example, using medical care, the model does relate to interesting real questions.

II. The Model

What follows is a greatly simplified and slightly restructured version of a model set out in Ehrlich and Chuma (1990). (See also Moore and Viscusi (1988), Kenyon and McCandless (1984), and Jones-Lee (1976) for somewhat similar models.)

I use the following notation:

LU	$=$	lifetime utility
x	$=$	rate of spending on consumption, assumed constant over the lifetime
y	$=$	money spent by the subject on precaution (e.g., medical care), assumed spent in one lump sum at time zero
s	$=$	subsidy rate on precaution expenditure, $0 \leq s < 1$
z	$=$	bequest, made in one lump sum at time of death
$\overline{x}$	$=$	$x + y + z =$ cash endowment
A	$=$	a bequest parameter. $A = 0$ indicates the individual has no interest in the size of his bequest, whereas $A > 0$ indicates the individual gets utility from his bequest.
T	$=$	length of life.
$u(x)$	$=$	instantaneous utility function.

I assume in this paper that the instantaneous utility function $u(\cdot)$ is constant over the lifetime and has the usual nice properties. The expression $u(x)$ represents the rate of flow of utility from consumption. For simplicity, consumption x is also assumed to be undertaken at a constant rate over the lifetime. Therefore, it is not necessary to include a time variable or subscript in either $u(\cdot)$ or x. Again for simplicity, the individual's discount rate is assumed to be zero. Therefore, lifetime utility from consumption is given by

$$\int_{t=0}^{T} u(x)dt = Tu(x).$$

To make the model easy to solve, I assume that utility from the bequest z at death is simply $Au(z)$.

The subject's objective function is now:

$$LU = \int_{0}^{T} u(x)dt + Au(z) = Tu(x) + Au(z). \tag{1}$$

The choice variables to this point are x, y and z. There are two constraints. First is a more-or-less standard budget constraint. Consumption takes place at a rate x over a lifetime of length T. Precaution expenditure y is made at one moment, at time 0. The bequest z is made at one moment, the time of death. For simplicity, there is no discounting in the budget, just as there is no discounting in the utility function. Therefore, the budget constraint is:

$$\overline{x} = xT + y + z. \tag{2}$$

The second constraint is the crucial ingredient of the model. I assume that greater expenditure on precaution (e.g., medical expenditures) will result in longer life. That is, increasing y will increase T. Consequently, T is also a choice variable.

Furthermore, I assume that precaution expenditure may be subsidized. That is, the government may pay a fraction of the total spent on precaution (as it does, for instance, in the enormous Medicare and Medicaid programs). The variable y represents the subject's *own* precaution expenditure. Let s be the government subsidy rate, so, for example, $s = .5$ if the government matches each dollar spent by the subject with one dollar of its own, and $s = .9$ if the government matches each dollar spent by the subject with nine of its own. Then total precaution expenditure on behalf of the subject is $\frac{y}{1-s}$; his own expenditure on his behalf is y; and the government's expenditure on his behalf is $\frac{y}{1-s} - y = \frac{sy}{1-s}$. I assume that the individual takes s as a parameter. I assume further that the length of life T is a function of total precaution expenditure $\frac{y}{1-s}$ made on behalf of this individual. Therefore, the second constraint is:

$$T = f\left(\frac{y}{1-s}\right). \tag{3}$$

The subject is assumed to maximize lifetime utility LU (in equation (1)), subject to the constraints of equations (2) and (3).

III. Maximization Conditions and the Value of Time

The Lagrangian first-order conditions for an interior maximum are as follows:

(A) $\left[\frac{d}{dy} f\left(\frac{y}{1-s}\right)\right]^{-1} = \frac{u(x)}{u'(x)} - x$

(B) $Au'(z) = u'(x)$

(C) $\overline{x} = xT + y + z$

(D) $T = f\left(\frac{y}{1-s}\right)$.

Let x^*, y^*, z^*, T^* denote a solution to these equations.

It is useful to interpret the left-hand side of equation (A). Note first that:

$$\frac{d}{dy} f\left(\frac{y}{1-s}\right) = \frac{\text{Extra length of life}}{\text{Extra own expenditure on precaution}}.$$

Therefore, inverting both sides gives:

$$\left[\frac{d}{dy}f\left(\frac{y}{1-s}\right)\right]^{-1}=\frac{\text{Extra own expenditure on precaution}}{\text{Extra length of life}}.$$

The expression on the right-hand side is the subject's (marginal) value of an extra unit of lifetime, e.g., a dollar value (in terms of his *own* dollars) for an extra year of life. This will be called the individual's *value of time, private,* or *VOTP*. Hence:

$$VOTP=\left[\frac{d}{dy}f\left(\frac{y}{1-s}\right)\right]^{-1}. \tag{4}$$

$VOTP$ is the individual's willingness-to-pay, out of his own pocket, for an extra time unit of life.

At this point it is useful to start to put some structure on the $f(\cdot)$ function. I will assume that an individual who spends zero on precaution will live a life of length $T_{\min}\geq 0$, and that the length of life $T=f(\cdot)$ asymptotically approaches a maximum $T_{\max}>T_{\min}$. Figure 1 below illustrates a length-of-life function, plotted against $y/(1-s)$ on the horizontal axis.

Insert Figure 1 here.

In Figure 1, it is assumed that $T_{\min}=40$ years, $T_{\max}=90$ years, $s=0.5$, y varies between 0 and 20, and $f(\cdot)$ is the nicely behaved function given by equation (8) below. Assume that $y^*=5$. The line L in the figure is tangent to the $f(\cdot)$ function at $y^*=5$ and $T^*=73$. The inverse of the slope of L is equal to $\frac{1}{1-s}\,VOTP$, as a consequence of equation (4).

The value of time, private, is the natural measure of an individual's extra year of life. However, if his own outlay is *VOTP* for an extra unit of his lifetime, that outlay brings forth an additional government expenditure of $VOTP \times s/(1-s)$ for the extra unit of time. The total extra expenditure, private plus public, for this individual's extra year of life will be called the *value of time, social,* and it is given by:

$$(5) \qquad VOTS = \frac{1}{1-s} VOTP.$$

Referring back to Figure 1, the inverse of the slope of the line L is evidently *VOTS*.

The terminology "value of time, social" may be misleading. In this model the private agent does the choosing of x^*, y^*, z^* and T^*; his choice is made subject to a given exogenous subsidy rate s. The subsidy rate exists, for whatever reason, and I do not presume to imply that "society" has "decided" an extra year of his life would have a certain value. It is simply the case that society, wisely or unwisely, is paying *VOTS* for an extra year of this agent's life.

Both *VOTP* and *VOTS* are marginal value of time measures; they represent the value of, e.g., an *additional* year of life. There are two ways to approach the value of the *entire* lifespan. Consider *VOTP*. The first way to measure the value of the lifespan would be to multiply *VOTP* by T^*. But this is mathematical nonsense; it makes little sense to multiply the value of the marginal unit of time by the total number of units. The second way would be to integrate *VOTP* from $t = 0$ to $t = T^*$. That is, sum all the (differing) *VOTP*'s over the all the additional time units the subject has chosen to buy. Call the result *total value of time, private,* or *TVOTP*. This is the aggregate willingness-to-pay for all those additional time units beyond $T_{\min}$. It is a simple mathematical exercise to show that:

$$(6) \qquad TVOTP = \int_{t=0}^{T^*} \left[\frac{d}{dy} f\left(\frac{y}{1-s}\right) \right]^{-1} dt = y^*.$$

That is, the subject's aggregate willingness-to-pay for extra years of life, above and beyond $T_{\min}$, equals what he does in fact choose to pay for those extra years of life. Your years of life, like your car, are worth to you what you pay for them.

The above may seem like an obvious conclusion. But it stands in stark contrast to the usual value of life theorizing in the uncertainty context, where many authors find that the value of a statistical life far exceeds total lifetime income or endowment. (See, e.g., Bergstrom (1982), Conley (1982), Jones-Lee (1984), Smith (1990), Viscusi (1993), among many others. The common view is that the willingness-to-pay-to-reduce-risk value of life is around an order of magnitude larger than the human capital value of life.)

IV. Special Assumptions

I will now focus the model by making some specific assumptions about functional forms and parameter values. The purpose is to allow computation of solutions.

Assumption (1) (Power Utility Function).

(7) $$u(x) = x^{\alpha}, \quad \text{where} \quad 0 < \alpha < 1.$$

Assumption (1) is convenient because it simplifies first-order conditions (A) and (B) into:

$$\text{(A1)} \quad VOTP = \left[\frac{d}{dy} f\left(\frac{y}{1-s}\right)\right]^{-1} = x\left(\frac{1-\alpha}{\alpha}\right)$$

$$\text{(B1)} \quad z = xA^{\frac{1}{1-\alpha}}.$$

Next I introduce a somewhat special but computationally easy length-of-life function:

Assumption 2 (Length of Life).

(8) $$f\left(\frac{y}{1-s}\right) = T_{\max} - \frac{T_{\max} - T_{\min}}{1 + \beta\left(\frac{y}{1-s}\right)}, \quad \text{where} \quad \beta > 0.$$

In this formula, β is a parameter that scales the effect of total precaution expenditure $y/(1-s)$ on the length of life. Note that the function in equation (8) is graphed in Figure

1 above; where, $f(0) = T_{\min} = 40$ and f approaches $T_{\max} = 90$ asymptotically as y approaches infinity. Assumption (2) can be combined with the definitions of the value of time, private and social, to get:

(9) $$VOTP = \frac{1}{\beta(T_{\max} - T_{\min})}\left(1 + \frac{\beta y}{1-s}\right)^2 (1-s), \quad \text{and}$$

(10) $$VOTS = \frac{1}{\beta(T_{\max} - T_{\min})}\left(1 + \frac{\beta y}{1-s}\right)^2 .$$

At this stage the model can be solved analytically for y^*. Equation (9) is substituted into first-order condition (A1), condition (B1) is used as it stands, and the length of life assumption in equation (8) is substituted into first-order conditions (C) and (D). The result is a quadratic equation involving the choice variable y^* and the various parameters. The next assumption has the effect of getting rid of the linear term in the quadratic equation for y^*, and thereby making the computations especially easy:

Assumption 3 (An easy value for α).

Let $\alpha = \frac{1}{2}$.

With assumption (3), it is easy to show that the optimal precaution expenditure is the solution to the following equation:

(11) $$1 + \frac{\beta y^*}{1-s} = \left[\frac{T_{\max} - T_{\min}}{T_{\max} + A^2}\left(1 + \frac{\beta \overline{x}}{1-s}\right)\right]^{\frac{1}{2}} .$$

It is also easy to show that:

(12) $$VOTP = x^* = \frac{(\frac{1-s}{\beta})(1 + \frac{\beta \overline{x}}{1-s})}{T_{\max} + A^2}.$$

Using equations (11) and (12), as well as appropriate preceding equations, all the interesting variables in the model (including $z^*, T^*, VOTS,$ lifetime utility, and so on) can be calculated.

The next two sections of this paper present graphs computed from the solved model of this section. Computations and graphs are contained in *Quattro Pro for Windows* spreadsheets, which are available upon request.

V. Simulations

One of the most interesting issues raised by any discussion of the value of life, or the value of time, is the efficiency, or lack thereof, of a policy that subsidizes the individual's expenditures on precaution. In the model of this paper, every subsidy rate s results in a subsidy amount $\frac{sy}{1-s}$. Were the subject given $\frac{sy}{1-s}$ in cash, as an addition to his endowment $\overline{x}$, instead of as a subsidy to his consumption of one good, he would be better off. This is standard microeconomics welfare analysis, and what follows largely elaborates on the standard result.

For the purpose of examining the efficiency question in detail, I calculate several variables. First is lifetime utility contingent on the precaution subsidy, i.e., the *LU* of the model. Second is lifetime utility if a subsidy at rate s were replaced with a program where $s = 0$ but the associated subsidy cost $\frac{sy}{1-s}$ were added as a cash grant to the individual's wealth. This produces the variable "lifetime utility if the subsidy were replaced by a cash grant," or *LU If*, for short.

Third, for any given subsidy rate s, there is a gain in utility, when compared to no subsidy and no cash grant. This will be called *LU Gain* in what follows. Then, for a given subsidy rate s and a resulting *LU Gain*, one can ask this question: what cash grant, in the *absence of a subsidy,* would produce the *same LU gain*? This is the compensating variation measure of consumer's surplus which corresponds to the subsidy s. It will be abbreviated *CS* in the graphs below. It represents the minimum the subject would demand to give up a subsidy at rate s on precaution expenditure. It is thus a willingness-to-accept measure of the value of the subsidy.

Fourth and finally, a subsidy at rate s produces a cost of $sy/(1-s)$, but would be traded by the individual for the corresponding consumer's surplus. I will call the ratio of the consumer's surplus to the subsidy cost $sy/(1-s)$ the "efficiency" of the subsidy.

With these observations we can turn to some simulations. In what follows I use these parameter values:

$$
\begin{array}{lcll}
\overline{x} & = & 100 & \text{(money units)} \\
T_{\max} & = & 90 & \text{(time units -- years)} \\
T_{\min} & = & 40 & \text{(time units -- years)} \\
\beta & = & 0.2. &
\end{array}
$$

The $\overline{x}$ value assumption can be interpreted as meaning that one money unit corresponds to one percent of an average person's lifetime endowment. $T_{\max}$ is set at a plausible value for maximum average human lifespan with very high expenditures on precaution. $T_{\min}$ is set at a plausible minimum lifespan with no expenditure on precaution. (These numbers are of course arguable. There is evidence that humans are genetically programmed with a maximum lifespan of around 115 years (Nesse and Williams (1994), chapter 8). Life expectancy at birth in Massachusetts in the year 1850 was around 40 years (*Historical Statistics of the U.S.*). So this $T_{\max}$ may be slightly low, but $T_{\min}$ may be close to "right." However, even large changes in the $T_{\max}$ and $T_{\min}$ assumptions would have no effect on the qualitative results below.) The β parameter is chosen to get plausible variability in length of life as the subsidy rate changes. Figures 2 through 6 below are calculated from the solved model under the assumptions listed above.

For notational simplicity I drop the * superscripts in the rest of this section.

I first consider the effect of the A parameter. (For other relevant discussions of bequest motives, see Jones-Lee (1974) and Bernheim (1991), among others.) Intuitively, as the bequest parameter increases, all else held constant, one expects spending on consumption x to decline, spending on precaution y to decline, and the bequest z to rise. Figure 2 illustrates this, for A variable between 0 and 3.0, and for s fixed at 0.5. Note that as A increases, x declines very marginally, y declines noticeably, and z increases dramatically. For instance, at $A = 0$, the subject is spending around 1.1 money units per year on consumption x; he is spending around 9.4 money units, one time only, on precaution y; and he is bequeathing 0. At $A = 3$, he is spending around 1.0 money units per year on consumption x; he is spending around 8.9 money units, one time only, on precaution y; and he is bequeathing 9.3 money units.

Insert Figures 2 and 3 around here.

The change in expenditure pattern as we go from $A = 0$ to $A = 3$ results in a small change in lifespan T. This is shown in Figure 3. When $A = 0$, the subject chooses to live around 79.5 years. When $A = 3$, he chooses to live 79.0 years. Note that as A rises from 0 to 3, although the subject is planning to bequeath much more (z rising from 0 to 9.3 money units), and is planning to live a shorter life, his (marginal) value of an extra year of life ($VOTP$) decline only slightly. Under assumption (2), $VOTP = x$, which, as was noted above, declines from around 1.1 to around 1.0.

Next I turn to the crucial subsidy rate parameter. For the Figure 4 through 7 simulations I assume A is fixed at 2.0. (With a 50 percent subsidy on precaution, this would result in a bequest of around four times per-year consumption, a plausible "ball-park" number.)

The purpose of a subsidy on precaution is of course to increase life, to buy more time. Figure 4 shows the effect. As s rises from 0 to .99, the chosen lifespan rises from 75.0 years to 86.5 years. However, the extra lifespan is bought at considerable cost.

Insert Figures 4 and 5 around here.

Figure 5 shows the subject's three expenditure variables x, y and z. Note first that x is almost constant as the subsidy rate rises from 0 to .99. (In fact, it falls slightly, from around 1.12 to around 1.06.) Note second that the desired bequest z is also almost constant. (In fact, it falls slightly, from around 4.47 to around 4.26.)

But note third that there is a striking decline in the subject's own precaution expenditure y : It falls from around 11.7 to around 1.6. It falls because of the subsidy. The subsidy program is replacing the subject's own precaution expenditure; in fact, much more than replacing it. Total precaution expenditure on behalf of the subject is $y/(1-s)$, and, as Figure 5 shows, that rises dramatically. It rises from around 11.7 to around 158.1 (off the graph), as s rises from 0 to .99. (In this simulation, lifetime expenditure on consumption, or xT, equals total precaution expenditure $y/(1-s)$ when $s = .97$. That is, at a 97 percent subsidy rate, subject's lifetime consumption equals social spending on increasing his lifespan.) The recipient of the subsidy is of course better off as the subsidy rate increases, and in Figure 5 the consumer's surplus CS is also shown. It rises from 0 at $s = 0$ to 26.6 at $s = .99$.

Next, consider the value of time measures, private and social, that is *VOTP* and *VOTS*. Based on the simplifying assumption that $\alpha = 1/2$, we already know that

$$VOTP = x, \quad \text{and}$$

$$VOTS = x/(1-s).$$

In the simulation at hand, x is almost constant as s varies, and, therefore, so is *VOTP*. The individual's willingness-to-pay for an additional years of life is virtually flat. However, the value of life, social, increases dramatically. If one naively assumed that *VOTS* represented "society's" willingness-to-pay for an additional year of life, it would appear that "society" is willing to pay more and more for additional years, as s gets large. In fact, as s rises from 0 to .99, *VOTS* rises from around 1.1 money units (= *VOTP*) to around 106 money units (= 100 times *VOTP*). Figure 6 shows *VOTP*, *VOTS*, and also the subsidy $sy/(1-s)$ that is being paid toward the subject's precaution.

Insert Figures 6 and 7 around here.

Finally, consider a cost-benefit assessment of the subsidy. As noted above, the "efficiency" of the subsidy is defined in this section as consumer's surplus CS divided by subsidy cost $sy/(1-s)$. Figure 7 shows efficiency in this simulation. Note that it starts at 1, when the subsidy is least. (This is a limiting value, since it is actually not defined for $s = 0$.) It approaches 0 as the subsidy approaches 1. (This is another limiting value.)

The conclusion is that high subsidy rates on expenditures to prolong life create significant inefficiency. The cash required to buy the same utility increment soon becomes a small fraction of the precaution subsidy. (See K. K. Fung (1993) for a related (and unpolitic) proposal, to bribe the terminally ill to forego treatment.)

VI. Pareto Optimality, Miscellaneous Paradoxes, and Time-Consistency

I turn to some thoughts on Pareto optimality, various potential welfare economics paradoxes, and related issues. I will assume for now that there are two individuals, each of whom has a utility function, length of life function, and budget of the type discussed above. (Generalization to three or more people is straightforward.)

I will now drop the assumption that $\alpha = \frac{1}{2}$, and I will drop the values assumed for $\overline{x}$, A, α, β, $T_{\max}$, and $T_{\min}$ used in the simulations of section V. However, for ease of analysis in this section, I will now assume, for both individuals, that $A = 0$. That is, there is no bequest motive.

Therefore, we now have persons 1 and 2, with given (and possibly differing) $\overline{x}_i$, α_i, β_i, $T_{i,\max}$, and $T_{i,\min}$. Government subsidy rates s_i lurk in the wings, as does a government budget constraint. For the time being, however, assume $s_i = 0$.

This model is somewhat similar to a standard pure exchange economy model. Each person has an endowment $\overline{x}_i$ (of money, a single good). Each one has a complex budget (equations (2) and (8) together) that relates his endowment to the utility producing "goods" x_i and T_i. There are no externalities; i's utility does not depend on x_j or T_j.

On the other hand, this differs from a standard exchange model because (1) years of life cannot be traded between i and j, and (2) even x_i cannot be directly shifted into x_j, because the x_i's are rates of consumption per year and the T_i's may differ between i and j. Thus one natural Edgeworth box diagram, which has x_i's on one axis and T_i's on the other, is not useful.

To create a usable Edgeworth box diagram, we proceed as follows: First, consider our basic optimization problem:

$$\begin{aligned} &\max_{x,y,T}\ x^{\alpha}T, \quad \text{subject to} \\ \overline{x} &= xT + y \quad \text{and} \\ T &= T_{\max} - \tfrac{T_{\max}-T_{\min}}{1+\beta y}. \end{aligned}$$

It is easy to show this is equivalent to:

$$\max_{y,xT} \qquad xT(\overline{x}-y)^{\alpha-1}\left(T_{\max} - \tfrac{\Delta T}{1+\beta y}\right)^{-\alpha+1},$$

$$\text{subject to} \quad \overline{x} = y + xT.$$

Now the function being maximized depends on y, lifetime precaution expenditure, and xT, lifetime consumption expenditure, with those two variables appearing in a single very simple linear constraint. We can easily calculate indifference curves, budget constraint, expansion paths, and competitive equilibria, all in a convenient y vs. xT space.

Figure 8 is calculated assuming $\overline{x}_1 = 100$, $T_{1,\max} = 115$, $T_{1,\min} = 40$, $A_1 = 0, \alpha_1 = .4$, $\beta_1 = .2$, and $s_1 = 0$. It shows three indifference curves in y vs. xT space, as well as the subject's expansion path, i.e., the locus of y and xT combinations he chooses as $\overline{x}$ rises. Note that along the expansion path, the indifference curves are tangent to budget lines (not drawn), with slopes of -1. Note also that the farthest-out indifference curve, with utility level $LU = 91.7$, happens to be the equilibrium level for this individual with endowment $= 100$.

Insert Figures 8 and 9 around here.

Next, we add a second person to show an equilibrium for the economy. Let person 1 be the individual illustrated in Figure 8. Assume the following for person 2: $\overline{x}_2 = 50$, $T_{2,\max} = 80$, $T_{2,\min} = 30$, $A_2 = 0$, $\alpha_2 = .6$, $\beta_2 = .2$ and $s_2 = 0$.

Figure 9 is the resulting useful Edgeworth box diagram. Two of person 1's indifference curves are drawn, these are the higher two from Figure 8. Two of person 2's indifference curves are also now shown. The graph is scaled so that the total y dimension equals y_1^* plus y_2^*, and so that the total xT dimension equals $x_1^* T_1^*$ plus $x_2^* T_2^*$. The equilibrium occurs at E, where the two expansion paths cross and where the indifference curves of persons 1 and 2 are tangent. Note that at E, each person's marginal rate of substitution (of xT for y) must equal 1.

Figure 9 thus shows a computed equilibrium outcome when each person, given his $\overline{x}_i$ and his other parameters, chooses a y_i^* and an $x_i^* T_i^*$. Each one is starting with a certain quantity of money $(\overline{x}_i)$. So the interpretation of Figure 9 is not exactly like that of a typical exchange economy Edgeworth box diagram, wherein two people are endowed with quantities of two distinct goods. However, with the figure we can visualize potential Pareto improving trades. For instance, from Z the two could trade to E (with 1 giving up some lifetime consumption expenditure, in exchange for some of 2's precaution expenditure).

Figure 9 illustrates a general result:

An interior equilibrium in the buying time economy is Pareto optimal if $s_i = 0$ *for all* i.

This general result can be demonstrated fairly easily for interior equilibria, and I will, therefore, omit the demonstration.

Now consider the subsidy rates. The discussion of "efficiency" in the last section indicates that whenever $s > 0$, the consumer's surplus corresponding to a given subsidy rate is less than the subsidy amount. Therefore, replacing a subsidy at rate s and associated cost to the government of $\frac{sy}{1-s}$, by a cash grant of $\frac{sy}{1-s}$, will make the subject better off. From this we have the next general result:

An equilibrium in the buying time economy is not Pareto optimal if $s_i > 0$ *for any* i.

Next consider some of the standard welfare economics paradoxes. First is the Boadway (1974) paradox, the essence of which is the following: Suppose there are two competitive equilibria x and y in a two-person two-goods exchange economy. Suppose the price ratio at x differs from the price ratio at y. Consider a move from x to y. If consumers' surplus is defined in terms of the equilibrium x price ratio, the move from x to y may result in an increase in total consumers' surplus, but if it is defined in terms of the equilibrium y price ratio, the move may result in a decrease in total consumers' surplus. Thus, measured one way, x is better than y, but measured the other way, y is better than x. It should be noted that the Boadway paradox depends crucially on different price ratios (or marginal rates of substitution) at x and y; without such differences there can be no such paradox. Now consider an equilibrium in the buying time economy, and assume that $s_i = 0$ for all i. Since in y vs. xT space the price ratio must be 1, we have the following result:

For interior equilibria in the buying time economy, if $s_i = 0$ *for all* i, *there are no Boadway paradoxes.*

The Boadway paradox is, of course, only one version of inconsistency of the Kaldor (1939) welfare improvement criterion. Other versions of Kaldor inconsistency can be constructed, in which non-Pareto optimal points are compared (see, e.g., Feldman (1980), chapter 7). The buying time model is subject to such paradoxes based on comparisons of non-Pareto optimal points. But since buying time equilibria will be optimal if $s_i = 0$ for all i, we need to worry about these paradoxes if and only if there are subsidies.

Next consider the *ex ante* and *ex post* paradoxes of Broome (1978), Ulph (1982), and others.

Since there is no ex ante and ex post in the buying time economy, there are no ex *ante/ex post paradoxes in it either.*

To conclude this section, I shall consider another potential paradox in this type of model, namely time-consistency. I now focus again on one individual, so the i subscript is dropped.

Assume for simplicity that $s = 0$, and, as above, that $A = 0$. Suppose the subject, given his endowment $\overline{x}$, has chosen x^*, y^* and T^* to maximize his utility. That is, he has solved the problem:

$$\max_{x,y,T} \; x^\alpha T, \text{ subject to}$$
$$\overline{x} = xT + y \text{ and}$$
$$T = T_{\max} - \frac{T_{\max} - T_{\min}}{1 + \beta y}.$$

To lighten the notation, I again drop the $*$ superscripts from x, y and T. From equations (A1) and (9), we know that the following must hold for the optimal point:

$$\frac{(1+\beta y)^2}{\beta \Delta T} = x\left(\frac{1-\alpha}{\alpha}\right). \tag{13}$$

Let us say that the lifetime choices are made at time zero. Assume that at some later time $t < T$, the subject has an opportunity to re-examine his choice and possibly change it. Would he choose to buy more time?

At time 0 he spent y on precaution. He now has an opportunity to spend more. If he follows the original plan, he will live $T - t$ more years, and his cash on hand (to pay for consumption over those years) is $x(T-t)$. His remaining utility is $x^\alpha(T-t)$.

Suppose he were to transfer a small increment $\epsilon > 0$ of money from consumption to additional precaution. Then y would increase by the increment ϵ. Therefore, his remaining lifetime would increase. It is possible to show that x would then have to decline by the following increment, approximately, and exactly in the limit as $\epsilon \to 0$:

$$dx = -\frac{\epsilon}{(1-\alpha)(T-t)}. \tag{14}$$

Taking the total differential of remaining utility $x^\alpha(T-t)$ gives

$$d\text{ (remaining utility)} = \alpha x^{\alpha-1}(T-t)\cdot dx + x^\alpha \frac{\beta\Delta T}{(1-\beta y)^2}\cdot dy. \tag{15}$$

substituting from (14) and (15) then gives d (remaining utility) $= 0$. This is a first order condition for the maximization of remaining utility. So he will not transfer ϵ.

In short, having chosen x, y and T at time zero, the subject will stick by his choice, even if he has a later opportunity to shift funds between consumption and precaution. For instance, consider a 90-year old subject with one month to live. If he has an opportunity to modify his precaution/consumption decision, so as to live a little longer (white consuming low), or so as to live less long (white consuming more), he will not make either change.

The general result is:

There are no regrets in the buying time model. Choices made at time zero are still optimal at time $t < T$.

Conclusions

To briefly summarize, the purpose of this paper is to develop a simple model of the value of extra years of life, a model that can be easily solved, manipulated and analyzed.

Some of the important results include:

(1) $TVOTP = y^*$. The individual's total value of time, private, equals what he chooses to pay for precaution. Over the entire chosen lifespan, his willingness to pay for extra years of life equals what he pays.

(2) $VOTP = (\frac{1-\alpha}{\alpha})x$. The value of time, private (at the margin) is proportional to the rate of consumption expenditure. When $\alpha = 1/2$, it equals consumption expenditure.

(3) $VOTS = (\frac{1}{1-s})(\frac{1-\alpha}{\alpha})x$. The value of time, social (at the margin), exceeds $VOTP$, and as the subsidy rate approaches 1, $VOTS$ grows without bounds.

The following results are from the simulations:

(4) As the bequest parameter A rises, consumption x declines slightly, precaution y declines, planned bequest z rises markedly, and the planned lifespan declines.

(5) As the subsidy rate s rises, the value of time, private, stays almost constant, but the subject's own precaution expenditure falls substantially, and value of time, social, rises dramatically.

(6) As the subsidy rate s rises, efficiency drops sharply. That is, as s gets larger, the cash grant required to buy the same utility level becomes a sharply smaller fraction of the money spent on subsidizing precaution.

The following results are independent of the simulations, and are related to broader welfare economics issues:

(7) An interior equilibrium in a buying time economy is Pareto optimal if and only if the subsidy rate is zero for everyone.

(8) There is no Boadway paradox in the buying time economy, nor are there *ex ante*/*ex post* paradoxes.

(9) There are no time-inconsistency paradoxes in the buying time economy. Subjects approach the end of life believing they have made the right decisions about the lengths of their lives.

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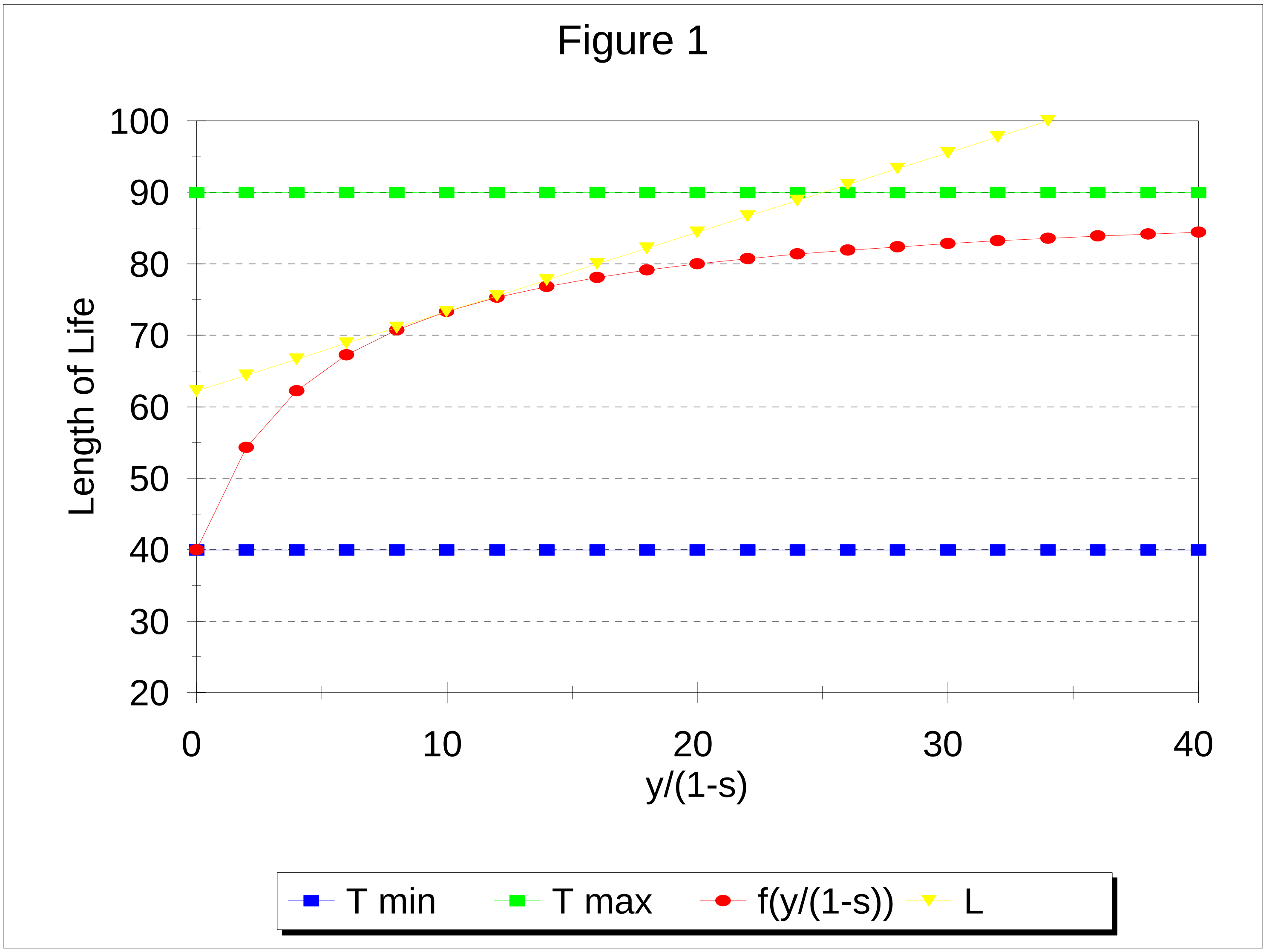
Figure 1
Length of Life
100
90
80
70
60
50
40
30
20
0
10
20
30
40
y/(1-s)
T min
T max
f(y/(1-s))
L

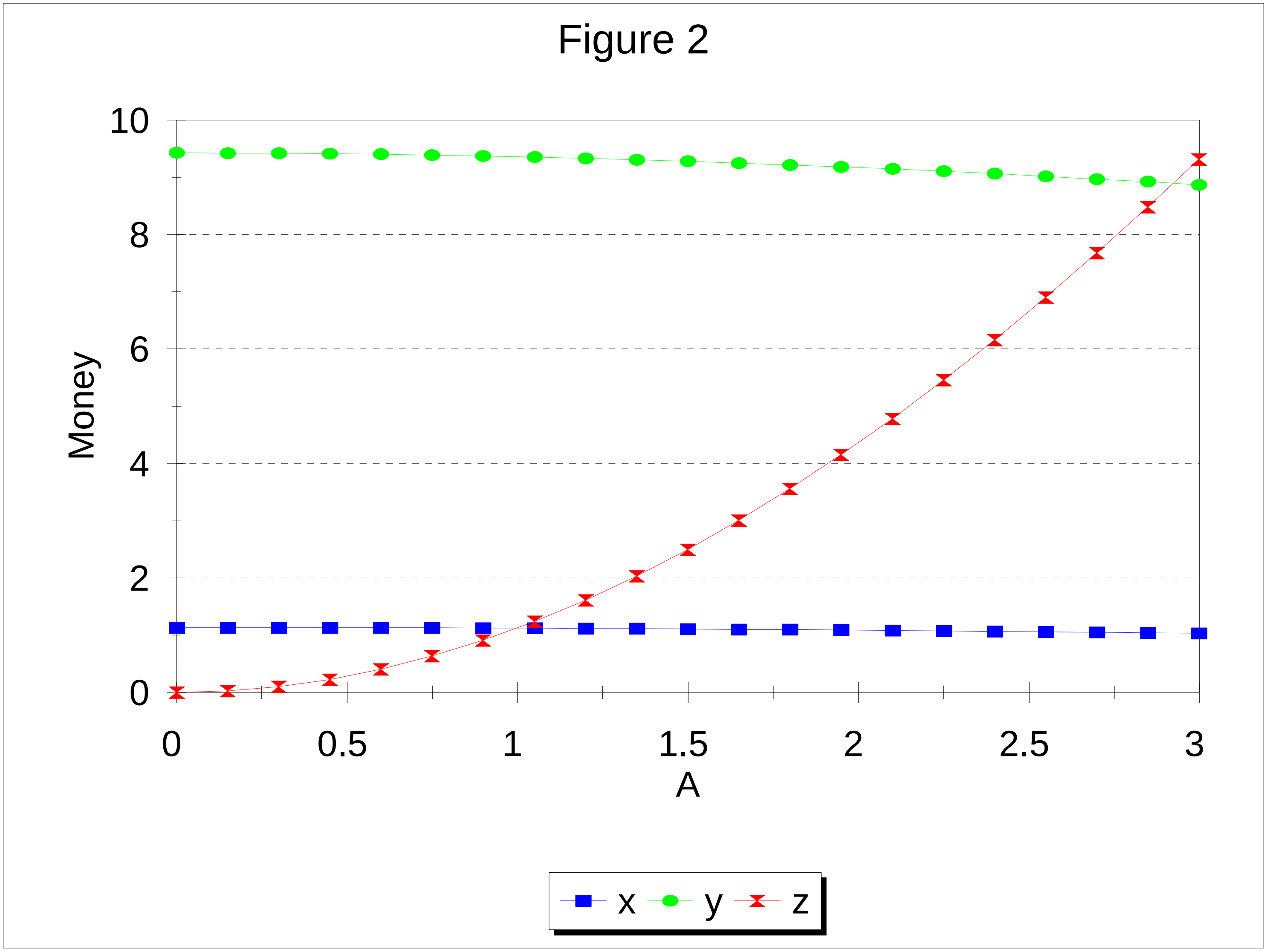

Figure 2
Money
10
8
6
4
2
0
0
0.5
1
1.5
2
2.5
3
A
x
y
z

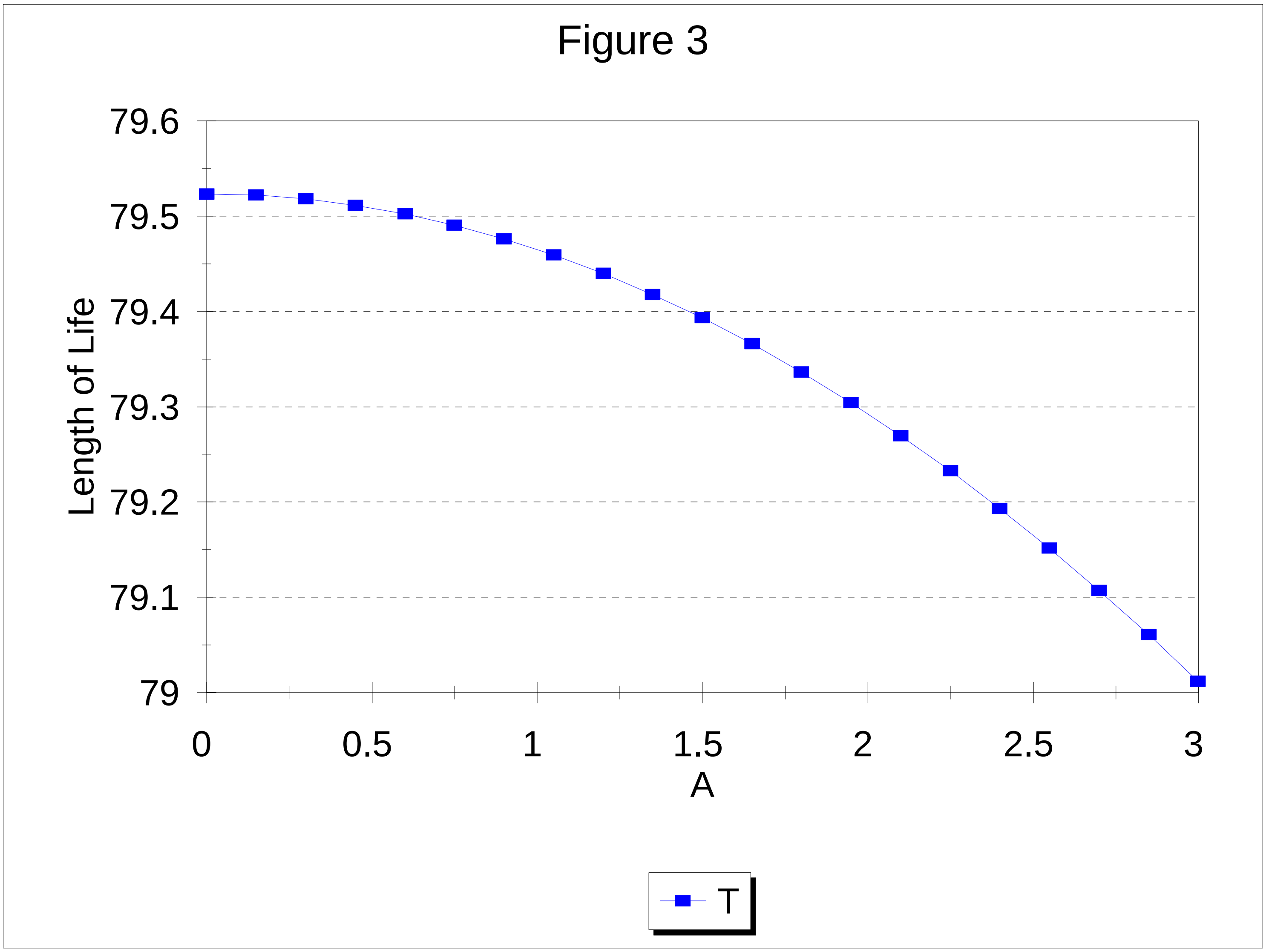

Figure 3
79.6
79.5
79.4
79.3
79.2
79.1
79
Length of Life
0
0.5
1
1.5
2
2.5
3
A
T

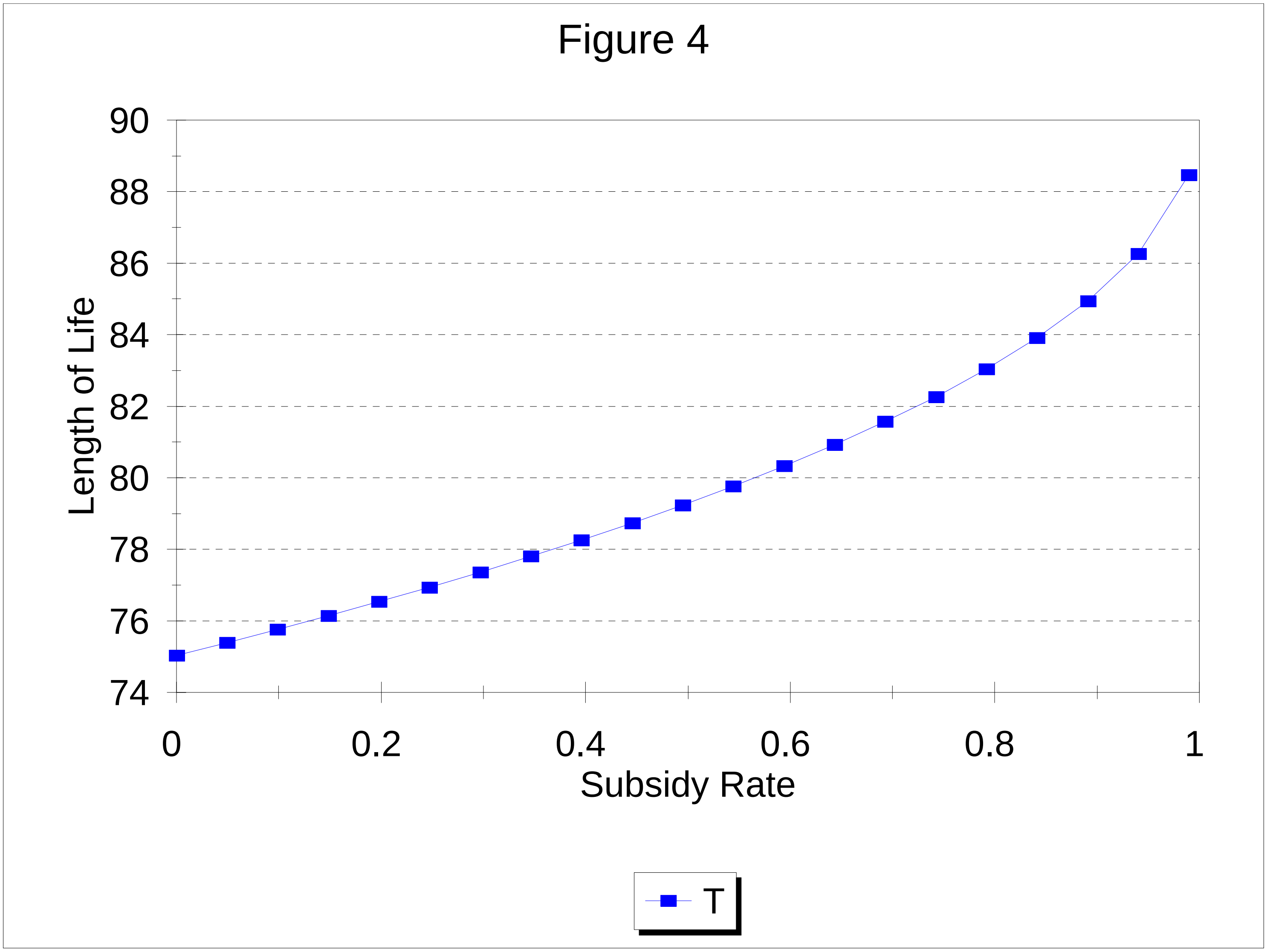
Figure 4
90
88
86
84
82
80
78
76
74
Length of Life
0
0.2
0.4
0.6
0.8
1
Subsidy Rate
T

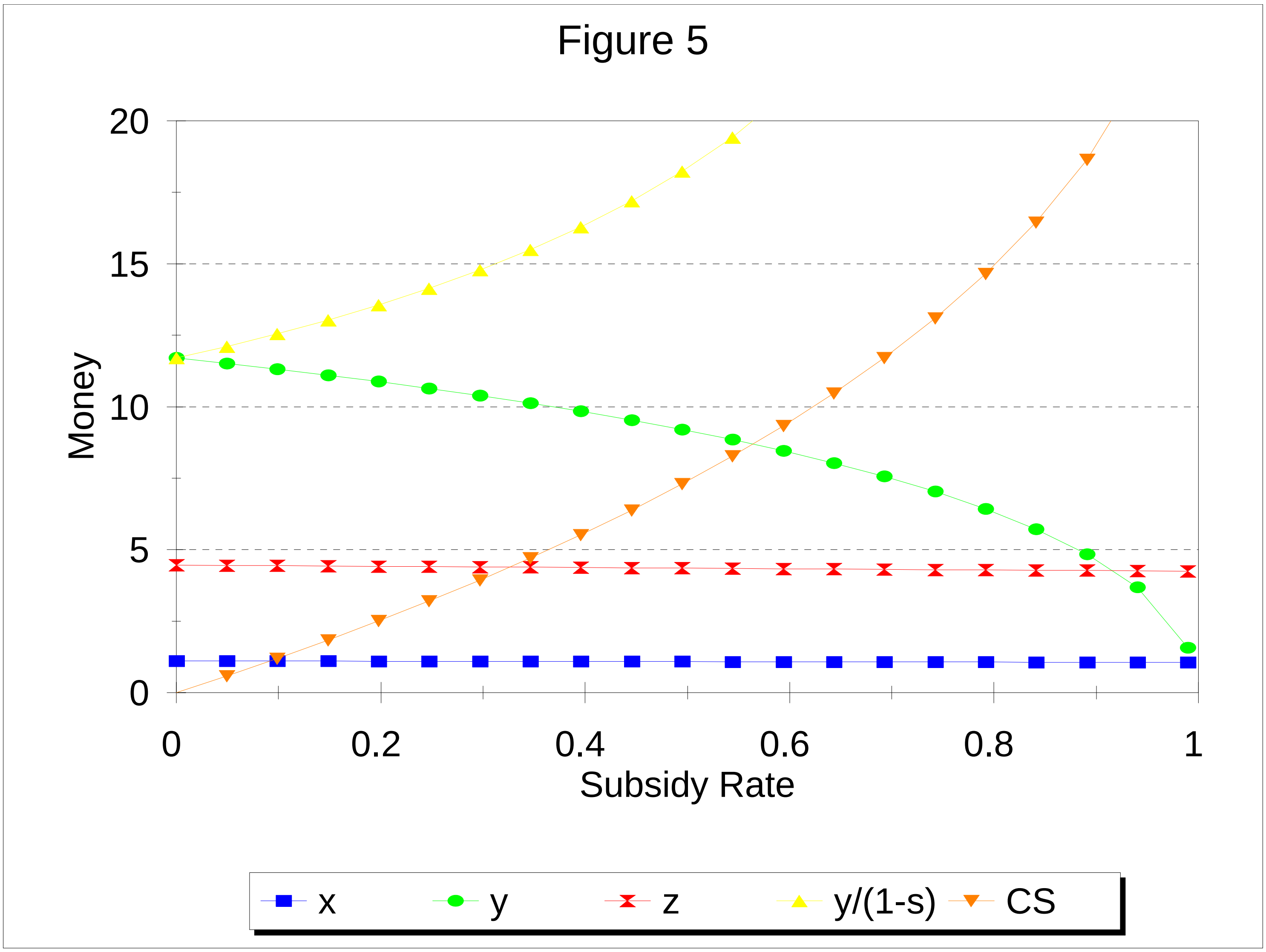
Figure 5
Money
Subsidy Rate
20
15
10
5
0
0
0.2
0.4
0.6
0.8
1
x
y
z
y/(1-s)
CS

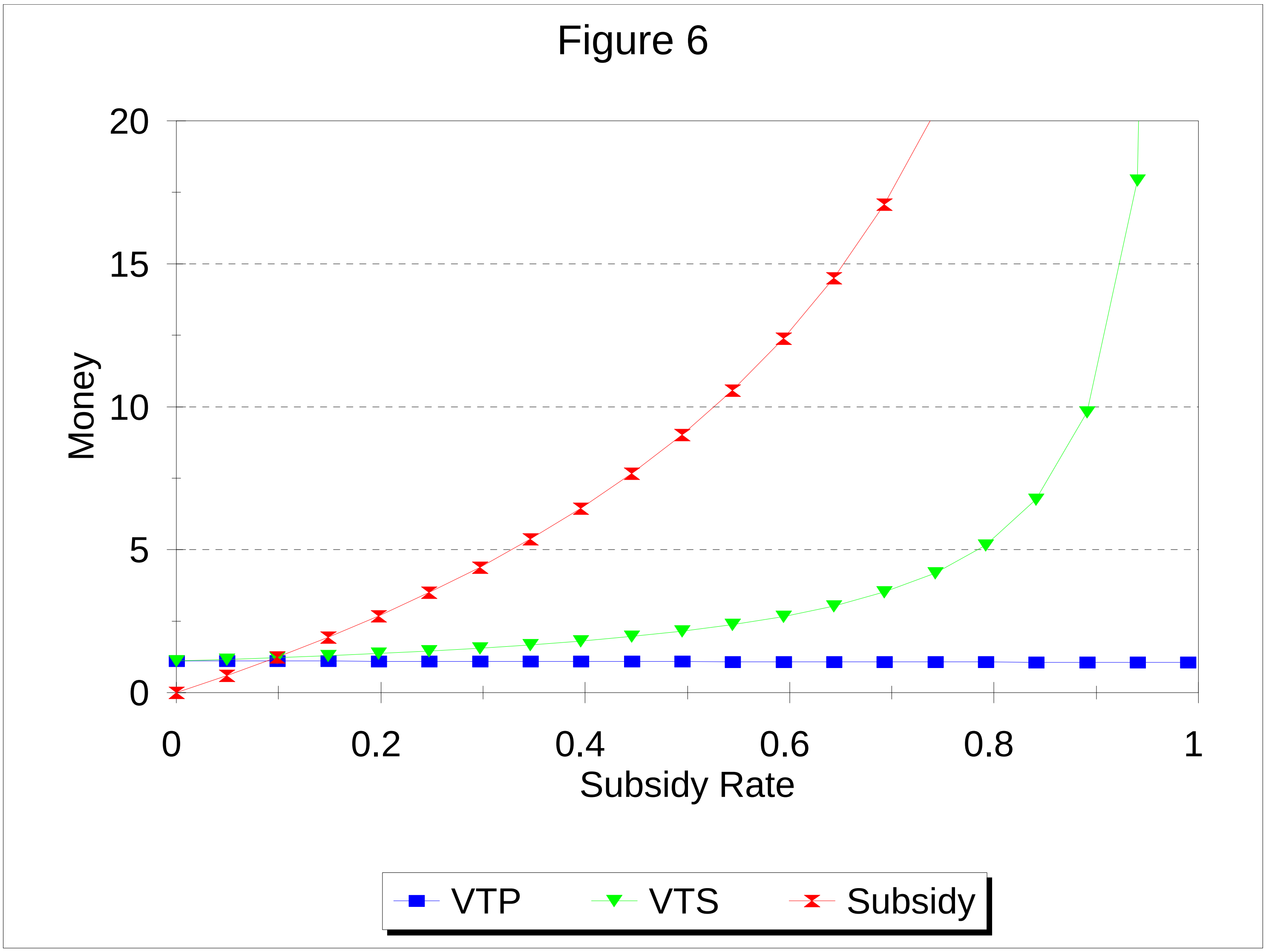
Figure 6
Money
20
15
10
5
0
0
0.2
0.4
0.6
0.8
1
Subsidy Rate
VTP
VTS
Subsidy

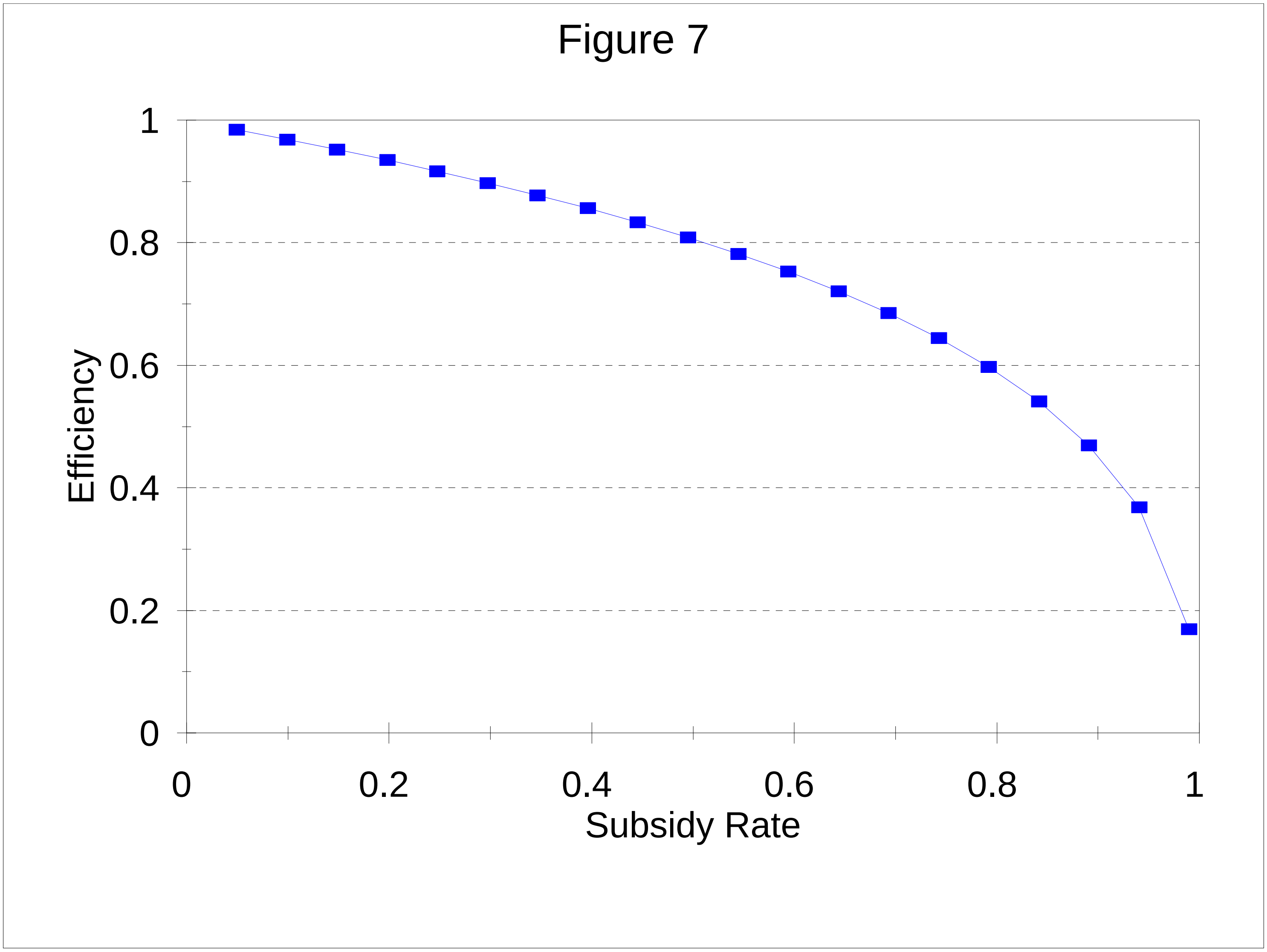

Figure 7
1
0.8
0.6
0.4
0.2
0
Efficiency
0
0.2
0.4
0.6
0.8
1
Subsidy Rate

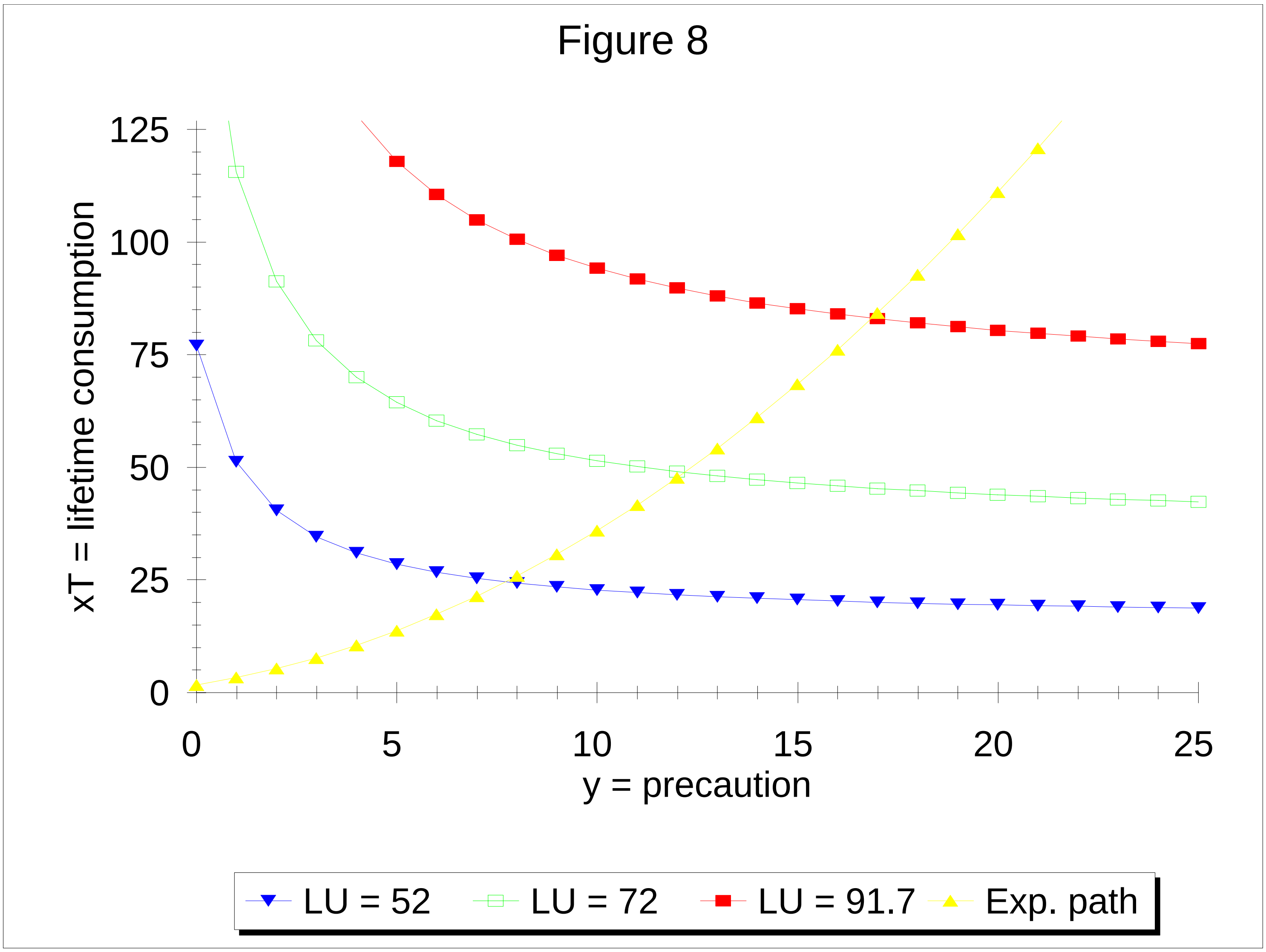
Figure 8
125
100
75
50
25
0
xT = lifetime consumption
0
5
10
15
20
25
y = precaution
LU = 52
LU = 72
LU = 91.7
Exp. path

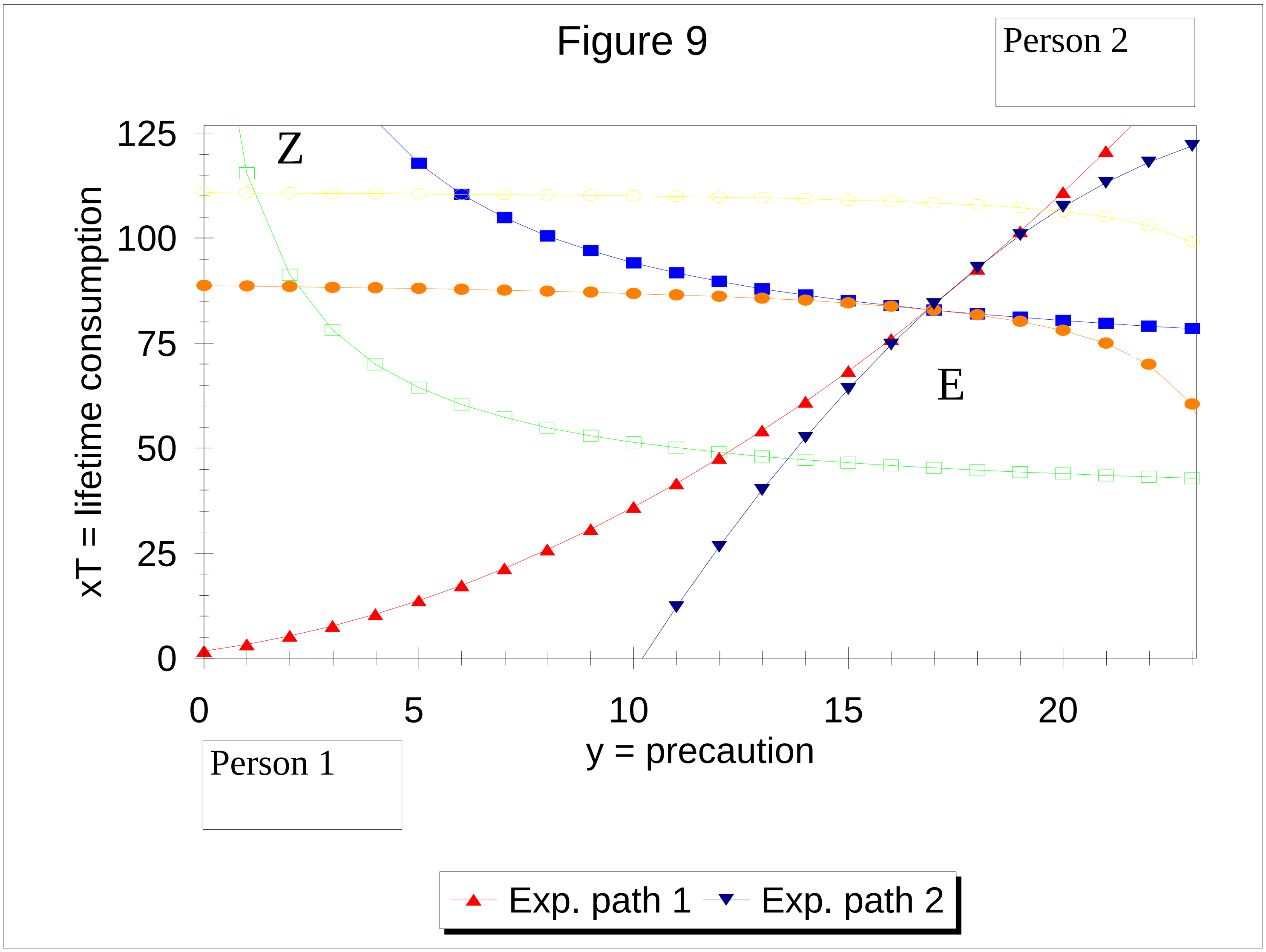

Figure 9
Person 2
Z
E
125
100
75
50
25
0
xT = lifetime consumption
0
5
10
15
20
y = precaution
Person 1
Exp. path 1
Exp. path 2