The Value of Life
Revisited

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Working Paper 96-2, Revised

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January, 1997

Abstract

This paper re-examines the standard willingness-to-pay value of life model of Jones-Lee and others. A consumer decides on allocating his income between consumption, life insurance, and precaution — which affects his probability of survival in the one future period. The innovation of the paper is to assume a simple exponential utility function, a simple form for the survival probability function, and to solve analytically for various measures of the value of life.

I show that for plausible parameter magnitudes, the various willingness-to-pay measures differ by several orders of magnitude. Moreover, the standard marginal willingness-to-pay measure has some strikingly bad features, including a strong negative connection with altruism. I conclude by suggesting that the standard probabilistic model of the value of life ought to be replaced by a certainty model, a model of the value of time.

Keywords

Value of life, value of time, willingness-to-pay, Broome paradox, death

Classification Code

D11, D61, D6, D91

"Cowards die many times before their deaths;
The valiant never taste of death but once.
Of all the wonders that I yet have heard,
It seems to me most strange that man should fear;
Seeing that death, a necessary end,
Will come when it will come."

Shakespeare: *Julius Caesar,* 1599.

I. Introduction

What is the money value of a human life? This has been an interesting question for as long as we know. In the Law Code of Hammurabi, for instance, we read that "if a citizen has struck a citizen in a quarrel, and has inflicted on him a wound, [and] if he has died as a consequence ⋯, he shall swear [he struck him unwittingly], ⋯ and, if he was of citizen stock, he shall pay a half-mina of silver."(Thomas, 1961, lines 206–207). (A similar legal remedy – money damages – is suggested in some lines of *Exodus* (Chapter XXI, lines 28–30), but only in the special case of a fatal ox-goring, when the ox's propensity to gore is known to its owner. The similarity of these *Exodus* lines to lines 250–251 in Hammurabi's Code suggest that the biblical lines are a borrowing. The standard ancient Jewish remedy to accidental homicide was exile for the killer, not money paid by him to the clan or family of the victim.)

Islamic law clearly distinguished between intentional and accidental killing, and provides for the paying of *diyah* (blood-wit) in the case of accidental homicide. One source places the value of diyah at 30.63 kg of silver, or half that amount if the victim is a female, a Christian or a Jew (Niazi, 1985, p. 159).

The Justinian Code provided for money payments to be made in cases of accidental killings of slaves (Kolbert, 1979). Early tribal customs in pre-Norman Britain put a compensatory price, or *wirgild* (man-price), on wrongful deaths (Seebohm, 1972), and similar blood money compensation customs continued after 1066.

The modern Anglo-American legal treatment of value of life starts with Lord Campbell's Act in 1846. This law and its successors, called wrongful death statutes, provide that the dependents of a deceased person may recover for the pecuniary losses they suffer, particularly lost wages the deceased would have provided them. The legal alternative to a wrongful death statute is a survival statute, according to which the legal representative of the deceased can pursue the damages that the deceased himself suffered through his death, principally loss of wages, but also pain and suffering before death, and, possibly, loss of enjoyment of life. Losses in many jurisdictions of the United States are combinations of, or hybrids of, wrongful death and survival laws. (For surveys, see, e.g., Keeton (1984), Speiser (1975), Brookshire (1987).)

In short, in the legal sphere the common, standard approach to the value of a life is the *human capital* approach. The deceased is primarily viewed as a money making machine. The value of his life is mainly given by lifetime income or earnings, possibly net of expenses needed to maintain the machine, possibly discounted to present value, and possibly augmented by imputed income for non-market services.

However, this is not the standard approach to value of life taken by many economists, and it is no longer the standard approach of government policy makers. The human capital approach is derided because it does not measure how much the deceased valued his own life. To measure how much people actually value their own lives, it is said, we should see how much they are willing to pay to avoid total risks. This is the *willingness-to-pay* approach, and it is now the standard economic theory of value of life (Jones-Lee (1974), Jones-Lee (1976), Mishan (1971), Mishan (1982a), Schelling (1968), Schelling (1987), Viscusi (1992), and others).

The willingness-to-pay approach essentially proceeds by posing the following question: Suppose a consumer has an opportunity to make his life safer, but at a cost. Assume he can reduce his probability of dying by ϵ if he buys some government program. What is the maximum, say C, he would pay for the extra survival probability? The consumer decides on some C for the given ϵ. Then his valuation of the survival probability increment ϵ is C. The willingness-to-pay value of his *whole life* is said to be C/ϵ. When there are n

identical individuals being asked the same sort of question, they reveal that they would pay a total of $n \cdot C$ to prevent $n \cdot \epsilon$ expected death, which gives C/ϵ per expected death prevented.

Alternatively, a consumer may be faced with an increase in the probability of dying ϵ, and he may be asked for the minimum C that he would accept, in exchange for the greater risk of death. The value of life is again C/ϵ, and this is, strictly speaking, a *willingness-to-accept* measure. In what follows I will formally define several alternative value of life measures, some of which are, strictly speaking, willingness-to-pay, and some of which are, strictly speaking, willingness-to-accept. For simplicity, I will call them all willingness-to-pay measures, in contrast to the human capital measure.

There have been many studies which attempt to estimate C/ϵ. Good surveys can be found in Fisher et al. (1989); Jones-Lee (1989), Chapter 2; Viscusi (1992), Chapters 3 & 4; and Viscusi (1993) among others. This is not an empirical paper, and it is not my intention to criticize the empirical work. However, I will make a few observations: First, several of the empirical studies find values of life that exceed the human capital measure by roughly an order of magnitude. Second, the studies get wildly differing estimates for the value of life, and many anomalous results (for example, that union workers value their lives much more highly than non-union workers).

Third, the empiricists seem to have overlooked many common behaviors people pay for that appear to be *risk-seeking*. Behaviors like skiing, whitewater canoeing and kayaking, sky-diving, automobile and motorcycle racing, bungee jumping, rock climbing, mountaineering, and spelunking all come to mind, not to mention warfare. (For instance, skiers and alpine climbers at Mont Blanc in France suffer more than two hundred fatalities per year. Climbing Mt. Everest appears to carry a risk of death of approximately 20 percent, and yet, according to recent news stories, some people pay $60,000 to guides and expedition outfitters for the opportunity!) Perhaps if the effort were expended, empirical evidence would be amassed that "proves" people are willing to pay to *increase* risk of death.

Fourth, in the United States the willingness-to-pay approach has started to leak into courtrooms under the name "hedonic damages." Here willingness-to-pay is used to attach a dollar value to the life of an individual already dead, with the damages going to his estate or survivors. Several authors have observed that this is a misuse of the method (e.g., Dickens (1990), and Viscusi (1992)).

The willingness-to-pay approach has been attacked on philosophical and social-choice-theory grounds. (See particularly Broome (1978, 1982, 1985), Ulph (1982), Blackorby and Donaldson (1986), and Blackorby, Bossert and Donaldson (1993).) The most telling philosophical attack is what I would call the Broome paradox (Broome, 1978): Suppose the government plans a project that will cost some money and also create a risk to some lives (for instance, a long tunnel). Assume there are n identical people whose lives are put at risk, and the project creates an additional risk of death for these people of ϵ. Assume that they would require at least C each to accept that additional risk of death. (Here C is a willingness-to-accept number, strictly speaking.) Then by reasoning like what's above the willingness-to-pay value of life is C/ϵ. Now assume that the government project would produce a net benefit to society of B, aside from the cost of lost lives. The usual economic cost benefit analysis (e.g., Mishan (1971 and 1982a)) would compare society's benefit of B to the expected cost of $n\ \epsilon$ lives at C/ϵ per life, or a life-cost of $n\ C$. If $B > n\ C$, benefits exceed costs, and the project would be approved. But, argues Broome, what if we know which $n\ \epsilon$ were to die, what if we know their names? And what if we approached the $n\ \epsilon$ named people, and asked what amount of money they would require to compensate for the certainty of death? They would each answer with an extremely high figure (perhaps $\$ +\infty$), and if we then compared the life-cost to the project benefit of B, the project would fail the cost benefit test by a spectacular margin. So Broome argues that the willingness-to-pay approach is philosophically unacceptable. It leads to conclusions based on *ex ante* information which almost surely would be rejected with *ex post* information.

Like the papers cited above, the purpose of this paper is to critique the theoretical foundation of willingness-to-pay value of life measures. I use a simple two-period, two-state expected utility maximization model, similar to what has been used traditionally. (See, e.g., Jones-Lee (1974 and 1976) among others.) The innovation here is to make a few assumptions that allow analytic solutions to the model. Using the solved model, I define various alternative willingness-to-pay value of life measures. I show that willingness-to-pay "value of life" is not uniquely defined, and that, in fact, there are at least six ways to approach the willingness-to-pay value of life concept, all giving numbers that differ by several orders of magnitude. It seems to me that if theory suggests one individual's "value of life" varies by orders of magnitude, depending on exactly which definition is used, then the willingness-to-pay "value of life" concept is dangerously ill-defined, with great potential for misuse.

The model used here incorporates insurance (as does, e.g., Bailey (1978), Bergstrom (1982), Conley (1976), Cook & Graham (1977), Dehez & Dreze (1982), Jones-Lee (1980)), as well as bequest-motives (as does, e.g., Jones-Lee (1974), and many others). It incorporates precaution as a choice variable, which is somewhat unusual.

The model has two parameters that are especially important. The first is a measure of the individual's concern for his dependents (or his beneficiaries); this is the *bequest parameter*, or what I call *altruism.* The second is a measure of the individual's *fear of death.* I show that for the standard willingness-to-pay measure of the value of life, value of life will depend crucially on these two parameters. The standard willingness-to-pay value of life measure is strongly negatively related to altruism, and strongly positively related to fear of death. I suggest that these associations between the value of life measure and altruism (often considered a virtue), and fear of death (sometimes considered a vice), make the standard value of life measure suspect.

Finally, I briefly discuss which value of life measure is "correct," if any. I suggest and outline an alternative to the usual probabilistic willingness-to-pay approach, which may bridge the gap between the legal tradition and the human capital measure, on the one hand, and willingness-to-pay, on the other, and which may avoid some of the pitfalls of the willingness-to-pay approach. The alternative is a deterministic *value of time* model.

II. The Model

This is a two-period model, with a "before" and an "after", or *ex ante* and *ex post.* A two-period model is clearly less realistic than a multi-period model or a continuous time model, but the lack of realism is compensated for by tractability.

In the first period, an individual is contemplating his state in the second period. In that future period, he will be either alive or dead. If alive, his utility (as contemplated from the first period) will depend on the amount of money he has to spend. If dead, his utility (as contemplated from the first period) will depend on the amount of money in his estate, and on a constant factor that represents his "fear of death." This is, of course, utility as seen *ex ante,* for if he is dead *ex post,* he then has no utility (as far as we know).

Our individual is endowed with a given sum of money in period one. He may use it in three ways: (1) He may hold it to spend on consumption in period two, or to bequeath, as the case may be. The money he uses this way is called *consumption.* (2) He may spend it in period one on *precaution.* By spending it on precaution, he increases the probability that he will be alive in period two. (3) He may spend it in period one on *life insurance.* By spending it on life insurance, he increases his bequest by the face value of whatever life insurance policy he buys.

I assume that the individual has an expected utility function. His maximization problem is to choose the sums of money used in the three ways detailed above, so as to achieve the highest *ex ante* expected utility.

For the model I use the following notation:

x	$=$	amount of money to be spent on consumption (or bequeathed if dead) in period 2
y	$=$	money spent on precaution in period 1
z	$=$	money spent to purchase life insurance in period 1
$\overline{x}$	$=$	$x+y+z=$ cash endowment
$p(y)$	$=$	probability of life in period 2
$1-p(y)$	$=$	probability of death in period 2
$V(y,z)$	$=$	face value of life insurance policy
$v(y)$	$=$	price of insurance
$f(x)$	$=$	ex ante utility contingent on life
$g(x+V)$	$=$	ex ante utility contingent on death
A	$=$	bequest parameter, or "altruism"
K	$=$	"fear of death."

As in the standard model, I assume the subject chooses x, y and z so as to maximize expected utility

$$Eu = p(y)f(x) + (1-p(y))g(x+V(y,z)),$$

subject to the constraints

$$\overline{x} = x+y+z, \quad x, y \geq 0, \quad \text{and} \quad x+V \geq 0.$$

I don't require that z be nonnegative because it is analytically simpler to allow individuals to buy negative quantities of life insurance (i.e., rather like annuities, translated into the structure of this model). I do require that consumption, precaution, and the estate $x+V$ be non-negative.

To solve the model, I make the following special assumptions:

Assumption 1 (Actuarially fair life insurance).

$$V = \frac{z}{1-p(y)}.$$

That is, the premium for the life insurance policy z equals the expected payout, $V \cdot [1-p(y)]$. To put it slightly differently, the price for a dollar's worth of insurance is $v = 1/(1-p(y))$.

There are two possible alternative assumptions about insurance companies' pricing policies: They may be able to observe each individual's degree of precaution y, and price accordingly (offering each individual a schedule of rates, depending on y). Or they may not (offering each individual one price, $1/(1-p)$). The latter assumption produces a slightly simpler model, and I will follow it here. It implies that, when choosing x, y, and z, the utility-maximizing individual assumes $\frac{\partial V}{\partial y} = 0$, or, to say the same thing, he views the price v of a unit of insurance as a constant, rather than as a function of y.

Assumption 2 (Utility if dead)

$$g(x+V) = Af(x+V) - K,$$

where A and K are non-negative constants. That is, the utility from the dead state is comprised of (a) a scaled up (or down) version of utility if alive, minus (b) a *fear of death parameter* K. The parameter A is a measure of the subject's concern for his bequest. If he is very concerned about the welfare of his heirs, A might be larger than 1. It he simply wants to be sure that his dependents are no worse off financially if he dies than if he lives, he might have $A = 1$. If he has no dependents and no interest in a bequest, $A = 0$. A will be called *altruism* or the *bequest parameter.*

The fear of death parameter reflects the disutility of death per se, viewed ex ante. If the subject doesn't care whether he lives or dies (aside from the issue of a bequest), $K = 0$. If he does want to avoid death, $K > 0$.

Assumption 3. (Exponential utility).

$$f(x) = x^{\alpha}, \quad \text{for some } 0 < \alpha < 1.$$

The exponential utility function provides some degree of generality, but still permits analytic solutions of the model. Note that $1-\alpha$ is Arrow's relative risk aversion measure. (An alternative although slightly less general approach would be to let $f(x) = ln\ x$. This would produce a model quite similar to what is developed below.) Note that for some of the analysis below the exponential utility function assumption is not needed. When I do not use this assumption I will simply assume the following regularity conditions for $f(\cdot)$: that for $x \geq 0$, it is non-negative, differentiable, strictly increasing and strictly concave.

Assumption 4. (Survival probability).

$$p(y) = a - \frac{b}{1+y}, \text{ where } 0 < b \leq a \leq 1.$$

When plotted against y, this probability function has intercept $a - b$, and is asymptotic to a horizontal line at level a. The parameter a can be viewed as the maximum survival probability that can be achieved (approached in the limit, as $y \to +\infty$). The difference $a - b$ can be viewed as the minimum survival probability (realized when $y = 0$), and the parameter b can be viewed as the maximum possible gain in survival probability (achieved in the limit as $y \to +\infty$). Note that assuming an exact form for the survival probability function is not necessary for much of the discussion of value of life that will follow. I will indicate when and where it is used as I go along. If it is not necessary to assume an exact form, I will simply assume that $p(\cdot)$ is differentiable and strictly increasing.

III. Solving the Model

In general terms, our individual wants to maximize expected utility subject to certain constraints. Using only assumptions 1 and 2, the problem is to maximize:

$$E(u) = p(y)f(x) + (1-p(y))[Af(x+vz) - K], \quad \text{subject to}$$

$$x + y + z = \overline{x};\ x, y \geq 0, \text{ and } x + vz \geq 0.$$

First-order conditions for maximization lead to the following equations (assuming an interior maximum, and incorporating assumptions 1 and 2):

$$(1) \qquad f'(x) = Af'(x + vz).$$

$$(2) \qquad p' \; [f(x) - (Af(x + vz) - K)] = f'(x) = \lambda.$$

Here the primes denote derivatives, and λ is the Lagrange multiplier associated with the budget constraint.

Now consider equation (2). The intuitive interpretation of p' is important. This is the marginal increase in survival probability resulting from a marginal increase in precaution expenditure y, for the utility maximizing individual who is choosing precaution, along with x and z. The inverse of p' is the marginal increase in precaution expenditure corresponding to a marginal increase in survival probability.

That is, $1/p'$ *represents precisely the utility maximizing individual's marginally measured willingness-to-pay value of life.* It is exactly analogous to the C/ϵ measure discussed above. (See also Bailey (1978), and others.) Therefore, $1/p'$ *is the standard economic measure of the value of life, and in what follows, I let* VOL *stand for this measure.*

Hence, from (2)

$$(3) \qquad VOL = \frac{1}{p'} = \frac{1}{f'(x)} \, [f(x) - (Af(x + vz) - K)].$$

The interpretation of equation (3) is simple and obvious:

$$(4) \qquad VOL = \frac{\text{Utility advantage from being alive over being dead}}{\text{Marginal utility of money spent on consumption}}.$$

Now I use assumption 3, which, when combined with equation (1) above, gives:

$$x + vz \;=\; A^{\frac{1}{1-\alpha}}\, x, \text{ or } vz \;=\; (A^{\frac{1}{1-\alpha}} - 1)\, x. \tag{5}$$

It follows that the non-negative estate constraint $(x + vz \geq 0)$ is automatically satisfied as long as $x \geq 0$, because of the $A \geq 0$ assumption.

Next, substituting for $f(x)$ and for $x + vz$ in equation (2) gives:

$$VOL = \frac{1}{p'} = \frac{1}{\alpha x^{\alpha-1}}\, [x^{\alpha}\, (1 - A^{\frac{1}{1-\alpha}}) + K]. \tag{6}$$

In the event that $A = 0$ (no bequest motive),

$$VOL = \frac{1}{\alpha x^{\alpha-1}}\, [x^{\alpha} + K], \quad \text{and} \quad x + vz = 0. \tag{7}$$

In the event that $A = 0$ (no bequest motive), *and* no insurance market exists (so that z is constrained to be zero), first-order maximization conditions lead directly to

$$VOL \;=\; \frac{1}{p}\, \frac{1}{\alpha x^{\alpha-1}}\, [x^{\alpha} + K]. \tag{8}$$

Note the difference between equation (8), which makes VOL inversely related to survival probability, and equation (3) and (7), which do not. That is, *the inclusion of insurance (in this case negative insurance) cuts the link between survival probability and the value of life.* (This is a general result, for $A \geq 0$.)

Finally, in the event that $A = 1$, equation (6) reduces to a very simple and attractive form:

$$VOL = \frac{K}{\alpha x^{\alpha-1}}. \tag{9}$$

IV. Other Willingness-to-Pay Measures

The next step in this analysis is to introduce alternative willingness-to-pay measures of the value of life. Recall that the VOL measure defined above is the subject's willingness-to-pay for extra survival probability, per unit of extra survival probability; the units are $\Delta y / \Delta p$.

The simplest alternative measure is the non-marginal analog to VOL. Subject chooses a utility maximizing y (with corresponding survival probability $p(y)$) so as to enhance his survival probability. The total expenditure on precaution per unit of survival probability purchased is then y divided by $p(y) - p(0)$. I will call this the *average (non-marginal) value of life*, abbreviated $AVOL$, and it is defined as

$$(10) \qquad AVOL = \frac{y}{p(y) - p(0)}$$

Note that, unlike VOL, finding $AVOL$ requires a specific assumption about the form of the $p(\cdot)$ function. That is, it requires use of assumption 4.

The next two alternative measures rely on answers to hypothetical John Broome-style questions. Suppose a subject has some bequest motive, so that $A > 0$. Suppose he has chosen a utility maximizing x, y and z. Confront him and ask the following: What addition to your estate $\tilde{X}$ would you require, so as to be indifferent (ex ante), between the live state and the dead state? I'll call the answer to this question the *life-death indifference* measure of the value of life. In general, it is the solution to

$$(11) \qquad f(x) = A\, f(x + vz + \tilde{X}) - K.$$

Equation (11) has only incorporated assumption 2. Using assumptions 1, 2 and 3 together gives:

$$(12) \qquad \tilde{X} = A^{-\frac{1}{\alpha}}\,(x^{\alpha} + K)^{\frac{1}{\alpha}} \; - \; A^{\frac{1}{1-\alpha}}\, x.$$

If the subject has no money to spend (which means $x = 0$), this reduces to

$$X = (K/A)^{\frac{1}{\alpha}} \tag{13}$$

I will call the X defined in equation (11) the *destitute-life-death-indifference* measure of the value of life. The virtue of this willingness-to-pay measure is that it is independent of income; it only depends on the utility function parameters.

I shall define two more value of life measures. Suppose an individual has chosen x, y, and z and is asked the following (reasonable) hypothetical question: What is the maximum you would pay, out of your spendable income x, to eliminate the chance of death? Call his answer $\hat{X}$. $\hat{X}$ is another non-marginal willingness-to-pay measure. The person who pays $\hat{X}$ removes the death probability $1-p$, and his willingness-to-pay-per-statistical-death-prevented is therefore $\hat{X}/(1-p)$. I call $\hat{X}/(1-p)$ the *live-for-sure* measure. (See Albrecht (1992) for discussion of a similar measure.)

$\hat{X}$ is found by solving the equation

$$p(y)\ x^\alpha + (1-p(y))\ \{A^{\frac{1}{1-\alpha}}\ x^\alpha\ - K\} \ = \ (x-\hat{X})^\alpha. \tag{14}$$

A somewhat analogous measure can be defined as follows: Suppose an individual has chosen x, y and z and is asked this (unpleasant) hypothetical question: What is the minimum you would demand, to be added to your spendable (and bequeathable) x, to eliminate the chance of life? Call the answer $\overline{X}$. The subject who accepts $\overline{X}$ removes the survival probability p, and his demand-per-statistical-life-lost is $\overline{X}/p$. I call $\overline{X}/p$ the *die-for-sure* measure.

$\overline{X}$ is found by solving the equation

$$p(y)\ x^\alpha + (1-p(y))\ \{A^{\frac{1}{1-\alpha}}\ x^\alpha - K\} \ = \ A\ (A^{\frac{1}{1-\alpha}}\ x + \overline{X})^\alpha - K. \tag{15}$$

Summarizing the various willingness-to-pay to reduce the risk of death measures of the value of life, there are, it seems to me, at least six:

(i) $AVOL = \frac{y}{p(y)-p(0)}$. (What the subject actually pays for enhanced survival, but non-marginal.)

(ii) $VOL = \frac{1}{p'}$. (The economists' favorite. Willingness-to-pay at the margin.)

(iii) Life-death-indifference $\tilde{X}$. (Broome-inspired, non-marginal, willingness-to-accept).

(iv) Destitute-life-death-indifference X. (Broome-inspired, non-marginal, income-independent, willingness-to-accept).

(v) Live-for-sure $= \hat{X}/(1-p)$. (Plausible, non-marginal, willingness-to-pay).

(vi) Die-for-sure $= \overline{X}/p$. (Less plausible, non-marginal, willingness-to-accept).

If one knows the utility-maximizing x, as well as A, K and α, value of life measures (ii), (iii) and (iv) can all be calculated without using the parameters of the survival probability function. However, measure (i) does require knowledge of $p(\cdot)$, and (v) and (vi) may as well, although they can be nicely approximated when $A = 1$, even without the $p(\cdot)$ function.

V. Some Simple Analytical Solutions when $A = 1$, and Some Calculated Values

In this section I assume that $A = 1$. With this assumption the utility-maximizing individual choose no life insurance (so $z = 0$) and all of the six alternative value of life concepts have fairly neat and simple analytical solutions. There are shown in Table 1. (Note that the expressions for live-for-sure and die-for-sure are slightly complex, but have simple approximations when p is close to 1 or 0. Note also that only $AVOL$ requires the functional form assumption for $p(\cdot)$, that is, assumption 4.

Insert Table 1 Around Here.

Table 1

Analytical Solutions for Willingness-to-Pay Values of Life

Assuming $A=1$, Arbitrary α, K, and x, and Arbitrary p Function (Except $AVOL$)

	Measure	Exact Solution	Approximate Solution
(i)	AVOL	$\sqrt{\frac{1}{b}}\sqrt{\frac{K}{\alpha x^{\alpha-1}}}$	
(ii)	VOL	$\frac{K}{\alpha x^{\alpha-1}}$	
(iii)	Life-death-indifference	$(x^\alpha+K)^{\frac{1}{\alpha}}-x$	
(iv)	Destitute-l.d.-indifference	$(K)^{\frac{1}{\alpha}}$	
(v)	Live-for-sure	$\frac{x-(x^\alpha-(1-p)K)^{1/\alpha}}{1-p}$	$\begin{cases} \frac{K}{\alpha x^{\alpha-1}} & \text{if } p\cong 1 \\ x-(x^\alpha-K)^{1/\alpha} & \text{if } p\cong 0 \end{cases}$
(vi)	Die-for-sure	$\frac{(x^\alpha+pK)^{1/\alpha}-x}{p}$	$\begin{cases} (x^\alpha+K)^{1/\alpha}-x & \text{if } p\cong 1 \\ \frac{K}{\alpha x^{\alpha-1}} & \text{if } p\cong 0 \end{cases}$

To appreciate the significance of Table 1, consider some numbers. Empirical studies of willingness-to-pay value of life typically produce estimates in the \$1 to \$10 million range. Let us assume therefore that $VOL = \$1.0$ million. The current average annual wage in the United States is in the neighborhood of \$25,000; so let us assume $x = \$25,000$. Suppose the probability of survival for the individual who spends \$0 on precaution is 0.9, and the survival probability is asymptotic to 1.0 as $y \rightarrow +\infty$. Therefore, under assumption 4, $a = 1.0$ and $b = 0.1$. Finally, the α parameter is constrained to be between 0 and 1; let us assume a midpoint value of $\alpha = 1/2$. Then by equation (9) for VOL, we must have $K = 3162$.

At this point all the value of life measures (as well as all the other model variables) can easily be computed, and the results are shown in Table 2. (Note again that only $AVOL$, among the willingness-to-pay measures, requires the $p(\cdot)$ assumptions.)

Insert Table 2 Around Here.

Table 2

Calculated Values of Life

Assuming $A = 1,\ a = 1.0,\ b = 0.1,\ \alpha = 0.5,\ K = 3162,\ x = \$25,000.$

Approach		Measure	Value
Human Capital		x	\$25,000
Willingness-to-Pay	(i)	*AVOL*	\$3,162
	(ii)	*VOL*	\$1,000,000
	(iii)	Life-death ind.	\$11,000,000
	(iv)	Destitute l.-d. ind.	\$10,000,000
	(v)	Live-for-sure	\$996,800
	(vi)	Die-for-sure	\$10,996,800

To conclude this section, the model seems to lend support to the common empirical claim that marginal willingness-to-pay measures of value of life, such as VOL, are much larger than the human capital measure of the value of life. (See Jones-Lee (1989), and Fisher et al. (1989) for the empirical evidence, and Bailey (1978) and Bergstrom (1982) for related theory.)

However, it is more important to observe that the alternative willingness-to-pay measures, ranging from $AVOL$ to VOL to die-for-sure, *are vastly different from each other. In fact, the various willingness-to-pay measures differ by a factor of around 3500, between the lowest and the highest. The AVOL measure, which looks at what the subject has actually chosen to pay, in total, to enhance his survival chances, is an order of magnitude less than human capital. The marginal VOL measure is more than an order of magnitude greater than human capital. And the Broome-like measures are roughly 400 times greater than human capital. This huge variability of measures, in my view, is one reason to be very wary of willingness-to-pay measures of the value of life.*

Is it possible to consider six alternative value of life measures, differing by a factor of 3500, and remain unconcerned? Why not simply use the willingness-to-pay measure which is appropriate to the situation? One reason is that the multiplicity creates great confusion, with many scholars calling vastly different things "the value of life." The second is the possibility of logical paradox: One measure will find policy A superior to policy B, while another, equally plausible, will find B better than A. When an economic model implies A is superior to B, and B is superior to A, it may be time to search for a new model.

VI. How VOL Depends on K and A

In this section I return to the VOL willingness-to-pay measure of the value of life, without the $A = 1$ constraint of the previous section.

Consider the formulations for VOL given by equations (3) and (6):

$$VOL = \frac{1}{f'(x)} \left[f(x) - (Af(x+vz) - K) \right] \tag{3}$$

$$VOL = \frac{1}{\alpha x^{\alpha-1}} \left[x^{\alpha} \left(1 - A^{\frac{1}{1-\alpha}}\right) + K\right]. \tag{6}$$

Recall that equation (3) relies only on assumptions 1 and 2, whereas equation (6) also uses the exponential utility function specification of assumption 3.

Two obvious questions come to mind about VOL, the economists' favorite measure of value of life. First, how does VOL depend on K, the fear of death parameter? And second, how does VOL depend on A, the altruism parameter?

In general, with a fixed initial endowment $\overline{x}$, an individual will choose x, y, and z based on the parameters A, K, α, a and b. Normally, in order to calculate VOL's response to changes in K (or A), one would hold $\overline{x}$ fixed, and let x, y and z vary as the parameter K (or A) varies. However, it is much easier in this model to hold x fixed, and let $\overline{x}, y$ and z vary as A (or K) varies. Therefore, this is what I shall do for comparative statics. In the equations below, the derivatives hold x and the obvious parameters constant, and allow $\overline{x}, y$ and z to vary. I will also assume for this purposes of this discussion that $x > 0$.

First, how does the VOL measure vary with the altruism parameter A? From equation (3) we get

$$\frac{dVOL}{dA}|_x \text{ constant} = -\frac{1}{f'(x)} \left[f(x+vz) + Af'(x+vz)\frac{d}{dA}(x+vz)\right]. \tag{16}$$

Using equation (1) it is easy to show that

$$\frac{d(x+vz)}{dA}|_x \text{ constant} = -\frac{f'(x+vz)}{Af''(x+vz)}.$$

Based on the regularity assumptions made for $f(\cdot)$ (which imply that $f(x) > 0$, for $x > 0$; and $f'(x) > 0$ and $f''(x) < 0$, when $x \geq 0$), we can conclude that:

$$\frac{dVOL}{dA}|_x \text{ constant} \leq 0,$$

with the strict inequality holding if $x + vz > 0$. Note that this conclusion has not used the functional form assumption $f(x) = x^{\alpha}$.

With assumption 3, we can use equation (6) for VOL, and get the following:

$$\frac{dVOL}{dA}|_x \text{ constant} \; = \; -x \left(\frac{\alpha}{1-\alpha}\right) A^{\frac{\alpha}{1-\alpha}} \leq 0,$$

with the strict inequality holding if $A > 0$. In short, as altruism rises, the (marginal) willingness-to-pay value of life falls.

But it is worse than that. Not only does VOL fall as A rises, but it falls rather rapidly to zero. For example, with the parameters assumed for Table 2 $(A = 1, \alpha = 0.5, K = 3162)$, and with the assumed level for utility maximizing consumption $(x = \$25,000)$, we have $VOL = \$1,000,000$. Maintaining all of these values except A, and increasing A to 3.0, for instance, causes VOL to drop to \$600,000; increasing A to 4.0 causes VOL to drop to \$250,000; and VOL reaches \$0 at around $A = 4.58$. (To see what might be a plausible value for A, recall from equation (5) that the ratio of face value of insurance policy to consumption x should be equal to $A^{\frac{1}{1-\alpha}} - 1$. With $A = 4$, $\alpha = 0.5$, and $x = \$25,000$, an individual would have a \$375,000 insurance policy.)

In my view, this strongly negative relationship between A and VOL suggests that basing social policies on VOL might be unwise.

Second, how does VOL vary with the fear of death parameter K?

In the general case (without assumption 3, but with the regularity assumptions on $f(\cdot)$), we differentiate equation (3) to get

$$\frac{dVOL}{dK}|_x \text{ constant} \; = \; \frac{1}{f'(x)} \; > \; 0. \qquad (18)$$

With assumption 3, we differentiate equation (6) to get

$$\frac{dVOL}{dK}|_x \text{ constant} \; = \; \frac{1}{\alpha x^{\alpha-1}} \; > \; 0. \qquad (19)$$

In short, as fear of death rises, the (marginal) willingness-to-pay value of life measure rises.

This positive relationship between K and VOL is of course to be expected. After all, willingness-to-pay models of value of life get their impetus from the natural distaste for the dead state. But I find something philosophically unattractive about a theory that leans so much on fear of an unknown and unknowable outcome. There is *ex ante* disutility from the dead state but *ex post* there is no known disutility or utility. To paraphrase the ancient Greek philosopher Epicurus, we fear death awfully when we are alive, but when we fear it we *are alive*, and when we are dead we will not fear it any more.

VII. What Is the Correct Value of Life Measure?

The sections above suggest that, first, there are several plausible candidates for the willingness-to-pay measure of the value of life, and they differ enormously. This can lead to unfortunate confusion, and paradoxes of the Broome type. For a given individual, the willingness-to-pay value of life measure will greatly depend on definition chosen (e.g., marginal vs. average, willingness-to-pay vs. willingness-to-accept, probability increment vs. life/death question, and so on).

Second, the standard marginal willingness-to-pay value of life measure, or VOL, has some unpleasant characteristics, particularly its negative connection with altruism and its positive connection with fear of something unknown.

It seems to me that when empirical studies of what is called the "value of life" produce numbers ranging between £50,000 and £11,700,000 (Jones-Lee (1989)), and when a theoretical paper suggests that one individual with one set of parameters has a "value of life" ranging between \$3,162 and \$11,000,000 depending on what definition is used (Table 2 above), it is time to pause and question the model.

In my opinion, the willingness-to-pay approach should be put on a firmer foundation. A subject should not be asked to hypothetically evaluate death, which he does not know. A subject *should* be asked to evaluate additional years of life, which he *does* know. The model should not be about buying extra probability of survival, it should be about buying extra years of life. Uncertainty coupled with an unknowable state should be set aside, and replaced with certainty, coupled with choice of additional units of a known state.

I am suggesting the buying time approach to the value of life. The model below is based in part on models in Jones-Lee (1976), Kenyon and McCandless (1984), and Moore and Viscusi (1988); and most particularly on certainty models in Ehrlich and Chuma (1990), and Feldman (1995). In the simplest buying time model, there is no uncertainty and hence no insurance, the subject decides on consumption, precaution and bequest; and precaution expenditures, instead of increasing survival probability, increase the length of life. So the subject reveals his willingness-to-pay for years of life.

At this point, I will replace the notation and definitions of section II. I will use the following revised notation and definitions:

Lu	$=$	lifetime utility
x	$=$	instantaneous rate of spending on consumption, assumed constant over the lifetime
y	$=$	money spent by the subject on precaution, assumed spent in one lump sum at time zero
z	$=$	bequest, made in one lump sum at time of death
$\overline{x}$	$=$	$xT + y + z =$ cash endowment
A	$=$	the bequest parameter. $A = 0$ indicates the individual has no interest in the size of his bequest, whereas $A > 0$ indicates the individual gets utility from his bequest.
T	$=$	length of life.
$u(x)$	$=$	instantaneous utility function.

Assume for simplicity that the utility function $u(\cdot)$ is constant over the lifetime, and has the usual nice properties; assume further that the individual's discount rate is zero. To make the model easy to solve, suppose utility from the anticipated bequest z at death is $Au(z)$. (This is analogous to assumption 2 in section II.) Note that there is no life insurance; one simply sets money aside for a bequest.

Now the subject chooses $x, y,$ and z so as to maximize (certain) lifetime utility:

$$(20) \qquad Lu = \int_0^T u(x)dt + Au(z) = Tu(x) + Au(z).$$

There are two constraints. First is a more-or-less standard budget constraint. Consumption takes place at a rate x over a lifetime of length T. Precaution expenditure y is made at one moment, at time 0. The bequest z is made at one moment, the time of death. For simplicity, there is no discounting in the budget, just as there is no discounting in the utility function. Therefore, the budget constraint is:

$$\overline{x} = xT + y + z. \tag{21}$$

The second constraint is crucial. In the probabilistic model, the more that was spent on precaution, the greater was the survival probability in the next period. In this deterministic model, the more that is spent on precaution, the longer is the lifespan. That is:

$$T = \ell(y), \tag{22}$$

where $\ell(\cdot)$ is some positive and increasing function with reasonable regularity properties.

Recall that in the probabilistic model, the standard economic willingness-to-pay value of life was $VOL = \frac{1}{p'}$.

The analogous concept is the deterministic model is *value of time*, which I will abbreviate VOT. *It is the marginal increase in precaution expenditure per extra unit of length of life, for the lifetime utility-maximizing individual who is choosing* x, y *and* z. It is defined as:

$$VOT = \frac{1}{\ell'}. \tag{23}$$

It is easy to show that first-order conditions for an interior maximum for this model include the following equations:

$$u'(x) = A\, u'(z), \quad \text{and} \tag{24}$$

$$VOT = \frac{1}{\ell'} = \frac{u(x)}{u'(x)} - x. \tag{25}$$

Note the similarity between equation (24) and equation (1) of the probabilistic model, and note the simple formula for the value of time.

Now we can make an assumption similar to assumption 3 in the probabilistic model.

Assumption 5 (Exponential instantaneous utility)

$$u(x) = x^{\alpha}, \quad \text{for some } \ 0 < \alpha < 1.$$

When assumption 5 is incorporated in equation (25), the result is:

$$VOT = \left(\frac{1-\alpha}{\alpha}\right) x. \tag{26}$$

If it so happens that $\alpha = 0.5$ (as in Table 2 above), then the willingness-to-pay value of one extra year of life is exactly equal to the annual rate of expenditure on consumption!

The virtues of this model and this equation, compared to the probabilistic model and, say, equation (6) for VOL, are many. First, the problematic fear of death parameter disappears in the value of time model. Subject likes to live, and plans accordingly, but is aware that life ends, and that extending it costs money. He chooses his optimal lifespan, knowing that when the light goes out, it goes out. Second, for a given x, the value of time is not negatively related to altruism A. In fact, A does not appear in equation (26). Third, there aren't multiple definitions of value of time lying in wait, to be uncovered by Broome-style questions. Fourth and finally, the magnitude of VOT is comparable to what the human capital measure of life would be (that is, x or $\overline{x}$). You don't have the great gap that arises in the probabilistic models. The value of a lifetime by this theory is back in the ballpark of the legal theories, from Hammurabi's to our own.

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