Abstract

Past racial subordination resulting in significant inequality creates a "transition problem" of determining how to deal fairly with the legacy of an unjust history. I show that – in the presence of continued social segregation and when human capital spillovers within social networks are important – the consequences of past discrimination may persist indefinitely, absent some racially egalitarian intervention. I conclude that under such conditions "color-blindness" – i.e. official indifference to race in the formulation of public policies – is NOT an adequate response to this problem, and that "affirmative action" (i.e. policies whose explicit objective is to create more equal social outcomes between racial groups) is ethically justified.

Acknowledgement 1 This paper has been prepared for the session at the 2013 AEA annual meetings entitled: "Whither Affirmative Action?" It is a preliminary draft and should not be quoted or cited without the author’s permission. This paper draws heavily on my joint work with Sam Bowles and Rajiv Sethi (Bowles et al. forthcoming), though I am solely responsible for the analysis to follow, for the opinions expressed herein, and for any errors.

1 Introduction:

A history of racial/ethnic subordination resulting in significant economic inequality between identifiable social groups has left many societies facing what I will call a "transition problem." (Think of racial inequality in South Africa after the fall of Apartheid, or in the United States after the civil rights revolution; or, of caste disparities in India after Independence.) In the aftermath of reforms that sought to end overtly discriminatory practices against members of a disadvantaged population subgroup, these and other societies have had to figure out what kind of "transition policies" constitute a fair way to deal with the legacy of those immoral historical practices. In my view, critical assessment of the future of racial affirmative action in the US should acknowledge this problem and address it explicitly. That is my goal in this essay.

I argue in what follows that "color-blindness" – by which I mean official indifference to racial/ethnic identity in the formulation of policy – is NOT an adequate basis for reckoning with the transition problem, and that some kinds of affirmative actions are ethically justified, and may be ethically required. My argument is based on the fact that – due to continued social segregation and the importance of human capital spillovers within social networks – the consequences of historical discrimination can persist into the indefinite future, absent some racially egalitarian intervention. In this discussion I will take a broad view of "racial affirmative action", understanding it to encompass any policy whose explicit objective is to create more equal social outcomes between racial groups.
Of course, doubts about the propriety of racial affirmative action are not new. Such reservations have been voiced since the 1960s. Indeed, I can recall addressing this very topic as a young economist, thirty-two years ago (!), at the 1980 AEA Meetings (Loury, 1981.) There, though I did not then use the language of "transition problems," I nevertheless raised what I believe remains a critical question: Can something approximating equality between racial groups be expected from purely color-blind (i.e., non-racially discriminatory) policies and practices, given a history of race-based exclusion and social hierarchy? My central point, then and now, is that in general one must answer that question in the negative. There are limits to the ability of non-discrimination policy by itself to bring about genuinely equal opportunity between social groups. My book, *The Anatomy of Racial Inequality* (Loury 2002), develops this point by emphasizing the distinction between what I called there discrimination in contract and discrimination in contact: Equal Opportunity (EO) policies aim to prohibit racial “discrimination in contract” – that is, in the formal sectors of employment, credit, and housing markets, in government contracting, college admissions and the like. But, given the autonomy in the choice of social affiliations which individuals expect to enjoy in a free society, racial “discrimination in contact” – in the formation of friendship networks, households, business partnerships and professional ties, for instance – is not and cannot be reached by EO policies. And yet, because human development always and everywhere takes place within some social context, when network-mediated human capital spillovers are important "discrimination in contact" can cause racial disparities to persist into the indefinite future. This observation, it seemed to me in 1980 and still does now, has enormous implications for conceiving of what genuine “equality of opportunity” between racial/ethnic groups in a stratified society should entail. My principle claim is that if EO policies cannot be relied upon to eventually undo the inegalitarian consequences of a blatantly unjust history then, at least in principle, some kind of "affirmative actions" to promote that end are morally justified. I will explore a simple economic model to illustrate how racial segregation in social affiliation (i.e., ongoing discrimination in contact) bears on the legitimacy of racial affirmative action.

In the aftermath of the Civil Rights Movement, at mid-20th century, the US faced a classic "transition problem." The Supreme Court’s decision in *Brown v. Board of Education* (1954) struck down *de jure* racial segregation of public schools on the grounds that ‘separate educational facilities are inherently unequal’. But *de facto* segregation in the schools has persisted to this day. Moreover, as the data to be reviewed briefly below suggest, the "dream" – that the demise of legally enforced segregation and discrimination against blacks, coupled with an apparent reduction in racial prejudice among whites, would cause the significant social and economic disadvantage of African Americans to fade – has not been fulfilled. As I have previously emphasized (see, e.g., Loury 1995, 1998), one of the principle barriers to achieving greater racial equality in the post-civil rights era in the US is racial assimilation in social networks. The real economic opportunities of any individual depend not only on his own income, but also on the incomes of those with whom he is socially affiliated.
Such patterns of affiliation, in a society like the US, are not arbitrary but depend in part on race and ethnic identity. Given a history of open, widespread and severe racial discrimination, group differences in economic success may persist across generations without ongoing discrimination against the less affluent group because racial segregation of friendship networks, mentoring relationships, neighborhoods, workplaces and schools leaves the less affluent group at a disadvantage in acquiring the things – contacts, information, cognitive skills, behavioral attributes – that contribute to economic success.

Two philosophical approaches to assessing the legitimacy of racial affirmative action may be contrasted: (i) A procedural/rule-oriented approach: “Let us establish equal treatment without regard to race, allowing the chips to fall as they may”. This approach implicitly assumes either that convergence to something approximating equality will obtain in the absence of overt economic discrimination, or that persisting group disparity that originates in historical injustice is morally irrelevant. I maintain that neither assumption is tenable. Alternatively, there is: (ii) A substantive/group-redistributive approach: "Let’s redress racial inequality via direct group-egalitarian interventions like affirmative action policies, recognizing that this may be the only way to overcome historical inertia so as to achieve genuine equality over the long term." I claim that when social segregation is extensive and human capital spillovers are important, only the substantive/group redistributive approach is morally adequate.

2 Evidence of persistent racial inequality in contemporary US society

Briefly, here are some data on trends in Black/White socioeconomic disparities in the US over the forty-year period 1968-2008. My only point in presenting these data is to show that convergence to economic equality between “blacks” and “whites” in the US is not in sight. Thus, consider the trends in four-year college completion rates for men and women by race:

And, look at how the wage and salary earnings have varied by race and gender since the late 1960s:

Finally, consider how inequality in family incomes and childhood poverty rates between racial groups has evolved since the late 1960s. A substantial racial gap persists into the 21st century, showing no tendency to wither away:

So, African Americans continue to be significantly disadvantaged relative to whites in the US. This substantial disparity is a persistent feature of the American social scene. It is also the case that race and class-based segregation are pervasive features of the American social structure (see, e.g., Anderson 2010). Of course, nothing in my theoretical argument will be able to demonstrate a causal connection between these empirical phenomena. But it is not implausible to hypothesize such a linkage, and there is some evidence (e.g., Cutler and Glaeser 1996, among much else) to support that view. In any event, this is not the place to attempt such an empirical demonstration. Rather, in what
Percent of Native-Born, Non-Hispanic Men and Women Aged 25 to 34 Reporting a Four-Year College Education
follows I advance a simple theoretical model that illustrates how, in principle, the fact of persistence of racial group inequality can be related to the fact of social segregation by race in the US.

3 A simple model of persistent group inequality

Consider, then, the following dynamic model of inequality between social groups:¹ Imagine that a set of economic agents who belong to distinct racial groups are embedded in a social structure which affects the ease with which they can acquire skills. Specifically, these agents make choices about investing in human capital. Their choices affect their later-life economic outcomes, and the cost of an agent’s human capital investment varies with the quality of an agent’s social network. Specifically, I envision a society that exists over an infinite series of periods, \( t = 0, 1, 2, \ldots \). Overlapping generations of agents live for two periods, acquiring human capital (or not) when young, and earning a wage when old. A continuum of new agents of unit measure enters society at each date, belonging to one of two social groups: \( A \) or \( B \). (I will think of group \( B \) as being less advantaged.) A demographic parameter \( \beta \in (0, 1) \) gives the relative number of group \( B \) agents in each generation. There are two jobs – a high skilled \((H)\) and a low skilled \((L)\) position – which, when performed in conjunction, permit output to be produce. To keep things simple I assume all agents have the same economic abilities. Agents differ only with respect to

¹ The "exogenous-wage" model in this section borrows directly from Bowles et al. (forthcoming). The "endogenous-wage" model to follow is being presented here for the first time.
their social group membership (and, as will be made clear in a moment, in the resulting quality of their social networks.)

A costly investment in human capital while young equips a worker for the high-skilled job \((H)\). All workers are assumed able to do the low-skilled job \((L)\). Markets for human capital loans are assumed to be complete (so we ignore credit access issues.) In the first period of life either a discrete human capital investment is made, or not: that is, either \(h = 1\) or \(h = 0\). In the second period of life, agents inelastically supply one unit of labor, either \(H\) or \(L\) to the market, receiving the going wage. Competitive firms pay wages equal to marginal products. Initially I allow an individual’s marginal product to depend on his skill level, but not on the aggregate HC investment rate. Later I will relax this assumption, allowing for diminishing returns to the two factors, \(H\) and \(L\). The focus throughout is on how technology, demography and social structure determine the dynamics of group inequality.

Taking wages as exogenous this situation can be represented without further loss of generality as follows: The net gain from \(h = 0\) may be normalized at zero. And the net gain from \(h = 1\) may be denoted by the number \(-c \in \mathbb{R}\). So, the rational agent chooses \(h = 1\) \((h = 0)\) if and only if \(c < 0\) \((c > 0)\). Letting \(\sigma \in [0, 1]\) denote the fraction of high-skilled older individuals within a young agent’s social network, I posit that (due to social spillover effects) the cost of human capital to a young agent varies with the quality of his/her social network, such that: \(c = c(\sigma)\), where \(c'(\sigma) < 0\), and \(c(1) < 0 < c(0)\). Thus, there is coordination issue in that, if everyone in a young person’s social network is highly skilled then it pays for that agent to invest in human capital, while if nobody in a young person’s network is highly skilled then it does not pay to invest in human capital. Finally, in this very simple framework, denote by \(\sigma^*\) the critical quality (that is, fraction of older agents who are highly skilled) of a young agent’s social network such that human capital investment is a breakeven proposition: \(c(\sigma^*) = 0\). Clearly then, young agents choose \(h = 1\) \((h = 0)\) whenever \(\sigma > \sigma^*\) \((\sigma < \sigma^*)\).

I will now indicate more precisely how social networks are structured in this model society. Let the quality of an agent’s social network depends on group identity and the generation of birth. Specifically, an agent born at date \(t + 1\) is imagined to have a large number of social ties to generation \(t\) agents. Each of these ties is, with probability \(\eta \in [0, 1]\) drawn at random from the agent’s social group \((A\) or \(B)\), and is drawn at random from among the general population of agents without regard to group identity with probability \(1 - \eta\). Let \(x^t_i\) be the fraction of generation \(t\) agents in group \(i\) who are high skilled, and let \(\alpha^t_{i+1}\) denote the quality of the social network of a generation \(t + 1\) agent in group \(i\). It follows that:

\[
\alpha^t_{i+1} = \eta x^t_i + (1 - \eta)(1 - \beta)x^t_a + \beta x^t_b
\]

The parameter \(\eta\) represents the degree of in-group bias (homophily) in the society. Now, since \(\alpha^t_{a+1} - \alpha^t_{b+1} = \eta(x^t_a - x^t_b)\), it follows that generation \(t + 1\) agents from the two groups can enjoy social networks of the same quality only if \(\eta = 0\) (no in-group bias in social affiliations), or if \(x^t_a = x^t_b\) (no inequality
between the groups in the previous generation.) Otherwise, the young agents
in the less skilled group will have social networks of lower quality.

Given this specification, I conclude that the probability that an older associate of a younger group $A$ agent belongs to group $A$ equals

$$\alpha_1 \equiv \eta + (1 - \eta)(1 - \beta)$$

while the probability that an older associate of a younger group $B$ agent belongs to group $A$ equals

$$\alpha_0 \equiv (1 - \eta)(1 - \beta)$$

It is obvious (in this extremely simple model with exogenous wages) that $(x_a, x_b) = (1, 1)$ and $(x_a, x_b) = (0, 0)$ are stable symmetric steady states of this dynamical system, and that $(x_a, x_b) = (\sigma^*, \sigma^*)$ is an unstable symmetric steady state. Moreover, the asymmetric allocation $(x_a, x_b) = (1, 0)$ is a stable steady state if $\alpha_1 \geq \sigma^* \geq \alpha_0$. A bit of algebra demonstrates the following:

**Theorem 2** There exists a minimal degree of in-group bias in associational behavior, $\eta(\beta, \sigma^*)$ such that whenever $\eta > \eta(\beta, \sigma^*)$ then the initial condition of group inequality $(x_a^0, x_b^0) = (1, 0)$ is a stable steady state equilibrium. Moreover,

$$\eta(\beta, \sigma^*) \equiv \text{Max}\{1 - \frac{\sigma^*}{1 - \beta}, 1 - \frac{1 - \sigma^*}{\beta}\}$$

Furthermore, when $\eta < \eta(\beta, \sigma^*)$ the system converges, from the initial state $(x_a^0, x_b^0) = (1, 0)$, in one period, to a stable steady state with group equality. This steady state is Pareto superior to the initial state $[(x_a^1, x_b^1) = (1, 1)]$ if the initially disadvantaged group is not too big (i.e., if $\beta < 1 - \sigma^*$), and is Pareto inferior to the initial state $[(x_a^1, x_b^1) = (0, 0)]$ when the initially disadvantaged group is sufficiently large (i.e., if $\beta > 1 - \sigma^*$.)

4 **Endogenous wages**

The foregoing analysis illustrates how human capital spillovers combined with social segregation can produce a situation where "equal opportunity is not enough" to deal adequately with the "transition problem." This happens when the impact of human capital spillovers is strong enough $[c(0) < 0 < c(1)]$, and the extent of social segregation great enough $[\eta > \eta(\beta, \sigma^*)]$. But this simple demonstration is limited by the strong assumption that the return to becoming skilled is fixed, regardless of the skill intensity of the workforce. I now show that this qualitative result, and the associated intuition, carry over when the assumption of exogenous wages is relaxed.

As before, let there be a continuum of workers of unit measure who enter the model at each date and who live for two periods. For the sake of simplicity, continue to assume that individuals do not differ with respect to economic abil-
ity,\(^2\) and that demography \((\beta)\) and social structure \((\eta)\) determine the human capital spillovers that affect the cost of skill-acquisition by a young agent just as before. But I now allow wages to vary with relative factor supplies. Specifically, and with no further loss of generality, I will normalize the net return to a low skilled agent \((h = 0)\) at zero, and posit that the gross return to a high skilled agent \((h = 1)\) is given by \(w(x) > 0\), where \(x\) is the share of all workers with high skill. I will assume, not unreasonably, that \(c(\sigma)\) is a strictly decreasing, convex function [i.e., \(c'(\sigma) < 0\), and \(c''(\sigma) > 0\)]; and, that \(w(x)\) is a strictly decreasing concave function [i.e., \(w'(x) < 0\), and \(w''(x) < 0\)].\(^3\) Finally, to rule out corner solutions (which needlessly complicate the analysis without adding anything to the point I am trying to make), I further assume that \(w(0) > c(0)\) and \(w(1) < c(1)\). These functions are depicted in the diagram above.

As before, agents make a human capital investment choice when young, and earn wages when old. Their investment costs depend on the quality of their social networks. The benefit of acquiring human capital is the (correctly anticipated) wage premium to highly skilled workers that obtains in the next period. Given homogeneous abilities, either all agents in a group wish to become highly skilled, none do, or they are all indifferent. So, to understand how this model society will evolve from an historically given initial condition of group inequality in the quality of young agents’ social networks, \((\sigma^0_a, \sigma^0_b)\) with \(\sigma^0_a > \sigma^0_b\),

\(^2\) The assumption of homogeneous ability at the individual level is not critical for the result. What matters for my argument, if ability is taken to be variable in the population, is that the distribution of endowed aptitudes be the same for the both groups. This is what Loury (2002) calls the Axiom of Anti-Essentialism.

\(^3\)Convexity of \(c(\sigma)\) amounts to assuming diminishing marginal returns to network quality. And, concavity of \(w(x)\) follows when workers are paid their marginal products if high and low skilled labor are complements in production.
we need to consider three cases, based on which group is on the margin of human capital acquisition. Accordingly, define the critical network quality \( \sigma^* \) by the requirement:

\[
 w(1 - \beta) = c(\sigma^*).
\]

(If all group \( A \) and no group \( B \) agents invest in human capital, then the wage premium for highly skilled workers in the next period will be \( w(1 - \beta) \).)

Consider first the case in which \( \sigma^+ > \sigma_a^0 > \sigma_b^0 \). In this case, first period labor market equilibrium must be such that some group \( A \) agents and no group \( B \) agents invest in human capital:

Second, there is the case in which \( \sigma_a^0 \geq \sigma^+ \geq \sigma_b^0 \). Now first period labor market equilibrium entails all group \( A \) agents and no group \( B \) agents choosing \( h = 1 \):

Finally, there is the case in which \( \sigma_a^0 > \sigma_b^0 > \sigma^+ \), where first period labor market equilibrium has all group \( A \) agents and some group \( B \) agents becoming high skilled workers:

From simple perusal of these three mutually exclusive and collectively exhaustive cases it is readily seen that, if \( (\sigma_a^t, \sigma_b^t) \) denotes the quality of social influences on human capital acquisition for young agents in groups \( A \) and \( B \) respectively in any period \( t \), then labor market equilibrium for these agents when they become workers in the next period, \( (x_a^{t+1}, x_b^{t+1}) \), must satisfy:

\[
 x_a^{t+1} = \min \{ 1; \frac{\phi(\sigma_a^t)}{1 - \beta} \}, \text{ and }
\]

\[
 x_b^{t+1} = \max \{ 0; \frac{\phi(\sigma_b^t) - (1 - \beta)}{\beta} \}
\]
Case 2: $\sigma_0 \leq \sigma_a$; All A's and no B's in high-skilled positions

\[ x_a = 1; \quad x_b = 0 \]

Case 3: $\sigma_a > \sigma_0 > \sigma^*$; B's on margin; all A's employed in high-skilled positions

\[ x_a = 1; x_b = \frac{w^{-1}(c(\sigma_b)) - (1 - \beta)}{\beta} \]
where the function \( \phi(\cdot) \) is such that:

\[
\phi(\sigma) \equiv w^{-1}(c(\sigma)), \quad \sigma \in [0, 1]; \quad \text{and} \quad \phi(\sigma^*) \equiv 1 - \beta
\]

The assumptions that \( w(0) > c(0) \) and \( w(1) < c(1) \) guarantee \( \phi(0) > 0 \) and \( \phi(1) < 1 \). Moreover, a bit of calculus shows that \( \phi(\sigma) \) is an increasing, concave function if \( w(x) \) is decreasing/convex and \( c(x) \) decreasing/concave, as we have assumed.\(^4\) Therefore, there exist a unique point \( x^* \in (0, 1) \) for which \( x^* = \phi(x^*) \). This is the skill intensity which obtains in the model’s unique, group-symmetric steady state [where \( w(x^*) = c(x^*) \)].

Now, given these definitions and assumptions, define the set \( \Sigma \subset [0, 1]^2 \) as follows:

\[
\Sigma \equiv \{(\sigma_a, \sigma_b) = (x_a, x_b) : \begin{bmatrix} \alpha_1 & \alpha_0 \\ 1 - \alpha_1 & 1 - \alpha_0 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} \mid 0 \leq x_b \leq x_a \leq 1\}.
\]

and consider the mapping \( \Psi : \Sigma \rightarrow \Sigma \) given by:

\[
\Psi(\sigma_a, \sigma_b) \equiv \left( \min\{1; \frac{\phi(\sigma_a)}{1 - \beta}\}, \max\{0; \frac{\phi(\sigma_b) - (1 - \beta)}{\beta}\} \right) \cdot \begin{bmatrix} \alpha_1 & \alpha_0 \\ 1 - \alpha_1 & 1 - \alpha_0 \end{bmatrix}
\]

The idea here is that, if \( (\sigma_a^t, \sigma_b^t) \) are given as the quality of social networks for generation \( t \) agents in the two groups, and if labor market equilibrium obtains, then generation \( t + 1 \) workers will have social network qualities \( (\sigma_a^{t+1}, \sigma_b^{t+1}) \) such that:

\[
(\sigma_a^{t+1}, \sigma_b^{t+1}) = \Psi(\sigma_a^t, \sigma_b^t)
\]

with the function \( \Psi \) being as given above. So, the steady states of this simple dynamical system are the fixed points of \( \Psi \). As noted, the unique symmetric steady state (i.e., one with no group inequality) must have the property that:

\[
\sigma_a^\infty = \sigma_b^\infty = x^* \quad \text{for} \quad x^* \quad \text{the unique fixed point of} \quad \phi: \quad x^* = \phi(x^*)
\]

However, there are also asymmetric steady states. In what follows I will show that when racial segregation of social networks is sufficiently strong (i.e., when \( \eta \) is big enough), this unique symmetric steady state is unstable, and a unique asymmetric steady state favoring group \( A \) exists which is reached from any initial position where the group \( A \) workforce is more highly skilled than group \( B \). In this way, I will show that the qualitative behavior of this system with endogenous wages mirrors the earlier finding about the significance of social segregation for the persistence of group inequality.

\(^4\)Thus \( \phi(\sigma) \equiv w^{-1}(c(\sigma)) \), so

\[
\phi'(\sigma) = \frac{c'(\sigma)}{w'(\phi(\sigma))} > 0
\]

and

\[
\phi''(\sigma) = \frac{c''(\sigma)}{w'(\phi(\sigma))} - \frac{c'(\sigma)w''(\phi(\sigma))\phi'(\sigma)}{[w'(\phi(\sigma))]^2} < 0
\]
Consider, then, the diagram above which can be used to illustrate the dynamics. Think of $\Sigma$ as the state space of the implied dynamical system. It can be partitioned into three mutually exclusive and collectively exhaustive subsets, $\Sigma_i$, $\Sigma_{ii}$, and $\Sigma_{iii}$, and we can then study the motion of the system on each of these subsets. As should be obvious, $(t_a; t_b) \in \Sigma_i$ corresponds to our case 1 previously mentioned; while $(t_a; t_b) \in \Sigma_{ii}$ corresponds to case 2; and $(t_a; t_b) \in \Sigma_{iii}$ corresponds to case 3. So the mapping $\Psi$ may be written as follows:

\[
(\sigma_a^t, \sigma_b^t) \in \Sigma_i \Rightarrow \Psi(\sigma_a^t, \sigma_b^t) = \left( \frac{\phi(\sigma_b^t)}{1-\beta} \right) \cdot (\alpha_1, \alpha_0) \text{ with } \frac{\phi(\sigma_b^t)}{1-\beta} < 1;
\]

\[
(\sigma_a^t, \sigma_b^t) \in \Sigma_{ii} \Rightarrow \Psi(\sigma_a^t, \sigma_b^t) = (\alpha_1, \alpha_0); \text{ and}
\]

\[
(\sigma_a^t, \sigma_b^t) \in \Sigma_{iii} \Rightarrow \Psi(\sigma_a^t, \sigma_b^t) = (1, 1) - \left( \frac{1 - \phi(\sigma_b^t)}{\beta} \right) \cdot (1 - \alpha_1, 1 - \alpha_0) \text{ with } \frac{1 - \phi(\sigma_b^t)}{\beta} < 1.
\]

(Recall $\alpha_0 = (1-\beta)(1-\eta)$ is the probability that a member of a group B agent’s social network belongs to group A; and $\alpha_1 = \eta + (1-\beta)(1-\eta)$ is the probability that a member of a group A agent’s social network belongs to group A.)

Now, it is easy to see that if $(\alpha_1, \alpha_0) \in \Sigma_{ii}$ [that is, if case 2 obtains when $(\sigma_a^0, \sigma_b^0) = (\alpha_1, \alpha_0)$] then the initial condition of stark group inequality $(x_a^0, x_b^0) = (1, 0)$ will persist as a steady state. Moreover, we have the following Lemma:

**Lemma 3** Suppose that $(\alpha_1, \alpha_0) \in \text{Interior}(\Sigma_{ii})$. Then the implied dynamical system describing group inequality in this model society converges to $(\sigma_a^\infty, \sigma_b^\infty) = (\alpha_1, \alpha_0)$ and $(x_a^\infty, x_b^\infty) = (1, 0)$ from arbitrary initial conditions $(\sigma_a^0, \sigma_b^0)$ with $\sigma_a^0 > \sigma_b^0$, in a finite number of periods.

**Proof.** The claim is obvious for initial condition $(\sigma_a^0, \sigma_b^0) \in \Sigma_i$. Suppose then that $(\sigma_a^0, \sigma_b^0) \in \Sigma_i$. Then $(\sigma_a^1, \sigma_b^1) = \Psi(\sigma_a^0, \sigma_b^0)$ lies on the line segment
connecting the origin to the point \((\alpha_0, \alpha_1) \in \Sigma_{ii}\). Thus, either \((\sigma_a^1, \sigma_b^1) \in \Sigma_{ii}\), and the claim follows; or, \((\sigma_a^1, \sigma_b^1) \in \Sigma_i\) in which case, given our assumptions, \(\sigma_a^1 > \sigma_a^0\).\(^5\) Iterating we see that, moving away from the origin along the line segment connecting the origin to \((0,1)\), eventually a date \(t = 2\) will be reached when \(\sigma_a^t\) exceeds \(\sigma^*\), at which point the state of the system passes into the region \(\Sigma_{ii}\) and comes to rest in the next period at \((0,1)\), as claimed. Moreover, an identical argument establishes that if \((0,0) \in \Sigma_{iii}\) then \((1,1) = \Psi(\sigma_a^0, \sigma_b^0)\) lies on the line segment connecting the point \((1,1)\) to the point \((\alpha_0, \alpha_1) \in \Sigma_{ii}\), with \(\sigma_b^1 < \sigma_b^0\); and that, iterating, eventually \(\sigma_b^1\) must fall short of \(\sigma^*\), at which point the state of the system passes into the region \(\Sigma_{ii}\) and comes to rest in the next period at \((\alpha_0, \alpha_1)\), as claimed. \(\blacksquare\)

Thus, much depends on whether \((\alpha_1, \alpha_0) \in \text{Interior}(\Sigma_{ii})\) – that is, on whether \(\alpha_0 < \sigma^* < \alpha_1\). This is determined entirely by the values of the demographic and social structure parameters \(\beta\) and \(\eta\). To see how, consider the diagrams below. Inspection of these diagrams makes clear that \((\alpha_1, \alpha_0) \in \text{Interior}(\Sigma_{ii})\) if and only if

\[
\begin{align*}
B_{\min}(\eta) &< 1 - \beta < B_{\max}(\eta) \quad \text{where} \\
B_{\min}(\eta) &\text{ is defined by: } B_{\min} = \phi((1 - \eta)B_{\min}) \quad \text{while} \\
B_{\max}(\eta) &\text{ is defined by: } B_{\max} = \phi(\eta + (1 - \eta)B_{\max})
\end{align*}
\]

Now, as \(\eta\) rises from 0 to 1 it is clear that \(B_{\max}(\eta)\) rises from \(x^*\) to \(\phi(1)\), and \(B_{\min}(\eta)\) falls from \(x^*\) to \(\phi(0)\). These relations may be depicted as follows:

\(^5\)To see that \(\sigma_a^1\) must be greater than \(\sigma_a^0\) on \(\Sigma_{ii}\) recall that:

\[\sigma_a^1 = \left(\frac{\alpha_1}{1 - \beta}\right)\phi(\sigma_a^0).\]

The RHS above is a concave function of \(\sigma_a^0\) under our assumptions, which exceeds zero when \(\sigma_a^0 = 0\), and which exceeds \(\sigma^*\) when \(\sigma_a^0 = \sigma^*\) [because \(\phi(\sigma^*) = 1 - \beta\), and \(\alpha_1 > \sigma^* > \alpha_0\) given the hypothesis of the Lemma.] Therefore, the RHS must exceed \(\sigma_a^0\) everywhere on the interval \([0, \sigma^*)\).
\[ \alpha_0 \leq \sigma^* \leq \alpha_1 \text{ if and only if } B_{\min}(\eta) \leq 1-\beta \leq B_{\min}(\eta) \]
\(x^* = \phi(x^*)\) denotes the unique symmetric steady state skill intensity of the generalized model with endogenous wages, guaranteed to exist given our assumptions.

One may conclude from inspection of the diagrams above that the previous Theorem generalizes as follows:

**Theorem 4** In the model with endogenous wages, given our assumptions on \(c(x)\) and \(w(x)\), for every demographic parameter \(\beta \in (1 - \phi(1), 1 - \phi(0))\) there exists a minimal degree of in-group bias in associational behavior, \(\eta(\beta) \in (0, 1)\), such that whenever \(\eta \geq \eta(\beta)\) then the initial condition of group inequality \((x_0^a, x_0^b) = (1, 0)\) is a locally stable steady state equilibrium. Moreover, when \(\eta > \eta(\beta)\) the dynamical system converges, from any unequal initial state \(1 \geq x_0^a > x_0^b \geq 0\), in a finite number of periods, to the steady state \((x^\infty_a, x^\infty_b) = (1, 0)\) – that is, to a condition of stark and persistent group inequality.

\[
5 \quad \text{Conclusion}
\]

Given the importance of the division of labor in modern economies, not all positions will yield the same remuneration. Technology and markets will generate a compensation hierarchy. But, absent social segregation and intergenerational human capital spillovers, this need not imply any long-term correlation between racial/ethnic identity and social positions, so long as all agents experience equal opportunity in the labor market. However what I have shown, in the context of a simple model, is that in the presence of spillovers and segregated social networks, equal opportunity need not be sufficient to resolve an historically generated "transition problem," even asymptotically. As has been emphasized as long ago as Loury (1977), segregation matters for assessing the sufficiency of a procedural approach to racial justice. My analysis suggests that in a segregated society with a history of racial oppression, the morality of color-blindness may be quite superficial because with sufficient social segregation historically engendered inequality between social groups can persist under color-blindness, and new group inequality can emerge from nearly group egalitarian structures. My model highlights the role that demography (relative size of the disadvantaged group) and social structure (relative strength of in-group bias in social affiliations) play in this process. By showing that group inequality in the wake of historic injustice can persist with no fundamental ability differences between individuals or groups and with no ongoing economic discrimination, I have sought to provide a principled defense of the legitimacy of some kinds of affirmative action policies intended to narrow economic disparity between racial groups in the US,
References


