

# GROUP INEQUALITY

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## Abstract

We explore the combined effect of segregation in social networks, peer effects, and the relative size of a historically disadvantaged group on the incentives to invest in market-rewarded skills and the dynamics of inequality between social groups. We identify conditions under which group inequality will persist in the absence of differences in ability, credit constraints, or labor market discrimination. Under these conditions, group inequality may be amplified even if initial group differences are negligible. Increases in social integration may destabilize an unequal state and make group equality possible, but the distributional and human capital effects of this depend on the demographic composition of the population. When the size of the initially disadvantaged group is sufficiently small, integration can lower the long-run costs of human capital investment in both groups and result in an increase the aggregate skill share. In contrast, when the initially disadvantaged group is large, integration can induce a fall in the aggregate skill share as the costs of human capital investment rise in both groups. We consider applications to concrete cases and policy implications. (JEL: D31, Z13, J71)

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## 1. Introduction

Technologically modern societies are characterized by a broad range of occupations, some of which require years of costly investment in the development of expertise, while others need only minimal levels of training. Since investments in human capital must be adequately compensated in market equilibrium, the persistence of substantial

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*The editor in charge of this paper was George Marios Angeletos.*

Acknowledgments: This material is based upon work supported by the National Science Foundation under Grant No. SES-0133483, the Russell Sage Foundation, the Behavioral Sciences Program at the Santa Fe Institute, and the Institute for Advanced Study. We thank Young Chul Kim, Dan O'Flaherty, Arthur Robson, John Roemer, and Rohini Somanathan for their comments, and are especially grateful to two anonymous referees and the editor, George-Marios Angeletos, for some very valuable suggestions.

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earnings disparities is a necessary consequence of a modern production structure. What technology does *not* imply, however, is that members of particular social groups (identified, for instance, by race or religion) must be concentrated at different points in the income distribution. In fact, such concentration is widespread, persistent, and arises in societies with widely varying histories and commitments to equal opportunity.

In this paper we contribute to the understanding of this phenomenon by investigating the combined effect of segregation in social networks, peer effects, and the relative size of a historically disadvantaged group on the incentives to invest in market-rewarded skills and hence on the dynamics of group inequality. We do so on the basis of a model with the following features. All individuals belong to one of two social groups, parents invest in the human capital of their children, and generations are overlapping. There are two occupational categories, one of which requires a costly investment in human capital while the other does not. The investment cost depends both on individual ability and on the level of human capital in one's social network. There are no credit constraints and investments are perfectly observable. Wages in each period are determined under competitive conditions by the overall distribution of human capital in the economy, and investment decisions are based on anticipated wages. There is equal opportunity in the labor market, so wages depend only on one's investment and not on one's group identity. Ability is identically distributed across groups.

Within this framework, we establish two main results. First, we identify conditions under which there exists no stable steady state with equality across groups. In this case, small initial group differences will be amplified over time, resulting in a magnified correlation between earnings and identity even if negligible correlation exists to begin with. These conditions depend on three factors: the extent of segregation in social networks, the strength of interpersonal spillovers in human capital accumulation, and the level of complementarity between high- and low-skill labor in the process of production. In particular, social segregation plays a critical role: group inequality cannot emerge or persist under conditions of equal opportunity unless segregation is sufficiently great. Furthermore, the relationship between group equality and social segregation is characterized by a discontinuity: for any steady state with group equality, there exists a critical level of segregation at which it becomes unstable. If segregation lies above this threshold, convergence over time to the steady state is possible only if groups are equal to begin with. Hence a small increase in social integration that takes the economy across the threshold may have large effects on long-run group inequality, while a large increase in integration that does not cross the threshold may have no persistent effect.

Building on this analysis, we consider the case of multiple symmetric steady states (each of which entails group equality). This raises the question of which steady state is selected when integration induces a transition to group equality. Here we find that the population share of the initially disadvantaged group plays a critical role. If this share is sufficiently small, integration can result not only in the equalization of income distributions across groups, but also a decrease in the costs of human capital

investment in *both* groups and a rise in the aggregate skill share in the economy. Under these conditions integration might be expected to have widespread popular support. On the other hand, if the population share of the initially disadvantaged group is sufficiently large, integration can give rise to an increase in the costs of human capital investment in both groups and a decline in aggregate human capital levels in the long run. To the extent that this consequence is anticipated, integration may therefore face widespread popular resistance.

The economics literature has identified two main channels through which group inequality may be sustained across generations. The first is based on discrimination, either through tastes, as in Becker (1957), or incomplete information about individual productivity as in the theory of statistical discrimination (Arrow 1973, Phelps 1972).<sup>1</sup> The second channel, initiated by Loury (1977), stresses the effects of segregation in sustaining group inequality. Loury's model contains many of the ingredients that we consider here, including identity groups, peer effects, and the endogenous determination of wages; he establishes that convergence to group equality occurs if there is complete integration, but need not do so under complete segregation. Significant contributions building on this work include Durlauf (1996) and Benabou (1996). Both Durlauf and Benabou consider local complementarities in human capital investment coupled with endogenous sorting across locations and show how persistent disparities across neighborhoods can arise. Along similar lines, Lundberg and Startz (1998) model social groups as essentially distinct economies, except for the possibility that the human capital of the majority group has a spillover effect on the minority; the size of this effect is interpreted as the level of integration. They show that the income levels of the two groups can diverge persistently when segregation is complete. Finally, Mookherjee, Napel, and Ray (2010a, 2010b) consider local spillovers with global complementarities, and characterize the existence of steady states with inequality across groups, where groups correspond to spatially contiguous clusters of individuals who make similar levels of investment in skills.<sup>2</sup>

Since we assume that human capital investments are perfectly observable, there is no scope for statistical discrimination in our model. Hence our work is a contribution to the strand of literature linking segregation to persistent inequality in the absence of discrimination. Our exploration of the role of elite size in mediating the relationship between segregation and group inequality constitutes the principal novel contribution of this paper. Considering the effects of demographic composition is important for at least two reasons. First, the pursuit of similar policies with respect to integration across a broad range of urban areas is likely to have widely varying effects, since population shares are highly variable across cities. And second, the timing of such policies matters a great deal when the demographic structure of a society is changing. We return to both these points in what follows.

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1. See also Coate and Loury (1993), Antonovics (2002), Moro and Norman (2004), and Chaudhuri and Sethi (2008) for discrimination-based analyses of group inequality.

2. There is also a large literature on the intergenerational dynamics of inequality in which neither discrimination nor segregation plays a central role (Becker and Tomes 1979; Loury 1981; Banerjee and Newman 1993; Galor and Zeira 1993; Mookherjee and Ray 2003).

## 2. The Model

Consider a society that exists over an infinite sequence of generations and at any date  $t = 0, 1, \dots$  consists of a continuum of workers of unit measure. The workers live for two periods, acquiring human capital in the first period of life and working for wages in the second. The generations overlap, so that each young worker (i.e. the child) is attached to an older worker (the parent). For convenience, we assume that each worker has only one child. There are two occupations, of which one requires skills while the other may be performed by unskilled workers. Total output in period  $t$  is given by the production function  $f(h_t, l_t)$ , where  $h_t$  is the proportion of workers assigned to high-skill jobs, and  $l_t = 1 - h_t$ . Only workers who have invested in human capital can be assigned to high skill jobs, so  $h_t \leq s_t$ , where  $s_t$  is the proportion of the population that is qualified to perform skilled jobs at date  $t$ . The production function satisfies constant returns to scale, diminishing marginal returns to each factor, and the conditions  $\lim_{h \rightarrow 0} f_1 = \lim_{h \rightarrow 1} f_2 = \infty$ . Given these assumptions the marginal product of high (low) skill workers is strictly decreasing (increasing) in  $h_t$ . Let  $\tilde{h}$  denote the value of  $h$  at which the two marginal products are equal. Since qualified workers can be assigned to either occupation, we must have  $h_t = \min\{s_t, \tilde{h}\}$ . Wages earned by high- and low-skill workers are equal to their respective marginal products, and are denoted  $w_h(s_t)$  and  $w_l(s_t)$  respectively. The wage differential  $\delta(s_t) = w_h(s_t) - w_l(s_t)$  is positive and decreasing in  $s_t$  provided that  $s_t < \tilde{h}$ , and satisfies  $\lim_{s \rightarrow 0} \delta(s) = \infty$ . Furthermore,  $\delta(s) = 0$  for all  $s \geq \tilde{h}$ . Since investment in human capital is costly,  $s_t \geq \tilde{h}$  will never occur along an equilibrium path.

The set of workers consists of two disjoint identity groups, labeled 1 and 2, having population shares  $\beta$  and  $1 - \beta$  respectively. Let  $s_{1t}$  and  $s_{2t}$  denote the two within-group (high) skill shares at date  $t$ . The mean skill share in the overall population is then

$$s_t = \beta s_{1t} + (1 - \beta) s_{2t}. \quad (1)$$

The costs of skill acquisition are subject to human capital spillovers and depend on the skill level among one's set of social affiliates. These costs may therefore differ across groups if the within-group skill shares differ, and if there is some degree of segregation in social contact. As in Chaudhuri and Sethi (2008), suppose that for each individual, a proportion  $\eta$  of social affiliates is drawn from the group to which he belongs, while the remaining  $(1 - \eta)$  are randomly drawn from the overall population.<sup>3</sup> Then a proportion  $\eta + (1 - \eta)\beta$  of a group-1 individual's social affiliates will also be in group 1, while a proportion  $\eta + (1 - \eta)(1 - \beta)$  of a group-2 individual's affiliates will be in group 2.

3. Note that this specification does not require both groups to have the same preferences over the composition of their social networks. The observed level of segregation is the outcome of both sets of preferences (one group cannot become more segregated without the other also becoming more segregated) and the parameter  $\eta$  should be interpreted as a reduced-form measure that results from both sets of preferences without requiring that these be identical. Endogenizing the level of segregation, for instance along the lines of Currarini, Jackson, and Pin (2009), would be a worthwhile direction for future research but is beyond the scope of the present paper.

The parameter  $\eta$  is sometimes referred to as the *correlation ratio* (Massey and Denton 1988). In the Texas schools studied by Hanushek, Kain, and Rivkin (2009), for example, 39% of black third-grade students' classmates were black while 9% of white students' classmates were black. Thus, if schoolmates were the only relevant affiliates,  $\eta$  would be 0.3. The relevant social network depends on the question under study; for the acquisition of human capital, parents and (to a lesser extent) siblings and other relatives are among the strongest influences. Because family members are most often of the same group, the social networks relevant to our model may be very highly segregated.

Let  $\sigma_{it}$  denote the mean level of human capital in the social network of an individual belonging to group  $i \in \{1, 2\}$  at time  $t$ . This depends on the levels of human capital in each of the two groups, as well as the extent of segregation  $\eta$  as follows:

$$\sigma_{it} = \eta s_{it} + (1 - \eta) s_t. \quad (2)$$

In a perfectly integrated society, the mean level of human capital in one's social network would simply equal  $s_t$  on average, regardless of one's own group membership. When networks are characterized by some degree of positive assortment, however, the mean level of human capital in the social network of an individual belonging to group  $i$  will lie somewhere between one's own-group skill share and that of the population at large. Except in the case of perfect integration ( $\eta = 0$ ),  $\sigma_{1t}$  and  $\sigma_{2t}$  will differ as long as  $s_{1t}$  and  $s_{2t}$  differ.

The costs of acquiring skills depend on one's ability, as well as the mean human capital within one's social network. By "ability" we do not simply mean cognitive measures of learning capacity, but rather any personal characteristic of the individual affecting the costs of acquiring human capital, including such things as the tolerance for classroom discipline or the anxiety one may experience in school. The distribution of ability is assumed to be the same in the two groups, consistent with Loury's (2002) axiom of anti-essentialism. Hence, any differences across groups in economic behavior or outcomes arise endogenously in the model and cannot be traced back to any differences in fundamentals. The (common) distribution of ability is given by the distribution function  $G(a)$ , with support  $[0, \infty)$ . Let  $c(a, \sigma)$  denote the costs of acquiring human capital, where  $c$  is nonnegative and bounded, strictly decreasing in both arguments, and satisfies  $\lim_{a \rightarrow \infty} c(a, \sigma) = 0$  for all  $\sigma \in [0, 1]$ .<sup>4</sup>

The benefit of human capital accumulation is simply the wage differential  $\delta(s_t)$ , which is identical across groups. That is, there is no unequal treatment of groups in the labor market. Individuals acquire human capital if the cost of doing so is less than the

4. We are following Loury (1977) and Benabou (1993) in our assumptions about production structure (which satisfies global complementarity between high- and low-skill labor) and the costs of human capital investment (which satisfies positive spillovers from high human capital at the local level). All individuals are ex-ante identical in Benabou's model, so expected payoffs are the same; this need not be the case here. Our treatment of integration is also different; we treat this as parametric and continuously varying while it is endogenous and maximal in Benabou (1993). Benabou (1996), which we discuss further in what follows, considers exogenous groups and welfare differentials. For a parallel analysis of our model under the assumption of homogeneous ability, see Loury (2013).

wage differential. Note that the costs are incurred by parents while the benefits accrue at a later date to their children. Hence we are assuming that parents fully internalize the preferences of their children, do not discount the future, and are not credit constrained.

Given our assumptions, the skill shares  $s_{it}$  in period  $t$  are determined by the investment choices made in the previous period, which in turn depend on the social network human capital  $\sigma_{it-1}$  in the two groups, as well as the anticipated future wage differential  $\delta(s_t)$ . Specifically, for each group  $i$  in period  $t - 1$ , there is some threshold ability level  $\tilde{a}(\delta(s_t), \sigma_{it-1})$  such that those with ability above this threshold invest in human capital and those below do not. This threshold is defined implicitly as the value of  $\tilde{a}$  that satisfies

$$c(\tilde{a}, \sigma_{it-1}) = \delta(s_t). \quad (3)$$

Note that  $\tilde{a}(\delta(s_t), \sigma_{it-1})$  is decreasing in both arguments. Individuals acquire skills at lower ability thresholds if they expect a greater wage differential, or if their social networks are richer in human capital. It is also clear from (2) and (3) that for given levels of human capital investment in the two groups, increased segregation raises the costs of the disadvantaged group and lowers the costs of the advantaged group. The share of each group  $i$  that is skilled in period  $t$  is simply the fraction of the group that has ability greater than  $\tilde{a}(\delta(s_t), \sigma_{it-1})$ . Thus we obtain the following dynamics:

$$s_{it} = H(s_{it}, \sigma_{it-1}) := 1 - G(\tilde{a}(\delta(s_t), \sigma_{it-1})), \quad (4)$$

for each  $i \in \{1, 2\}$ . Given an initial state  $(s_{10}, s_{20})$ , a *competitive equilibrium path* is a sequence of skill shares  $\{(s_{1t}, s_{2t})\}_{t=1}^{\infty}$  that satisfies equations (1)–(4).

The following result rules out the possibility that there may be multiple equilibrium paths originating at a given initial state (all proofs are collected in the Appendix).

**PROPOSITION 1.** *Given any initial state  $(s_{10}, s_{20}) \in [0, 1]^2$ , there exists a unique competitive equilibrium path  $\{(s_{1t}, s_{2t})\}_{t=1}^{\infty}$ . Furthermore, if  $s_{10} \leq s_{20}$ , then  $s_{1t} \leq s_{2t}$  for all  $t$  along the equilibrium path.*

Proposition 1 ensures that the group with initially lower skill share—which we may assume without loss of generality to be group 1—cannot overtake the other group along an equilibrium path. Since there exists a unique competitive equilibrium path from any initial state  $(s_{10}, s_{20})$ , we may write (4) as a recursive system:

$$s_{it} = f_i(s_{1t-1}, s_{2t-1}), \quad (5)$$

where the functions  $f_i$  are defined implicitly by

$$f_i = H(\beta f_1 + (1 - \beta) f_2, \eta s_{it-1} + (1 - \eta)(\beta s_{1t-1} + (1 - \beta) s_{2t-1})) \quad (6)$$

A key question of interest here is whether or not, given an initial state of group inequality ( $s_{10} < s_{20}$ ), the two skill shares will converge asymptotically ( $\lim_{t \rightarrow \infty} s_{1t} = \lim_{t \rightarrow \infty} s_{2t}$ ).

### 3. Steady States and Stability

A competitive equilibrium path is a *steady state* if  $(s_{1t}, s_{2t}) = (s_{10}, s_{20})$  for all periods  $t$ . Of particular interest are *symmetric* steady states that satisfy the additional condition  $s_{1t} = s_{2t}$ . At any symmetric steady state, the common skill share  $s_t$  must be a solution to

$$s = 1 - G(\tilde{a}(\delta(s), s)). \quad (7)$$

Since costs are bounded and  $\lim_{s \rightarrow 0} \delta(s) = \infty$ , we have  $\lim_{s \rightarrow 0} \tilde{a}(\delta(s), s) = 0$ . And since  $\delta(1) = 0$ ,  $\lim_{s \rightarrow 1} \tilde{a}(\delta(s), s) = \infty$ . Hence there must exist at least one symmetric steady state. In general, there could be several such states, each of which may be stable or unstable under the dynamics (4).

Let  $\tilde{a}_1$  and  $\tilde{a}_2$  denote the partial derivatives of  $\tilde{a}$  with respect to its two arguments. If the following condition is satisfied at each symmetric steady state, then there will be exactly one such state:

$$G' |\tilde{a}_2| < 1 + G' \tilde{a}_1 \delta'. \quad (8)$$

We show in what follows that if this inequality is reversed at any symmetric steady state, then that steady state will be unstable. That is, the following condition is sufficient for local instability of a symmetric steady state:

$$G' |\tilde{a}_2| > 1 + G' \tilde{a}_1 \delta'. \quad (9)$$

This is intuitive, since (9) implies that peer effects are relatively strong compared with the wage effects of increasing skills, generating positive feedback in the neighborhood of the symmetric steady state.

Hence multiplicity of symmetric states arises only if at least one of these states is unstable. However, it is entirely possible for there to be a unique symmetric steady state that is also unstable (see Example 1 below). In other words, condition (8) is *not* sufficient for stability. A sufficient condition for stability is the following:

$$G' |\tilde{a}_2| < 1. \quad (10)$$

This condition requires that the positive effect of an increase in the level of human capital in one's peer group on the incentive to invest not be too large. This could be because the ability threshold is sufficiently nonresponsive to changes in peer group quality and/or because the distribution function is relatively flat at this state.

The conditions (9) and (10) are clearly mutually exclusive, and if either one holds at a symmetric steady state, then the stability properties of this state are independent of the level of social segregation, as in the following proposition.

**PROPOSITION 2.** *A symmetric steady state is stable if  $G' |\tilde{a}_2| < 1$  and unstable if  $G' |\tilde{a}_2| > 1 + G' \tilde{a}_1 \delta'$  at this state.*

On the other hand, if neither (9) nor (10) holds, then we have

$$1 < G' |\tilde{a}_2| < 1 + G' \tilde{a}_1 \delta'. \quad (11)$$

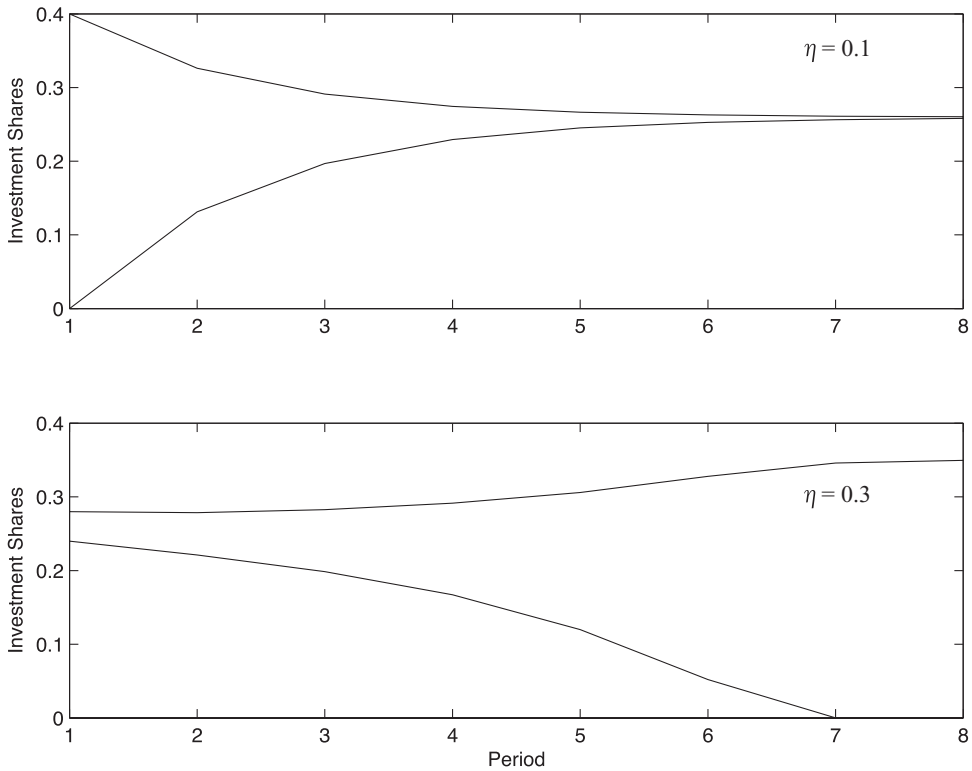


FIGURE 1. Dynamics of investment shares for two different segregation levels.

In this case the stability properties of the symmetric steady state depend on the level of segregation, as in the following proposition.

**PROPOSITION 3.** *Consider any symmetric steady state at which  $1 < G' |\tilde{a}_2| < 1 + G' \tilde{a}_1 \delta'$ . There exists a level of segregation  $\hat{\eta} \in (0, 1)$  such that this steady state is locally stable if  $\eta < \hat{\eta}$ , and unstable if  $\eta > \hat{\eta}$ .*

The following example illustrates the result, and also shows that the uniqueness of a symmetric steady state is consistent with local instability.

**EXAMPLE 1.** Suppose  $\beta = 0.25$ ,  $f(h, l) = h^{0.7}l^{0.3}$ ,  $G(a) = 1 - e^{-0.1a}$ , and  $c(a, \sigma) = 1 - \sigma + 1/a$ . Then there is a unique symmetric steady state  $(s_1, s_2) = (0.26, 0.26)$ . There exists  $\hat{\eta} \approx 0.21$  such that if  $\eta < \hat{\eta}$  the symmetric steady state is locally stable, and if  $\eta > \hat{\eta}$  the symmetric steady state is locally unstable. (Figure 1 shows the paths of investment shares for  $\eta = 0.10$  and  $\eta = 0.30$  respectively.) ■

The instability identified in Proposition 3 (and illustrated in the example) is of the saddle-point type: convergence to the steady state occurs if and only if the skill shares in the two groups are initially identical. If all symmetric steady states satisfy



either equation (11) or equation (9), then any initial disparities between groups will persist even under a regime of fully enforced equal opportunity unless segregation can be reduced below the critical threshold. Furthermore, as Example 1 shows, even with a unique symmetric steady state, initially small differences between groups can be amplified if segregation is sufficiently great. Under such conditions, redistributive policies can only maintain group equality so long as they remain permanently in place. Any temporary policy of redistribution will either be futile in the long run, or result in a reversal of roles in the social hierarchy.<sup>5</sup>

#### 4. Effects of Integration

We have focused to this point on the conditions giving rise to *unstable equality* rather than *stable inequality* between groups. We now turn attention to the latter question, and show that integration can cause an asymmetric steady state to disappear, resulting in convergence to equality across groups. But when there are multiple symmetric steady states, this raises the question of which one is selected when integration destabilizes group inequality. It turns out that the population share of the initially disadvantaged group plays a critical role in this regard.

##### 4.1. A Special Case

To clarify the logic of the argument, we begin with a simple case in which all individuals have the same ability  $\bar{a}$ , and relative wages are completely inelastic:  $\delta(s_t) = \bar{\delta}$  for all periods  $t$ . In this case the cost function is  $c(\bar{a}, \sigma)$  and the only stable steady states involve homogeneous skill levels within groups.<sup>6</sup> Suppose that

$$c(\bar{a}, 1) < \bar{\delta} < c(\bar{a}, 0), \quad (12)$$

which ensures that both  $(s_1, s_2) = (0, 0)$  and  $(s_1, s_2) = (1, 1)$  are stable steady states at all levels of segregation  $\eta$ . Condition (12) also implies that under complete segregation ( $\eta = 1$ ), the skill distribution  $(s_1, s_2) = (0, 1)$  is a stable steady state. Define  $\tilde{\beta}$  as the group 1 population share at which  $c(\bar{a}, 1 - \tilde{\beta}) = \bar{\delta}$ . This is the value of  $\beta$  for which, under complete integration, the costs of acquiring human capital are  $\bar{\delta}$  for both groups.

5. Benabou (1996) also finds that “there might be little role for redistributive policies” to affect the distribution of human capital, and that transfers and subsidized loans “may be poor substitutes for the actual integration of schools, neighbourhoods, and social networks”. In particular, Appendix 1 of his paper considers an overlapping generations model with two groups, local spillovers in human capital investment, and endogenous sorting which results in either perfect integration or complete segregation. In the latter case, group differences in human capital can diverge indefinitely. Here we have taken integration as parametric and continuously variable, and examined how changes in integration affect the stability of steady states with group equality. As we show next, the demographic composition of the population can have significant human capital effects when changes in integration occur.

6. There may exist steady states in which members of a group are all indifferent between acquiring human capital and not doing so, and make heterogeneous choices in the exact proportions that maintain this indifference, but such equilibria are dynamically unstable and we do not consider them here.

(This is because, if  $\eta = 0$  and  $(s_1, s_2) = (0, 1)$ , then  $\sigma_i = 1 - \beta$  for both groups.) There is a unique  $\tilde{\beta} \in (0, 1)$  satisfying this condition since  $c(\bar{a}, \sigma)$  is decreasing in  $\sigma$  and satisfies (12). We then have the following proposition.

**PROPOSITION 4.** *Given any  $\beta \in (0, 1)$ , there exists a unique  $\hat{\eta}(\beta)$  such that the stable asymmetric equilibrium  $(s_1, s_2) = (0, 1)$  exists if and only if  $\eta > \hat{\eta}(\beta)$ . The function  $\hat{\eta}(\beta)$  is positive and decreasing for all  $\beta < \tilde{\beta}$ , positive and increasing for all  $\beta > \tilde{\beta}$ , and satisfies  $\hat{\eta}(\tilde{\beta}) = 0$ .*

Hence group inequality can persist if segregation is sufficiently high, where the threshold level of segregation itself depends systematically on the population share  $\beta$  of the disadvantaged group. If segregation is reduced to a point below this threshold, group inequality can no longer be sustained. In this case, convergence to a symmetric steady state must occur. However, there are two of these in the model, since both  $(s_1, s_2) = (0, 0)$  and  $(s_1, s_2) = (1, 1)$  are stable steady states at all levels of segregation  $\eta$ . Convergence to the former implies that equality is attained through increased costs and hence declines in the human capital of the initially advantaged group. Convergence to the latter, in contrast, occurs through reductions in costs and therefore increases in the human capital of the initially disadvantaged group. The following result establishes that convergence to the high human capital state occurs if and only if the population share of the initially disadvantaged group is sufficiently low.

**PROPOSITION 5.** *Suppose that the economy initially has segregation  $\eta > \hat{\eta}(\beta)$  and is at the stable steady state  $(s_1, s_2) = (0, 1)$ . If segregation declines permanently to some level  $\eta < \hat{\eta}(\beta)$ , then the economy converges to  $(s_1, s_2) = (1, 1)$  if  $\beta < \tilde{\beta}$ , and to  $(s_1, s_2) = (0, 0)$  if  $\beta > \tilde{\beta}$ .*

Propositions 4 and 5 are summarized in Figure 2, which identifies three regimes in the space of parameters  $\beta$  and  $\eta$ . For any value of  $\beta$  (other than  $\tilde{\beta}$ ), there is a segregation level  $\hat{\eta}(\beta) \in (0, 1)$  such that group inequality can persist only if segregation lies above this threshold. If segregation drops below the threshold, the result is a sharp adjustment in human capital and convergence to equality. This convergence will result from a decline in the human capital of the initially advantaged group if the population share of the initially disadvantaged group is large enough (i.e.  $\beta > \tilde{\beta}$ ). Alternatively, it will result from a rise in the human capital of the disadvantaged group if its population share is sufficiently small. The threshold segregation level itself varies with  $\beta$  nonmonotonically. When  $\beta$  is small,  $\hat{\eta}(\beta)$  is the locus of pairs of  $\eta$  and  $\beta$  such that  $c(\bar{a}, \sigma_1) = \bar{\delta}$  at the state  $(s_1, s_2) = (0, 1)$ . Increasing  $\beta$  lowers  $\sigma_1$  and hence raises  $c(\bar{a}, \sigma_1)$ , which implies that  $c(\bar{a}, \sigma_1) = \bar{\delta}$  holds at a lower level of  $\eta$ . Hence  $\hat{\eta}(\beta)$  is decreasing in this range, implying that higher values of  $\beta$  require higher levels of integration before the transition to equality is triggered. When  $\beta$  is larger than  $\tilde{\beta}$ , however,  $\hat{\eta}(\beta)$  is the locus of pairs of  $\eta$  and  $\beta$  such that  $c(\bar{a}, \sigma_2) = \bar{\delta}$  at the state  $(s_1, s_2) = (0, 1)$ . Increasing  $\beta$  lowers  $\sigma_2$  and hence raises  $c(\bar{a}, \sigma_2)$ , which implies that  $c(\bar{a}, \sigma_2) = \bar{\delta}$  holds at a higher level of  $\eta$ . Hence  $\hat{\eta}(\beta)$  is increasing in this range, and

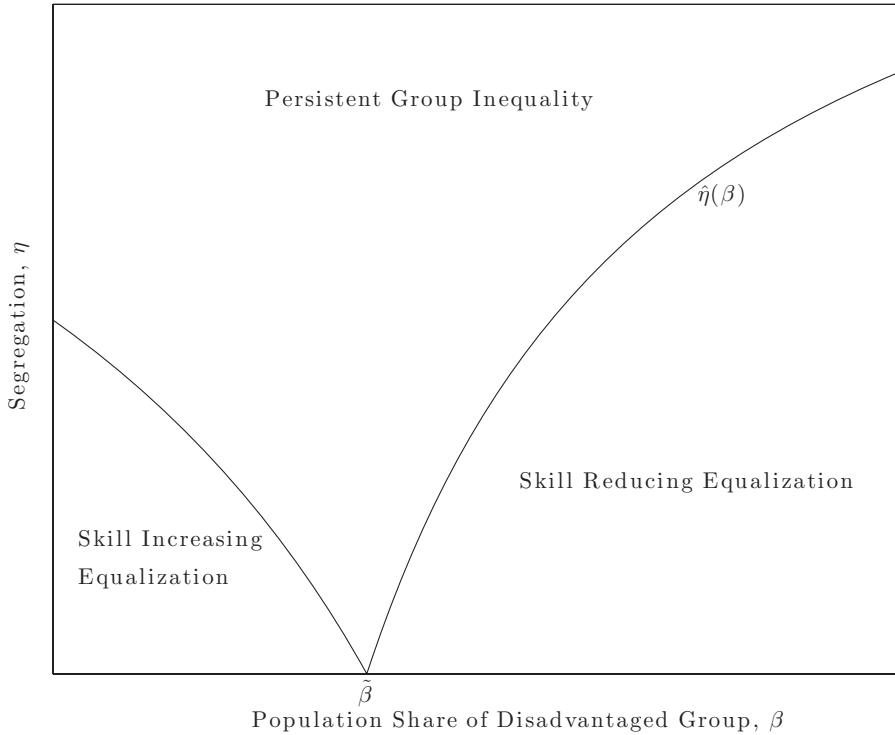


FIGURE 2. Segregation, population shares, and persistent inequality.

higher values of  $\beta$  require lower levels of integration in order to induce the shift to equality.

Greater integration within the regime of persistent inequality raises the costs to the advantaged group and lowers costs to the disadvantaged group. Hence one might expect integration to be resisted by the former and supported by the latter. Note, however, that this is no longer the case if a transition to a different regime occurs. In this case, when  $\beta$  is small, both groups end up investing in human capital as a consequence of integration and as a result enjoy lower costs of investment. But when  $\beta$  is large, integration policies that reduce  $\eta$  below  $\hat{\eta}(\beta)$  will result in higher steady state costs of human capital accumulation for both groups, with the consequence that no human capital investment is undertaken. Hence *both* groups have an incentive to support integrationist policies if  $\beta$  is small, and both might resist such policies on purely economic grounds if  $\beta$  is large.<sup>7</sup>

This simple model delivers a number of insights, but also has several shortcomings. Wages are completely insensitive to aggregate skill shares, and there is no heterogeneity within groups. As a result, all steady states are at the boundaries of the state space.

7. This effect arises also in Chaudhuri and Sethi (2008) which deals with the consequences of integration in the presence of statistical discrimination.

Changes in segregation affect human capital decisions if they result in a transition from one regime to another; within a given regime changes in social network quality affect costs but do not induce any behavioral response. Furthermore, even when transitions to another regime occur, human capital decisions are affected in only one of the two groups. Finally, convergence to a steady state occurs in a single period. These shortcomings do not arise when the model is generalized to allow for heterogeneous ability within groups and endogenous wages, which we consider next.

#### 4.2. *Endogenous Wages and Heterogeneous Ability*

Suppose that ability is heterogeneous within groups (though distributed identically across groups) and that wages are endogenously determined, so the skill premium  $\delta(s_t)$  is decreasing in the aggregate skill share as in Sections 2 and 3. Multiple steady states will exist under complete segregation if and only if there are multiple solutions to equation (7). Suppose that there are indeed multiple solutions, and let  $s_l$  and  $s_h$  respectively denote the smallest and largest of these. Then there are symmetric steady states  $(s_l, s_l)$  and  $(s_h, s_h)$  at all levels of segregation  $\eta$ .

Define the set

$$\Lambda := [0, s_l] \times [s_h, 1].$$

This is the set of asymmetric states at which the first group has skill share at most  $s_l$  and the second has skill share at least  $s_h$ . Given the multiplicity of symmetric steady states, there must also exist an asymmetric steady state within this set when segregation is complete. We thus have the following proposition.

**PROPOSITION 6.** *If  $\eta = 1$ , there exists an asymmetric steady state  $(s_1, s_2) = (x, y) \in \Lambda$ . The aggregate skill share  $s$  at this state satisfies  $s_l < s < s_h$ , and hence  $\delta(s_h) < \delta(s) < \delta(s_l)$ .*

That is, the asymmetric steady state must lie in a subset of  $\Lambda$  in which the aggregate skill share lies strictly between the two extreme symmetric steady states. Consequently, the wage premium for skilled workers at this asymmetric steady state is greater than that at the high-skill symmetric state but smaller than that at the low-skill symmetric state.<sup>8</sup>

Now consider the effects of increasing integration, starting from an asymmetric steady state  $(x, y) \in \Lambda$ . For any given population composition  $\beta$ , we shall say that integration is *equalizing* and *skill-increasing* if there exists some segregation level  $\hat{\eta}(\beta)$  such that for all  $\eta < \hat{\eta}(\beta)$  there is no stable asymmetric steady state, and the initial state  $(x, y)$  is in the basin of attraction of the high-investment symmetric steady state  $(s_h, s_h)$ . Similarly, we shall say that integration is *equalizing* and *skill-reducing* if there exists some segregation level  $\hat{\eta}(\beta)$  such that for all  $\eta < \hat{\eta}(\beta)$  there is no stable

8. Not every point in  $\Lambda$  has this property; there may be states in  $\Lambda$  where the skill premium is zero, and others at which it can be made arbitrarily large by varying  $\beta$ .

asymmetric steady state, and  $(x, y)$  is in the basin of attraction of the low-investment symmetric steady state  $(s_l, s_l)$ . We then have the following result.

**PROPOSITION 7.** *There exist  $\beta_l > 0$  and  $\beta_h < 1$  such that (i) integration is equalizing and skill-increasing if  $\beta < \beta_l$  and (ii) integration is equalizing and skill-reducing if  $\beta > \beta_h$ .*

Taken together, Propositions 6 and 7 imply that integration can result in a transition from group inequality to group equality, but the distributional and human capital effects of a such a transition depend on the population composition. When the size of the initially disadvantaged group is sufficiently small, convergence occurs to a high-investment state in which most of the adjustment involves increases in skills among the initially disadvantaged, with a decline in the wage premium for skilled workers. Those in the initially disadvantaged group gain through two effects: the costs of investment decline as peer groups become more human capital rich, and wages increase for those who continue to remain unskilled. Within the initially advantaged group, only those who were initially unskilled gain from the transition. However, the losses imposed on the rest are limited by the fact that wages and aggregate skills shares in the asymmetric equilibrium are close to those in the high-skill symmetric equilibrium when the size of the initially advantaged group is large.

When the size of the initially disadvantaged group is sufficiently large, however, transition occurs to the low-skill symmetric state, and the wage premium for skilled workers rises. Most of the adjustment now involves declines in investment among the initially advantaged. Unskilled members of the initially advantaged group experience wage declines. Even though wages for skilled workers rise, this need not result in a welfare gain since the costs of investment also rise. This induces many of them to cease investing in skills, driving down the aggregate skill share. Among the initially disadvantaged group there is some increase in skill shares as peer group human capital improves, but those who remain unskilled experience welfare declines through lower wages. Under these conditions, it is quite possible that there will be little appetite for integration in either group.

## 5. Discussion

In *Brown v. Board of Education* the US Supreme Court struck down laws enforcing racial segregation of public schools on the grounds that “separate educational facilities are inherently unequal”. Many hoped that the demise of legally enforced segregation and discrimination against African Americans during the 1950s and 1960s, coupled with the apparent reduction in racial prejudice among whites, would provide an environment in which significant social and economic racial disparities would not persist. But while substantial racial convergence in earnings and incomes did occur from the 1950s to the mid-1970s, little progress has been made since. For example, the strong convergence in median annual income of full-time year-round male and female

African American workers relative to their white counterparts that occurred between the 1940s and the 1970s has been greatly attenuated or even reversed since the late 1970s (President's Council of Economic Advisors 2006). Conditional on the income of their parents, African Americans receive incomes substantially (about a third) below those of whites, and this intergenerational race gap has not diminished appreciably over the past two decades (Hertz 2005). Similarly, the racial convergence in years of schooling attained and cognitive scores at given levels of schooling that occurred prior to 1980 appears not to have continued subsequently (Neal 2005). Significant racial differences in mortality, wealth, subjective well being, and other indicators also persist (Deaton and Lubotsky 2003; Wolff 1998; Blanchflower and Oswald 2004).

Enduring discriminatory practices in markets are no doubt part of the explanation. Even in the absence of any form of market discrimination, however, there are mechanisms through which group inequality may be sustained indefinitely. Racial segregation of parenting, friendship networks, mentoring relationships, neighborhoods, workplaces, and schools places the less-affluent group at a disadvantage in acquiring the things—contacts, information, cognitive skills, behavioral attributes—that contribute to economic success. But is the extent of segregation and the impact of interpersonal spillovers sufficient to explain the persistence of group differences? Preferentially associating with members of one's own kind is a common human trait (Tajfel et al. 1971) and is well documented for race and ethnic identification, religion, and other characteristics. In a nationally representative sample of 130 schools (and 90,118 students) same-race friendships were almost twice as likely as cross-race friendships, controlling for school racial composition (Moody 2001). Data from one of these schools studied by Currarini, Jackson, and Pin (2009) give an estimated  $\eta$  of 0.71. In the national sample, by comparison to the friends of white students, the friends of African American students had significantly lower grades, attachment to school, and parental socioeconomic status. Differing social networks may help explain why Fryer and Levitt (2006) found that while the white–black cognitive gap among children entering school is readily explained by a small number of family and socioeconomic covariates, over time black children fall further behind with a substantial gap appearing by the end of the third grade that is not explained by observable characteristics.

While there are many channels through which the racial assortment of social networks might disadvantage members of the less well-off group, statistical identification of these effects often is an insurmountable challenge. The reason is that networks are selected by individuals and as a result plausible identification strategies for the estimation of the causal effect of exogenous variation in the composition of an individual's networks are difficult to devise. Hoxby (2000) and Hanushek, Kain, and Rivkin (2009) use the year-to-year cohort variation in racial composition within grade and school to identify racial network effects, finding large negative effects of racial assortment on the academic achievement of black students. Studies using randomized assignment of college roommates have also found some important behavioral and academic peer effects (Kremer and Levy 2003; Sacerdote 2001; Zimmerman and Williams 2003). A study of annual work hours using longitudinal data and individual

fixed effects found strong neighborhood effects especially for the least well educated individuals and the poorest neighborhoods (Weinberg, Reagan, and Yankow 2004).

Racial inequality in the United States is rooted in a history of oppression backed by the power of the state. Similarly, in South Africa under Apartheid, group membership based on a system of racial classification was a critical determinant of economic opportunity. In the Indian subcontinent, formal caste-based hierarchies have been in place for centuries. However, not all instances of contemporary group inequality can be traced to historical oppression. Many immigrants of European descent arrived in the United States with little human or material wealth, but distinct ethnic groups have experienced strikingly different economic trajectories in subsequent generations. Descendants of Italian, Jewish, Slavic, Scotch-Irish immigrants have enjoyed very different paths toward economic and social equality, and substantial income, wealth, and occupational inequalities among them have persisted. There is evidence linking the degree of ethnic identification among midwestern immigrants of European descent in the mid-19th century with patterns of upward occupational mobility in the late 20th century, even though the range of actual occupations has changed dramatically over this period of time (Munshi and Wilson 2007). This is an example in which some level of segregation in social relations, mediated through institutions such as churches, could have played a role in generating and perpetuating ethnic occupational segregation across generations.

As another example, consider the case of group inequality based on regional origin in contemporary South Korea. The process of rapid industrialization drew large numbers of migrants to metropolitan Seoul from rural areas across the country. Those from the Youngnam region gained access to white collar jobs at a significantly higher rate than those from the Honam region, even after controlling for productive characteristics (Yu 1990). The importance of regional and other group ties in gaining high-level managerial positions went beyond discrimination based on economically irrelevant characteristics, but instead reflected the presumption that “social ties are tangible qualifications, and people with such ties ...are (presumptively) competent in the only relevant sense that counts” (Shin and Chin 1989, p. 19). While contemporary regional group identities and animosities originated almost two millennia ago, the advantages of the Youngnam region today have been attributed in part to the fact that the head of state at the time, Park Jung-Hee, was from the Youngnam region, and parochialism was instrumental in access to the most prized administrative and managerial positions. Despite the subsequent transition to democracy and widespread use of formally meritocratic selection methods in both the economy and school system, social identities and group inequalities based on regional origin remain significant, and may even have become more salient (Ha 2007). Disparities in the occupational richness of the respective social networks have allowed initial regional (and region of birth) differences to persist and even possibly to widen.

The theoretical arguments developed here apply quite generally to any society composed of social groups with distinct identities and some degree of segregation in social interactions. In cases involving a history of institutionalized oppression, segregation can prevent the convergence of income distributions following the end

of overt discrimination. And in cases with no such history, segregation can induce small initial differences to be amplified over time. Social integration can induce equalization of income distributions across groups but the distributional and human capital consequences of integration depend on the demographic composition. Group interests may be aligned in support of integration when the size of the elite is large, and may be aligned in opposition when the elite size is small. Thus the challenges facing policy makers in an urban area such as Baltimore are quite different from those faced in Bangor, Maine or Burlington, Vermont. Similarly, the challenges of assuring group-equal opportunity are quite different in New Zealand, where 15% of the population are Maori, and South Africa, where the disadvantaged African population constitutes 78% of the total. The timing of integrationist policies also matters in an economy in which the demographic structure is changing. For instance, in the United States more than half of new births are to nonwhite families. Under such circumstances, integrationist policies will be more effective in combating group inequality and more likely to have a positive impact on aggregate human capital levels if they are undertaken sooner rather than later.

## 6. Conclusions

While the vigorous enforcement of antidiscrimination statutes can substantially attenuate discrimination in markets and the public sphere, there are many important private interactions that lie outside the scope of such laws. For instance, a liberal judicial system cannot prohibit discrimination in an individual's choice of a date, a spouse, an adopted child, a role model, a friend, membership in a voluntary association, or residence in a neighborhood. Since so much of early childhood learning takes place in families and peer groups, segregation in the formation of social networks can have important implications for the perpetuation of group inequality across generations. Voluntary discrimination in *contact* can give rise to persistent group inequality even in the absence of discrimination in *contract*. Even in the complete absence of market discrimination and credit constraints, group inequality can emerge and persist indefinitely as long as significant social segregation endures in the presence of sufficiently strong peer effects.

Our main point is that declining segregation can have discontinuous effects on long-run group inequality, with these effects depending systematically on the demographic structure of the population. These findings are relevant to the debate over the appropriate policy response to a history of overt discrimination. Procedural or rule-oriented approaches emphasize the vigorous enforcement of anti-discrimination statutes and the establishment of equal opportunity. Substantive or results-oriented approaches advocate group-redistributive remedies such as affirmative action or reparations. There are conditions under which group inequality will persist indefinitely even in the presence of equal economic opportunity, and redistributive policies will be ineffective as long as they are temporary. In this case the only path to equality in income distributions across groups is an increase in social integration. But the location and



timing of such intervention matters: integration is most likely to be effective and skill-enhancing when and where the size of the initially disadvantaged group is sufficiently small. Under such circumstances the short-run costs to the elite may be outweighed by longer-term gains, and if individuals are sufficiently patient and forward-looking, integrationist policies can enjoy widespread popular support.

## Appendix: Proofs

### *Proof of Proposition 1*

Suppose  $(s_{10}, s_{20}) \in [0, 1]^2$  is given. Then, using equations (1) and (2),  $s_0 \in [0, 1]$  and  $(\sigma_{10}, \sigma_{10}) \in [0, 1]^2$  are uniquely defined. Define the function  $\varphi(s)$  as follows:

$$\varphi(s) = \beta(1 - G(\tilde{a}(\delta(s), \sigma_{1t-1}))) + (1 - \beta)(1 - G(\tilde{a}(\delta(s), \sigma_{2t-1}))).$$

Note that  $\varphi(0) = 1$ ,  $\varphi(1) = 0$  and  $\varphi(s)$  is strictly decreasing. Hence, given  $(\sigma_{10}, \sigma_{20})$ , there exists a unique value of  $s$  such that  $s = \varphi(s)$ . Note from equations (1) and (4) that in equilibrium,  $s_1$  must satisfy  $s_1 = \varphi(s_1)$ , so  $s_1$  is uniquely determined. The pair  $(s_{11}, s_{21})$  is then also uniquely determined from equation (4). The second claim follows from equations (4) and (2), since  $\tilde{a}$  is decreasing in its second argument.

### *Proof of Propositions 2 and 3*

The stability of the (unique) symmetric steady state under the dynamics (5) depends on the properties of the Jacobean

$$J = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}$$

evaluated at the steady state. Specifically, the state is stable if all eigenvalues of  $J$  lie within the unit circle, and unstable if at least one eigenvalue lies outside it. From equation (4), we get

$$f_{11} = -G'(\tilde{a}_1 \delta'(\beta f_{11} + (1 - \beta) f_{21}) + \tilde{a}_2(\eta + (1 - \eta)\beta)) \quad (\text{A.1})$$

$$f_{12} = -G'(\tilde{a}_1 \delta'(\beta f_{12} + (1 - \beta) f_{22}) + \tilde{a}_2(1 - \eta)(1 - \beta)) \quad (\text{A.2})$$

$$f_{21} = -G'(\tilde{a}_1 \delta'(\beta f_{11} + (1 - \beta) f_{21}) + \tilde{a}_2(1 - \eta)\beta) \quad (\text{A.3})$$

$$f_{22} = -G'(\tilde{a}_1 \delta'(\beta f_{12} + (1 - \beta) f_{22}) + \tilde{a}_2(\eta + (1 - \eta)(1 - \beta))). \quad (\text{A.4})$$

For  $i \in \{1, 2\}$ , define

$$\omega_i = (\beta f_{1i} + (1 - \beta) f_{2i}).$$

Then

$$\begin{aligned} \beta f_{11} &= -\beta G' (\tilde{a}_1 \delta' \omega_1 + \tilde{a}_2 (\eta + (1 - \eta) \beta)) \\ (1 - \beta) f_{21} &= -(1 - \beta) G' (\tilde{a}_1 \delta' \omega_1 + \tilde{a}_2 (1 - \eta) \beta). \end{aligned}$$

Adding these two equations, we get

$$\begin{aligned} \omega_1 &= -G' (\tilde{a}_1 \delta' \omega_1 + \tilde{a}_2 (1 - \eta) \beta + \beta \tilde{a}_2 \eta) \\ &= -G' (\tilde{a}_1 \delta' \omega_1 + \beta \tilde{a}_2) \end{aligned}$$

so

$$\omega_1 = \frac{-\beta G' \tilde{a}_2}{1 + G' \tilde{a}_1 \delta'}. \quad (\text{A.5})$$

Define  $\gamma \in (0, 1)$  as follows:

$$\gamma = \frac{G' \tilde{a}_1 \delta'}{1 + G' \tilde{a}_1 \delta'}. \quad (\text{A.6})$$

Hence, from (A.1) and (A.3),

$$\begin{aligned} f_{11} &= -G' \tilde{a}_2 (\eta + \beta (1 - \eta - \gamma)), \\ f_{21} &= -G' \tilde{a}_2 \beta (1 - \eta - \gamma). \end{aligned}$$

Now consider

$$\begin{aligned} \beta f_{12} &= -\beta G' (\tilde{a}_1 \delta' \omega_2 + \tilde{a}_2 (1 - \eta) (1 - \beta)) \\ (1 - \beta) f_{22} &= -(1 - \beta) G' (\tilde{a}_1 \delta' \omega_2 + \tilde{a}_2 (\eta + (1 - \eta) (1 - \beta))). \end{aligned}$$

Adding these two equations, we get

$$\begin{aligned} \omega_2 &= -G' \tilde{a}_1 \delta' \omega_2 - G' \tilde{a}_2 (1 - \eta) (1 - \beta) - (1 - \beta) G' \tilde{a}_2 \eta, \\ &= -G' \tilde{a}_1 \delta' \omega_2 - G' \tilde{a}_2 (1 - \beta), \end{aligned}$$

so

$$\omega_2 = -\frac{(1 - \beta) G' \tilde{a}_2}{1 + G' \tilde{a}_1 \delta'}.$$

Hence, from (A.2) and (A.4),

$$\begin{aligned} f_{12} &= -G' \tilde{a}_2 (1 - \beta) (1 - \eta - \gamma), \\ f_{22} &= -G' \tilde{a}_2 (1 - \gamma - \beta (1 - \eta - \gamma)). \end{aligned}$$

The Jacobean  $J$  is therefore

$$J = -G'\tilde{a}_2 \begin{bmatrix} \eta + \beta(1 - \eta - \gamma) & (1 - \beta)(1 - \eta - \gamma) \\ \beta(1 - \eta - \gamma) & 1 - \gamma - \beta(1 - \eta - \gamma) \end{bmatrix}.$$

It can be verified that the eigenvalues of  $J$  are

$$\lambda_1 = -G'\tilde{a}_2\eta, \quad (\text{A.7})$$

$$\lambda_2 = -G'\tilde{a}_2(1 - \gamma), \quad (\text{A.8})$$

both of which are positive since  $\tilde{a}_2 < 0$ . Using (A.6),

$$\lambda_2 = G'|\tilde{a}_2|(1 - \gamma) = \frac{G'|\tilde{a}_2|}{1 + G'\tilde{a}_1\delta'}, \quad (\text{A.9})$$

so  $\lambda_2 > 1$  if (9) holds. Hence (9) is sufficient for instability of the steady state. To see that (10) is sufficient for stability, note that it implies  $\lambda_2 < 1$  from (A.9) and  $\lambda_1 < 1$  for any  $\eta \in (0, 1)$  from (A.7). Since both eigenvalues are positive, the steady state is stable. This proves Proposition 2.

To prove Proposition 3, note that if (11) holds, then  $\lambda_2 < 1$  from (A.9). Hence the steady state is locally asymptotically stable if  $\lambda_1 < 1$  and unstable if  $\lambda_1 > 1$ . Applying the first inequality in (11) immediately yields the result.

#### ***Proof of Proposition 4***

At the state  $(s_1, s_2) = (0, 1)$ , the mean skill share is  $s = 1 - \beta$  from equation (1). Hence, using equation (2), we get

$$\begin{aligned} \sigma_1 &= (1 - \eta)(1 - \beta), \\ \sigma_2 &= \eta + (1 - \eta)(1 - \beta). \end{aligned}$$

Since  $c$  is decreasing in its second argument,  $c(\bar{a}, \sigma_1)$  is increasing in  $\eta$  and  $c(\bar{a}, \sigma_2)$  is decreasing in  $\eta$ . Under complete integration ( $\eta = 0$ ) we have  $\sigma_1 = \sigma_2 = 1 - \beta$ , and the costs of human capital accumulation are therefore  $c(\bar{a}, 1 - \beta)$  for both groups. Under complete segregation,  $\eta = 1$  and hence  $\sigma_1 = 0$  and  $\sigma_2 = 1$ . Hence under complete segregation, the costs of human capital accumulation are  $c(\bar{a}, 0)$  and  $c(\bar{a}, 1)$  for the two groups respectively, where  $c(\bar{a}, 1) < \bar{\delta} < c(\bar{a}, 0)$  by assumption.

First consider the case  $\beta < \tilde{\beta}$ , which implies  $c(\bar{a}, 1 - \beta) < \bar{\delta}$ . Since  $c(\bar{a}, \sigma_2)$  is decreasing in  $\eta$  and satisfies  $c(\bar{a}, \sigma_2) < \bar{\delta}$  when  $\eta = 0$ , it satisfies  $c(\bar{a}, \sigma_2) < \bar{\delta}$  for all  $\eta$ . Since  $c(\bar{a}, \sigma_1)$  is increasing in  $\eta$  and satisfies  $c(\bar{a}, \sigma_1) < \bar{\delta}$  at  $\eta = 0$  and  $c(\bar{a}, \sigma_1) > \bar{\delta}$  at  $\eta = 1$ , there exists a unique  $\hat{\eta}(\beta)$  such that  $c(\bar{a}, \sigma_1) = \bar{\delta}$ . For all  $\eta > \hat{\eta}(\beta)$ , we have  $c(\bar{a}, \sigma_2) < \bar{\delta} < c(\bar{a}, \sigma_1)$ , which implies that  $(s_1, s_2) = (0, 1)$  is a stable steady state. For all  $\eta < \hat{\eta}(\beta)$ , we have  $c(\bar{a}, \sigma_2) < c(\bar{a}, \sigma_1) < \bar{\delta}$ , which implies that  $(s_1, s_2) = (0, 1)$  cannot be a steady state. Note that any increase in  $\beta$  within the range  $\beta < \tilde{\beta}$  raises

$c(\bar{a}, \sigma_1)$ . Since  $c(\bar{a}, \sigma_1)$  is increasing in  $\eta$ , this lowers the value of  $\hat{\eta}(\beta)$ , defined as the segregation level at which  $c(\bar{a}, \sigma_1) = \bar{\delta}$ .

Next consider the case  $\beta > \tilde{\beta}$ , which implies  $c(\bar{a}, 1 - \beta) > 0$ . Since  $c(\bar{a}, \sigma_1)$  is increasing in  $\eta$  and satisfies  $c(\bar{a}, \sigma_1) > \bar{\delta}$  at  $\eta = 0$ , it satisfies  $c(\bar{a}, \sigma_1) > \bar{\delta}$  for all  $\eta$ . Since  $c(\bar{a}, \sigma_2)$  is decreasing in  $\eta$  and satisfies  $c(\bar{a}, \sigma_2) > \bar{\delta}$  at  $\eta = 0$  and  $c(\bar{a}, \sigma_2) < \bar{\delta}$  at  $\eta = 1$ , there exists a unique  $\hat{\eta}(\beta)$  such that  $c(\bar{a}, \sigma_2) = \bar{\delta}$ . For all  $\eta > \hat{\eta}(\beta)$ , we have  $c(\bar{a}, \sigma_2) < \bar{\delta} < c(\bar{a}, \sigma_1)$ , which implies that  $(s_1, s_2) = (0, 1)$  is a stable steady state. For all  $\eta < \hat{\eta}(\beta)$ , we have  $\bar{\delta} < c(\bar{a}, \sigma_2) < c(\bar{a}, \sigma_1)$ , which implies that  $(s_1, s_2) = (0, 1)$  cannot be a steady state. Note that any increase in  $\beta$  within the range  $\beta > \tilde{\beta}$  raises  $c(\bar{a}, \sigma_2)$ . Since  $c(\bar{a}, \sigma_2)$  is decreasing in  $\eta$ , this raises the value of  $\hat{\eta}(\beta)$ , defined as the segregation level at which  $c(\bar{a}, \sigma_2) = \bar{\delta}$ .

### *Proof of Proposition 5*

First consider the case  $\beta < \tilde{\beta}$ . Recall from the proof of Proposition 4 that if the economy is initially at  $(s_1, s_2) = (0, 1)$ , then for all  $\eta < \hat{\eta}(\beta)$ , we have  $c(\bar{a}, \sigma_2) < c(\bar{a}, \sigma_1) < \bar{\delta}$ . Hence all individuals in each of the two groups will find it optimal to invest in human capital, resulting in a transition to  $(s_1, s_2) = (1, 1)$ . This lowers both  $c(\bar{a}, \sigma_2)$  and  $c(\bar{a}, \sigma_1)$ , and hence maintains the condition  $c(\bar{a}, \sigma_2) < c(\bar{a}, \sigma_1) < \bar{\delta}$ . Hence the economy remains at  $(s_1, s_2) = (1, 1)$  thereafter.

Next consider the case  $\beta > \tilde{\beta}$ . Recall from the proof of Proposition 4 that if the economy is initially at  $(s_1, s_2) = (0, 1)$ , then for all  $\eta < \hat{\eta}(\beta)$ , we have  $\bar{\delta} < c(\bar{a}, \sigma_2) < c(\bar{a}, \sigma_1)$ . Hence all individuals in each of the two groups will find it optimal to remain unskilled, resulting in a transition to  $(s_1, s_2) = (0, 0)$ . This raises both  $c(\bar{a}, \sigma_2)$  and  $c(\bar{a}, \sigma_1)$ , and hence maintains the condition  $\bar{\delta} < c(\bar{a}, \sigma_2) < c(\bar{a}, \sigma_1)$ . Hence the economy remains at  $(s_1, s_2) = (0, 0)$  thereafter.

### *Proof of Proposition 6*

We need to show that when  $\eta = 1$ , there exists an asymmetric steady state within  $\Lambda$ . For any given  $(s_1, s_2) \in \Lambda$ , define  $x$  and  $y$  as the unique solutions to the following equations:

$$x = 1 - G(\tilde{a}(\delta(\beta x + (1 - \beta)s_2), s_1)) \quad (\text{A.10})$$

$$y = 1 - G(\tilde{a}(\delta(\beta s_1 + (1 - \beta)y), s_2)). \quad (\text{A.11})$$

Since the right-hand sides of both equations are decreasing in  $x$  and  $y$  respectively and are bounded between 0 and 1, the solutions are unique.

We claim that  $(x, y) \in \Lambda$ . To see this, suppose first that  $x > s_l$ . Then, since  $s_2 \geq s_h > s_l$ , we have  $\beta x + (1 - \beta)s_2 > s_l$ , and hence, since  $s_1 \leq s_l$  and  $(s_l, s_l)$  is a steady state, we have

$$x = 1 - G(\tilde{a}(\delta(\beta x + (1 - \beta)s_2), s_1)) < 1 - G(\tilde{a}(\delta(s_l), s_l)) = s_l.$$

This implies  $x < s_l$ , a contradiction. A similar argument may be used to show that  $y \geq s_h$ .

Now define the map  $L$  on  $\Lambda$  by  $L(s_1, s_2) = (x, y)$ . We have shown that  $L$  is a self-map on  $\Lambda$ . Since it is continuous and  $\Lambda$  is compact, the existence of a fixed point of  $L$  in  $\Lambda$  follows from Brouwer's fixed-point theorem. But examination of (A.10) and (A.11) shows that any such fixed point is an asymmetric steady state of the dynamics (4).

To prove the claim about skill shares and wages, note that since  $s_l$  is the smallest symmetric steady state, we must have

$$s > 1 - G(\tilde{a}(\delta(s), s))$$

for any  $s < s_l$ . But under complete segregation, this implies  $\Delta s_1 \equiv s_{1t} - s_{1t-1} > 0$  at any state  $(s_1, s_2)$  at which the aggregate skill share  $s$  satisfies  $s_1 < s \leq s_l$ , since

$$s_1 > 1 - G(\tilde{a}(\delta(s_1), s_1)) > 1 - G(\tilde{a}(\delta(s), s_1)).$$

Hence no such  $(s_1, s_2)$  can be a steady state. This proves that at any steady state  $(s_1, s_2) \in \Lambda$ , the aggregate skill share  $s$  must satisfy  $s > s_l$ . An analogous argument may be used to prove that  $s < s_h$ . The claim about wages follows immediately from this.

### *Proof of Proposition 7*

For each  $s_2 \in [0, 1]$ , define  $h_1(s_2)$  as the set of all  $s_1$  satisfying

$$s_1 = 1 - G(\tilde{a}(\delta(\beta s_1 + (1 - \beta)s_2), \eta s_1 + (1 - \eta)(\beta s_1 + (1 - \beta)s_2))).$$

This corresponds to the set of isoclines for group 1, namely the set of points at which  $\Delta s_1 \equiv s_{1t} - s_{1t-1} = 0$  for any given  $s_2$ . Similarly, for each  $s_1 \in [0, 1]$ , define  $h_2(s_1)$  as the set of all  $s_2$  satisfying

$$s_2 = 1 - G(\tilde{a}(\delta(\beta s_1 + (1 - \beta)s_2), \eta s_2 + (1 - \eta)(\beta s_1 + (1 - \beta)s_2))).$$

This is the set of points at which  $\Delta s_2 = 0$  for any given  $s_1$ . Any state  $(s_1, s_2)$  at which  $s_1 \in h_1(s_2)$  and  $s_2 \in h_2(s_1)$  is a steady state. Now consider the extreme case  $\eta = 0$ , and examine the limiting isoclines as  $\beta \rightarrow 0$ . In this case  $h_2(s_1)$  is the set of all  $s_2$  satisfying

$$s_2 = 1 - G(\tilde{a}(\delta(s_2), s_2)).$$

This equation has multiple solutions, the smallest of which is  $s_l$  and the largest is  $s_h$ . Hence there are at least two horizontal isoclines at which  $\Delta s_2 = 0$ . (Figure A.1 depicts the case of three solutions.)

Next consider the properties of  $h_1(s_2)$  for  $\eta = 0$  as  $\beta \rightarrow 0$ . This is the set of all  $s_1$  satisfying

$$s_1 = 1 - G(\tilde{a}(\delta(s_2), s_2)).$$

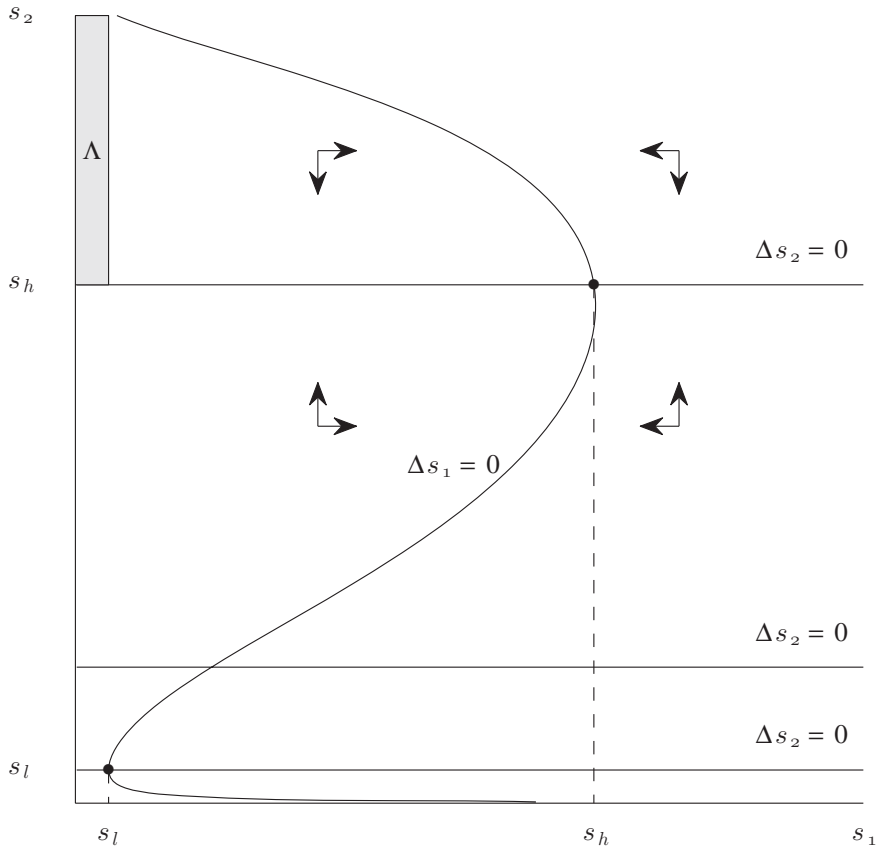


FIGURE A.1. Limiting Isoclines as  $\beta \rightarrow 0$  for  $\eta = 0$ .

Note that  $h_1(s_2)$  is single-valued and satisfies  $s_h \in h_1(s_h)$  and  $s_l \in h_1(s_l)$  since  $s_h$  and  $s_l$  correspond to symmetric steady states. Furthermore,  $1 \in h_1(0)$  and  $0 \in h_1(1)$  since  $\delta(1) = 0$  and  $\lim_{s \rightarrow 1} \delta(s) = \infty$ . These general properties are depicted in Figure A.1.

Now consider the asymmetric steady state when  $\eta = 1$ , which lies in  $\Lambda = [0, s_l] \times [s_h, 1]$  (the shaded area of Figure A.1). As is clear from the figure, any such initial state must lie in the basin of attraction of the high-investment symmetric steady state  $(s_h, s_h)$ . Since the isoclines are all continuous in  $\eta$  and  $\beta$  at  $\eta = \beta = 0$ , it follows that for  $\beta$  sufficiently small, integration is equalizing and skill-increasing.

The proof of the claim for large  $\beta$  is similar. Consider the limiting isoclines as  $\beta \rightarrow 1$  (maintaining the assumption that  $\eta = 0$ ). In this case  $h_1(s_2)$  is the set of all  $s_1$  satisfying

$$s_1 = 1 - G(\tilde{a}(\delta(s_1), s_1))$$

and  $h_2(s_1)$  is the set of all  $s_2$  satisfying

$$s_2 = 1 - G(\tilde{a}(\delta(s_1), s_1)).$$

The largest and smallest solutions to the former equation are  $s_l$  and  $s_h$  respectively, which correspond to vertical isoclines in  $(s_1, s_2)$  space; these are all points at which  $\Delta s_1 = 0$ . The latter equation generates a single isocline  $s_2 = h_2(s_1)$  which satisfies  $s_h \in h_2(s_h)$ ,  $s_l \in h_2(s_l)$ ,  $1 \in h_2(0)$  and  $0 \in h_2(1)$ .

It is easily verified that in this case, the asymmetric steady state with  $s_1 < s_l$  and  $s_2 > s_h$  must lie in the basin of attraction of the low-investment symmetric steady state  $(s_l, s_l)$ . (This can be seen from Figure A.1 by relabeling the axes and observing that the initial state now lies in the bottom right of the figure.) Since the isoclines are all continuous in  $\eta$  and  $\beta$  at  $\eta = 0$  and  $\beta = 1$ , it follows that for  $\beta$  sufficiently large, integration is equalizing and skill-reducing.

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