# THE LIMITATIONS OF PIGOUVIAN TAXES AS A LONG-RUN REMEDY FOR EXTERNALITIES*

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## I. INTRODUCTION

A central problem of economic policy is to assure efficiency of the competitive process when externalities are present. One traditional method of dealing with an externality is the imposition of a Pigouvian tax (per unit tax) on the externality-generating activity [Pigou, 1927, and Baumol, 1972]. This paper shows that contrary to widely held beliefs use of such a tax will not in general lead to an efficient allocation of resources in the long run. The source of the inefficiency of the tax is really quite simple. A per unit tax uniformly raises a firm's average cost curve, and therefore leads the firm in the long run to minimize average cost at the same output as in the pre-tax situation. In general, however, the output that is socially optimal in the presence of externalities is not the output that minimizes the firm's average production costs.[1] However, if one supplements the Pigouvian tax with a lump sum tax-subsidy scheme for participating firms, then a socially efficient allocation can be achieved.

## II. THE MODEL

We present a simple partial-equilibrium model to illustrate our arguments. Imagine a competitive industry consisting of identical firms producing a homogeneous product. Each firm incurs production costs $C(q)$ when producing $q$ units of output. Production in this industry also imposes additional damages on society due to some external effect. We assume these external damages resulting from in-

* This work was initiated while both authors were visiting the Economics group at Bell Laboratories. We thank John Panzar, A. Mitchell Polinsky, and an anonymous referee for helpful comments.

1. An example can illustrate this point simply. Suppose that steel is produced with a technology that generates a U-shaped average cost (AC) curve with the min AC point at output $q^*$. Suppose that steel production causes pollution damage. This pollution damage need not be constant per unit of steel produced by the firm. For example, starting up the steel furnace may generate pollution damage that is independent of output, in which case average pollution damage per ton of steel produced could fall initially, or alternatively the amount of smoke emitted per ton may depend on how many tons of steel the firm is producing. The minimum average pollution damage per unit of steel output will not in general occur at the output $q^*$ which minimizes average production costs. In this case, Pigouvian taxation alone cannot achieve efficiency.


*The Quarterly Journal of Economics,* November 1980 0033-5533/80/0095-0559$01.00

dustry production may be written as $D(n,q)$, where $n$ is the number of firms in the industry and $q$ is the output of each firm. Thus, we restrict ourselves to symmetric behavior by active firms, and implicitly assume that external damages are invariant with respect to changes in firms' employment of factors of production that keep output constant. Average production costs $C(q)/q$ are taken to be U-shaped with the minimum average production cost occurring at scale $q_s$. $P(Q)$ represents the industry's inverse demand function, where $Q = nq$ is total industry output. We assume that the efficient scale of firm production is "small" relative to the size of the market, so that the indivisibility of firms may be neglected and the number of firms $n$ can be taken to be a continuous variable. Finally, we assume interior solutions to all maximum problems.

The welfare criterion is the usual sum of consumer plus producer surplus.[2] The short run is defined as a situation in which the number of firms $n$ is fixed.[3] A Pigouvian tax $t$ is a charge to each firm of $\$t$ for each unit of output produced.

A short-run competitive equilibrium with Pigouvian tax $t$ and number of firms $n$ is a price-quantity pair $(p,q)$ such that supply equals demand and price equals private marginal costs. Using the definition of the inverse demand curve, we can summarize these two conditions by the single condition,

$$P(nq) = C'(q) + t. \tag{1}$$

A short-run social optimum with number of firms $n$ is a price-quantity pair $(p,q)$, which maximizes social welfare. If we again use the inverse demand curve to define $p$, we can completely characterize the optimal $q$ as the solution to

$$\max_{q \geq 0} \int_0^{nq} P(s)ds - nC(q) - D(n,q).$$

Hence $q$ satisfies the first-order condition,

$$nP(nq) = nC'(q) + \frac{\partial D}{\partial q}(n,q),$$

2. The results of this section are unchanged if instead of using a surplus criterion to measure social welfare, we use a Bergsonian social welfare function. Also, the results are unchanged if the problem is reformulated as one in which the number and output of firms of one industry affect the cost curves of other competitive industries.

3. We assume that in the short run, all $n$ identical firms operate. A rising marginal cost curve guarantees this condition, provided that any output is produced at all. See Polinsky [1977] for a discussion of the short-run effects of taxation when firms differ in their production technology.

or

$$P(nq) = C'(q) + \frac{1}{n}\frac{\partial D}{\partial q}(n,q). \tag{2}$$

The following result provides a frequently used justification for the use of Pigouvian taxation.

THEOREM 1. For the appropriately chosen tax $t$, the short-run competitive equilibrium coincides with the short-run social optimum.

*Proof.* It is necessary to show that if $q^*$ satisfies (2), it also satisfies (1) for the appropriately chosen $t$. Suppose that $q^*$ solves (2) and define $t = (1/n)(\partial D/\partial q)\,(n,q^*)$. Then, it follows immediately from (2) that $q^*$ also satisfies (1).

Q.E.D.

In the long run, entry or exit may occur, so the number of firms $n$ is free to vary. A long-run competitive equilibrium with tax $t$ (LRCE) consists of a price-quantity-number of firms triple such that supply equals demand, price equals private marginal cost, and profits equal zero. Again using the definition of the inverse demand curve, we can completely characterize a LRCE by

$$P(nq) = C'(q) + t, \tag{3}$$

and

$$qP(nq) = C(q) + tq. \tag{4}$$

A long-run social optimum (LRSO) is a price-quantity-number of firms triple that maximizes social welfare. Using the definition of the inverse demand curve, we can completely characterize a LRSO by the $q,n$ that solve

$$\max_{\substack{q \geq 0 \\ n \geq 0}} \int_0^{nq} P(s)ds - nC(q) - D(n,q).$$

The first-order conditions are

$$nP(nq) = nC'(q) + \frac{\partial D}{\partial q}(n,q)$$

or

$$P(nq) = C'(q) + \frac{1}{n}\frac{\partial D}{\partial q}(n,q), \tag{5}$$

and

$$qP(nq) = C(q) + \frac{\partial D}{\partial n}(n,q). \tag{6}$$

Theorem 1 asserts that any short-run social optimum may be attained as a short-run competitive equilibrium with the appropriately chosen Pigouvian tax rate. However, as competitive entry occurs, the tax rate must be adjusted to maintain short-run optimality. That this adjustment process need not result in a long-run social optimum is illustrated by the following result.

THEOREM 2. In general there exists no Pigouvian tax rate $t$ such that the long-run competitive equilibrium (LRCE) coincides with the long-run social optimum (LRSO).

*Proof.* It is necessary to show that if $t$ is the Pigouvian tax, then if $n^*, q^*$ satisfy (5) and (6) (i.e., $n^*, q^*$ are a LRSO), they will not also satisfy (3) and (4) (i.e., $n^*, q^*$ are not a LRCE). First, notice that if $t$ does not equate social to marginal cost, then it is obvious that (3) cannot be satisfied by $q^*, n^*$. To complete the proof, we must show that if $t$ is chosen to equate social to marginal cost, then (4) will not be satisfied by $n^*, q^*$.

Suppose that $q^*, n^*$ satisfy (5) and (6) and let the Pigouvian tax $t^* = (1/n^*)(\partial D/\partial q)(n^*,q^*)$. For such a $t^*$, it is clear from (5) that $(n^*,q^*)$ will also satisfy (3). However, for such a $(t^*,n^*)$, $q^*$ will not in general satisfy (4). To see this, rewrite (6) as

$$q^*P(n^*q^*) = C(q^*) + t^*q^* - t^*q^* + \frac{\partial D}{\partial n}(n^*,q^*),$$

or

$$q^*P(n^*q^*) = C(q^*) + t^*q^* + q^*\left[\frac{1}{q^*}\frac{\partial D}{\partial n}(n^*,q^*) - t^*\right], \tag{7}$$

or

$$q^*P(n^*q^*) = C(q^*) + t^*q^* + F^*,$$

where

$$F^* = q^*\left(\frac{1}{q^*}\frac{\partial D}{\partial n}(n^*,q^*) - t^*\right),$$

or using the definition of $t^*$,

$$F^* = q^*\left[\frac{1}{q^*}\frac{\partial D}{\partial n}(n^*,q^*) - \frac{1}{n^*}\frac{\partial D}{\partial q}(n^*,q^*)\right]. \tag{8}$$

It is immediately obvious from (7) that $t^*, n^*, q^*$ will satisfy (4) iff $F^* = 0$. But there is no reason why $F^*$ should always equal 0 (e.g., if $D(n,q) = nq^{1/2}$, then $F^* > 0$ for any $q > 0$).

Q.E.D.

The reasons for this failure of a pure Pigouvian tax instrument can be explained intuitively as follows. There are essentially two quantities, number of firms and scale of each firm, which the government is trying to control with just one instrument, the Pigouvian tax. In the short run the structure of the industry ($n$) is fixed and individual firm output is the only matter of concern. This last quantity can be completely controlled by a Pigouvian tax, which shifts marginal private costs so that they coincide with the marginal social costs at the socially optimal level of industry (and individual firm) production. In the long run, however, the industry structure is flexible and should be adjusted optimally along with individual firms' output. Yet the number of firms and the output of each firm cannot in general be varied independently in the long-run competitive equilibrium with just a Pigouvian tax. This is because the tax shifts the firm's average cost curve upward in a parallel manner. Minimum average tax-inclusive cost always occurs at $q_s$,[4] regardless of the tax rate $t$. By varying $t$, the government affects the number of firms but not their scale in the long run.

Only in the special case where, for any given industry output, externality damage is *independent* of individual firm scale (as occurs when constant returns to scale characterize the externality generation) will $q_s$ be the optimal scale of the firm in the long-run social optimum. In such a case, the scale of an individual firm has no effect on externality damages, and the optimal scale is determined solely by production considerations. In this very special case, Pigouvian taxes can lead to the long-run social optimum.[5] The following theorem illustrates this point.

THEOREM 3. If for any given industry output, externality damages are independent of individual firm output so that $D(n,q) = \overline{D}(nq)$, then with appropriate Pigouvian taxation the long-run competitive equilibrium (LRCE) coincides with the long-run social optimum (LRSO).[6]

4. Recall that $q_s$ is the output at which average production cost is minimized.

5. The intuition underlying this result should be clear. If for any industry output, externality damages are independent of firm scale, then only production cost considerations influence optimal firm scale. Competition takes these production considerations into account. Hence the tax need regulate only the number of firms, since competition takes care of optimal scale. One instrument can regulate one variable perfectly.

6. This result also appears in Schultze and D'Arge [1974].

*Proof.* Suppose that $n^*,q^*$ are a LRSO and therefore satisfy (5) and (6). If $D(n,q) = \overline{D}(nq)$, then from (8) it is obvious that $F^* = 0$, and hence it follows from (7) that if $t^* = (1/n^*)(\partial D/\partial q)\ (n^*,q^*)$, then $n^*$, $q^*$, and $t^*$ will satisfy (4), and it follows from (5) that they satisfy (3). Hence $(n^*,q^*)$ represents a LRCE.

Q.E.D.

If the government can levy a lump sum entry tax-subsidy on active firms in the industry along with the Pigouvian tax, then the long-run social optimum can be attained. Define a long-run competitive equilibrium with Pigouvian tax $t$ and lump sum tax $F$ as the triple $(p,q,n)$, which satisfies supply equals demand, price equals marginal cost, and profits equal zero. Using the inverse demand curve, we can write these conditions as

$$P(nq) = C'(q) + t, \tag{9}$$

and

$$qP(nq) = C(q) + tq + F. \tag{10}$$

We can now prove the following.

THEOREM 4. There is always a tax policy $(t,F)$ such that the long-run competitive equilibrium (LRCE) coincides with the long-run social optimum (LRSO).

*Proof.* It is necessary to show that if $q^*,n^*$ are a LRSO and hence satisfy (5) and (6), then there is a $t^*$ and $F^*$ such that $q^*,n^*$ are a LRCE and hence satisfy (9) and (10).

Choose $t^* = (1/n^*)(\partial D/\partial q)\ (n^*,q^*)$ and define $F^*$ as in (8). It is obvious from (5) that (9) is satisfied. It follows immediately from (7) that (10) is satisfied. Hence $q^*,n^*$ represent a LRCE.

Q.E.D.

This simple result confirms intuition and assures that a Pigouvian tax supplemented by a lump sum tax can restore full optimality. The optimal lump sum $F^*$ can be either negative, zero, or positive. However, as long as damages depend positively on output, the total tax payment $(F^* + t^*q^*)$ will always be positive. The optimal lump sum tax $F^*$ will be positive (negative) when average pollution damage is falling (rising) at the optimal firm output.[7] Moreover, when payment of a lump sum subsidy is indicated $(F^* < 0)$, it can be effected

7. The proof of these results is available on request.

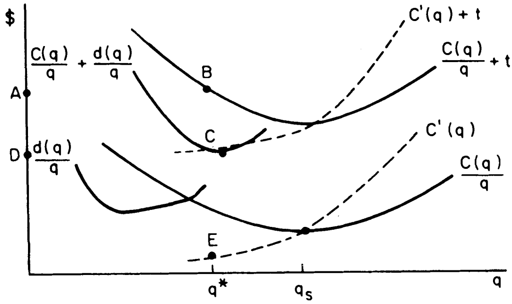


FIGURE I

by exempting the firm from payment of the Pigouvian tax on a certain amount (— $F^*/t^*$) of its output.[8]

## IV. A Special Case

The case $D(n,q) = nd(q)$, in which the damage cost from the externality is additive across firms, is of particular interest because of its simplicity. Here the social cost function for the firm is simply $C(q) + d(q)$. The socially efficient solution requires that firms should pay a lump sum tax if the average damage function is falling at the optimal output level ($d'(q^*) > d(q^*)/q^*$). For the case where $d(q) = q^\alpha$, $\alpha > 0$, a lump sum tax or subsidy is required as $\alpha$ is less or greater than one. When $\alpha > 1$, the optimal lump sum subsidy can be determined without further knowledge of demand or technology. For this damage function, the optimal subsidy is equivalent to a tax exemption of $1 - 1/\alpha$ of a firm's output.

Figure I illustrates the determination of the optimal tax rate and subsidy for the case where $d(q)/q$ is rising at the social optimum. As the figure suggests, only if $d(q)/q$ is constant or else just happens to achieve a minimum at the same point as $C(q)/q$, can a simple Pigouvian tax guarantee a long-run social optimum. Here $q^*$ is the socially optimal scale, while $q_s$ is the scale that would prevail under simple Pigouvian taxation. The optimal subsidy is *ABCD* and the optimal tax is *CE*.

8. Proof available on request. Notice that this way of distributing the subsidy completely avoids the problem often mentioned with regard to subsidies, that firms will simply collect the subsidy and produce zero output. Since it can be shown in general that total tax payments are always positive, this method of distributing subsidies can always be used. Moreover, if the exemption is interpreted as the property right allocation of the firm to pollute, it follows that only one assignment of property rights will be efficient.

## V. Summary

This paper has argued that Pigouvian taxes alone cannot be expected to correct the most common forms of externalities in the long run. Pigouvian taxes alone are incapable of providing firms in long-run equilibrium with an incentive to operate at a scale of plant other than the one that minimizes average private cost. It is only by using lump sum subsidy or entry fees that a policy maker could guarantee that both marginal production incentives *and* incentives for entry into the industry are efficient.

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