Racial Discrimination in Labor Markets with Posted Wage Offers

[Online Version]

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Abstract:

We develop a model of race discrimination in labor markets in which offered wage rates are posted along with job openings. If an employer with a job vacancy and a posted wage receives a pool of applicants, she chooses the most qualified and, if the best applicants are equally qualified, she is free to make an arbitrary, or perhaps a discriminatory, choice. We develop a game in which firms post wages, workers choose where to apply, and firms decide which workers to hire. The key result is that labor-market frictions can greatly amplify perceived racial disparities, so that mild discriminatory tastes or small productivity differences can produce large wage differentials between the races. Compared with the nondiscriminatory equilibrium, the discriminatory equilibrium features lower net output, lower wages for both white and black workers and greater profits for firms. We argue that this result generalizes to a number of different cases, including some in which blacks and whites are equally productive on average. Although based entirely on individualistic maximizing behavior without collusion or cooperation, the model has a flavor that is reminiscent of certain Marxian models in which capitalists increase profits by dividing workers against themselves.

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1 Introduction

Economic theory suggests that wage discrimination against groups of workers is unlikely to persist in a competitive economy, because in the presence of such discrimination profits can be had by hiring members of the discriminated-against groups. Consequently, in trying to account for differences in the treatment of worker groups, economists have tended to rely either on real productivity differences or else on market imperfections that tend to block the anti-discrimination market response.

In this paper, we offer a model of labor market friction which greatly magnifies weak discriminatory preferences or small productivity differences into large wage differentials. To do so, we analyze labor markets characterized by wage posting, wherein employers attach wage offers to announced job openings. Wage posting is a commonly observed labor-market phenomenon, probably because workers would be less likely to invest in the job application procedure for an unknown wage. In our scenario, the posted wage offers are binding on the employer, and they cannot be conditioned on the identity of the worker to be hired.

Below, we show that wage posting lends itself to persistent discrimination. The model has the flavor of monopsonistic competition, with a large number of firms that post wage offers and a large number of workers that respond to the posted wage offers. The intuition is straightforward. Because her wage has been posted, the employer cannot pay less to applicants who are less qualified or who are subject to discrimination elsewhere in the economy. Thus the anti-discrimination market response cannot function.

In our baseline model, employers find African American workers to be slightly less desirable employees than white workers, even when observable match-specific productivity differences are taken into account. Although perceived differences are small, they are sufficient to ensure that employers will always choose a white worker in preference to a black worker if both apply for the same job. Black workers have an incentive to avoid the cost of applying to firms that are likely to receive applications from whites. One way that blacks can accomplish this is to apply to firms with posted wage offers sufficiently low as to discourage white applicants. In equilibrium, blacks and whites will be employed by different firms (segregation), blacks will receive lower wages with the wage differential exceeding the taste or productivity differential (wage discrimination), and firms will retain higher profits.

Our argument shows that the labor-market structure we depict will amplify even modest racist tendencies or small productivity differences to yield highly visible economic outcomes with important social consequences. This is not to deny that active racism exists in the labor market; we mean to suggest only that even mild racist tendencies can produce segregated workplaces and wage discrimination against blacks. Indeed, in the limit, our equilibrium holds even when whites and blacks are equally productive and no employer has any racial preferences whatever, that is, even when firms are unwilling to pay anything more in order to hire a white in preference to a black worker.

Our results require that firms be committed to their posted wage offers, and that wage offers cannot be conditioned on the type of worker. Race-contingent posted wage offers would be an egregious and public violation of civil rights legislation that most employers would wish to avoid. Furthermore, in white racist social environments, wage discimination in favor of blacks would be a gross violation of social norms, and wage discrimination againt blacks would inevitably lead to hiring discrimination in their favor, also socially proscribed. Thus we should not expect to see race-contingent wage offers, even in the absence of civil rights legislation. Evidence from the 19th and early 20th century American South supports this view (Higgs, 1989).

Our principal conclusion is that in an economic environment with posted wage offers, segregation and wage discrimination against black workers can arise even when all information is symmetric, information about posted wage offers and about employers' discriminatory behavior is perfect and employers and workers lack substantial racist motives. This discrimination creates economic inefficiency, reduces total output, decreases wages for both black and white workers and increases profits.

Although based entirely on individualistic maximizing behavior without collusion or cooperation, the model has a flavor that is reminiscent of certain Marxian models in which capitalists increase profits by dividing workers against themselves.

2 The Wage-Posting Model in a Nondiscriminatory Regime

We analyze the wage-posting game without discrimination, or equivalently, with homogeneous workers. The solution of the game will serve as a benchmark, and additionally, it will yield the solution for white workers in the game with discrimination against blacks. We draw on a model sketched in Lang (1991) and formalized in Montgomery (1991). Proofs and much of the technical material are to be found in the appendix.

Suppose that all workers are equally productive, and firms make no distinctions between them. Each firm has one unfilled position and posts a wage in the hope of attracting an applicant. Workers observe the wage offers that have been posted and decide where to apply. Workers can apply for only a limited number of jobs (one in the formal model). Recognizing that higher wage offers are likely to attract more applicants for the limited number of openings at the firm (again one, in the formal model), workers trade off higher wage offers against a lower probability of employment. Firms recognize that raising the wage will increase the expected number of applicants and thus lower the probability of having to bear the cost of an unfilled job vacancy. For now we take the number of firms as fixed.

Consider a two-stage game with a large and fixed number N of identical firms and random number $\tilde{Z}$ of identical workers, where $\tilde{Z}$ is Poisson distributed with mean $Z \equiv E(\tilde{Z})$. This is the distribution that would arise if agents from a large population were to make independent and equally probable decisions to enter the job market. It will be important to our model that the realization of the random variable $\tilde{Z}$ not be observable, either to firms or to workers. By contrast, the mean Z of the distribution is assumed to be common knowledge.

In the first or wage-posting stage of the game, firms simultaneously announce their wage offers, which they are committed to pay each worker hired. In the second or worker-application stage, workers observe the profile of wage offers and simultaneously apply to firms for jobs. Each worker j applies to a single firm i. At the end of the game, firms apply the following hiring rules to their applicants: a firm that receives no applications cannot hire or produce; a firm that receives one application hires that applicant; and a firm that receives more than one application chooses hires one applicant at random.

Firm i's strategy consists of its choice of a single wage offer w_i. The vector $W \equiv \langle w_i \rangle$ denotes the profile of strategies for all firms. A worker's (mixed) strategy is a vector-valued function $q(W) \equiv \langle q_i(W) \rangle$, where each $q_i(W)$ is the probability that the worker will choose to apply to firm i. We restrict the worker's strategy choices to those consistent with the anonymity of firms: if $w_i = w_j$ then $q_i(W) = q_j(W)$. If all workers adopt the same mixed strategy, the number of workers that apply to a given firm i will have a Poisson distribution, whose mean we denote by z_i.

Firm i's payoff is its operating profits (revenue minus variable cost). When all workers adopt mixed strategy q, the expected operating profits of firm i are

$$\pi_i = (1 - e^{-z_i})(v - w_i), \tag{1}$$

where v is the value of the worker's output, and $1 - e^{-z_i}$ is the probability that the firm fills its vacancy. A worker's payoff is the firm's wage offer w_i if he is hired by firm i, and zero, otherwise.

We proceed to search for an equilibrium $\{W^*, q^*(W)\}$ of the wage-posting game that is symmetric among workers (all workers use the same mixed strategy $q^*(W)$). In the solution concept as applied to the wage-setting stage of the game, we substitute the common notion of a competitive equilibrium for that of a Nash equilibrium: the only difference being that in competitive equilibrium agents are required to be price-takers in a sense to be described below, whereas in Nash equilibrium agents are required to take into account even the very small effect that their own behavior may have on market prices. We will use the term "subgame-perfect competitive equilibrium" to describe a solution concept for a multistage game that is parallel to subgame-perfection, but with a competitive equilibrium substituted for a Nash equilibrium in the first stage.

We find the unique symmetric equilibrium of the worker-application game. Workers trade off each firm's wage offer against the expected number of competing job applicants so as to maximize their expected incomes. In equilibrium, workers obtain the "market expected income" at every firm to which they apply. Workers will apply with positive probability to any firm that offers a wage above the market expected income, and the expected number of applicants will rise to a level exactly sufficient to reduce expected income at that firm to the market level. Workers will not apply with positive probability to any firm that offers a wage less than or equal to the market expected income, because competition from other applicants (no matter how slight) would force expected income to fall below the market level. The market expected income is increasing in the wage offers of firms and decreasing in the number of workers in the pool of applicants. A firm's expected number of applicants (and the probability that each worker will apply) is a continuous function of its wage offer. If the wage offer is increased by a small amount, the expected number of applicants will rise until the expected income at the firm falls back to the market level (now very slightly higher). We proceed to model the situation more formally.

Let the wage-offer profile of firms be $W = \langle w_i \rangle$ with $W \neq 0$, and consider the worker-application subgame, in which workers apply for jobs. Suppose a firm has a pool of potential job applicants, each with the same non-negative probability of applying to that firm. Let z denote the expected number of applicants to the firm from that pool. Now imagine that an additional designated worker applies to the firm. The probability $f(z)$ that the additional applicant will be hired is given by

$$f(z) = \begin{cases} 1 & \text{for} \quad z = 0 \\ \\ (1 - e^{-z})/z & \text{for} \quad z > 0 \end{cases}, \tag{2}$$

which is derived in the appendix.

Given the wage-offer profile W, let $q(W)$ denote a symmetric equilibrium strategy for workers in the worker application subgame, where q_i is the probability that each worker applies to firm i. Note that the relation between z_i and Z is given by

$$z_i = q_i Z. \tag{3}$$

From the point of view of firm i, Z and z_i represent the expected number of job market entrants and applicants to the firm; from the point of view of a designated worker in the labor market, Z and z_i represent the expected number of *other* job market entrants and applicants to firm i *aside from himself.*[1] Thus, if K_i denotes the expected income or payoff that the designated worker can obtain by applying to firm i, we have

$$K_i \equiv w_i f(z_i). \tag{4}$$

Suppose now that firms have set wage offers $W \equiv \langle w_i \rangle$, and suppose that the worker application subgame has an equilibrium in which all workers adopt the same mixed strategy. Let $K \equiv \max\{K_i\}$ denote the maximum expected income available in that equilibrium. Because workers will choose to apply only to firms with $K_i = K$, we may think of K as the market expected income. If a firm i offers a wage w_i greater than K, then the expected number of applications z_i will be large enough

[1] The intuition is that a very large number of potential workers may independently enter the job market with a very small probability. The *ex ante* probability of entry by a designated worker is so small that, *ex post*, his realized entry (observed only by the worker himself) boosts the expected value of the total number of entrants by an amount close to 1. Thus, in the limit, the expected number of labor-market entrants from the point of view of a worker in the market is one greater than the expectation formed by outsiders. The same logic applies to applicants to a firm.

to reduce K_i to K. If a firm offers a wage w_i less than or equal to K, then K_i must be less than K, even when the expected number of applicants is very small. Thus no worker will apply to such a firm in equilibrium. We have

Proposition 1 *In any symmetric equilibrium of the worker application subgame, K_i is given by:*

$$K_i = \begin{cases} K & \text{for} \quad w_i \geq K \\ \\ w_i & \text{for} \quad w_i < K \end{cases}, \tag{5}$$

z_i satisfies

$$\begin{array}{l} z_i > 0 \quad \text{for} \quad w_i > K \\ \\ z_i = 0 \quad \text{for} \quad w_i \leq K \end{array}, \tag{6}$$

and

$$z_i = f^{-1}(\tfrac{K}{w_i}) \quad \text{for} \quad w_i \geq K \ . \tag{7}$$

Given W, the vector of the expected numbers of applicants $\langle z_i \rangle$ to the individual firms depends only on the ratio Z/N, the expected number of workers in the labor force divided by the number of firms. The larger is Z/N, the lower the probability that a given job applicant will be selected, so the lower will be market expected income K. Inasmuch as Z/N is parametrically specified in the model, only one value of $\langle z_i \rangle$, one value of K, denoted by $K^*(W)$, and one vector $\langle q_i \rangle$, denoted by the vector-valued function $q^*(W)$, can be consistent with a given W in a subgame equilibrium. It follows that there is a unique symmetric equilibrium of the worker application subgame.

Now we search for equilibria of the wage-posting game. Our solution concept will be the *subgame-perfect competitive equilibrium (SPCE)*, a simplification of standard subgame-perfection in which aggregate variables are assumed constant with respect to the changes in the strategy of an individual agent. We say that $\{W^*, q^*\}$ is a subgame-perfect competitive equilibrium if

i. each firm's w_i^* is a best response to the other components of W^* and to the the workers' strategy q^* on the assumption that the market expected income $K^*(W)$ remains fixed at $K^*(W^*)$ and is not sensitive to the firm's own wage;[2] and,

[2] This is a reasonable assumption for firms to make if N, the number of firms, and Z, the expected number of workers in the job-market, are both large. If firm i raises its offered wage to a level greater than w_i^*, expected income at firm i will increase. But then workers will increase the probability of applying to firm i until the expected income at firm i falls back to the expected income at all other firms. However, because applicants moved from other firms to firm i, each of the other firms will suffer a small net loss of applicants, which will in turn create a small increment in the expected income of each worker throughout the market. Just as competitive suppliers are assumed to ignore the effect of their own actions on market price, so our firms are assumed to ignore the the effect of their own wage movements on the market-wide expected income of workers.

In formal games, competitive equilibria is usually modeled with a continuum of agents, but for this case, our refinement yields a far simpler model.

ii. $q^*(W)$ is a best response of each worker to any vector of offered wages, W,and to the choice of $q^*(W)$ by all other workers.

The term "subgame-perfect" refers to the game-theoretic concept and not to perfect competition. Indeed the structure of the labor market we are analyzing is more akin to monopsonistic competition than to perfect competition. In this market, each of the many firms acts like a monopsonist: each weighs the benefit of higher wage offers for attracting workers against the increased cost if a worker is hired.

Let $r \equiv Z/N$ denote the ratio of the expected number of job applicants to the number of firms.

Proposition 2 *The game between firms and workers has a subgame-perfect competitive equilibrium $\{W^*, q^*\}$ that is unique among those in which all workers adopt the same mixed strategy. In this equilibrium, workers adopt the strategy q^*as defined in the preceding section. Firms adopt the strategy profile W^*, where for all firms i, the wage offer is given by*

$$w_i^* = \frac{vr}{e^r - 1}; \tag{8}$$

the expected income of workers, by

$$K^*(W^*) = ve^{-r}; \tag{9}$$

and operating profits, by

$$\pi_i^* = [1 - (1 + r)e^{-r}]v. \tag{10}$$

As r goes from 0 to ∞, π_i goes from 0 to v (here normalized to 1) and w_i^* and $K^*(W^*)$ go from v to 0.

A formal proof is available in the appendix, but the basic steps of the derivation are straightforward. We know from (7) that z_i satisfies $w_i = K^*(W)/f(z_i)$, and substitution into (1) yields

$$\pi_i = (1 - e^{-z_i})v - z_i K^*(W). \tag{11}$$

With $K^*(W)$ held constant, the first-order condition implies

$$z_i^*(W) = \log \frac{v}{K^*(W)}, \tag{12}$$

and it follows that $z_i^*(W)$ is the same for all i. Since each worker applies to exactly one firm, we have $z_i^* = Z/N = r$, so that (9) follows from (12). Substitution into (7) and (11) and the definition of f then yield (8) and (10).

The equilibrium in Proposition 2 is unique among those in which all workers have the same expected income. We believe that it is also unique among those in which any wage that is offered is offered by a large number of firms. We cannot rule out equilibria in which individual workers and firms are able to circumvent the anonymity of the labor market by coordinating on a unique wage that only one firm offers and for which only one worker applies. However, in the context of a large labor market, we find such equilibria implausible.[3]

3 The Wage-Posting Model in a Discriminatory Regime

In this section we generalize the model developed in the previous section in order to characterize statistical or rational discrimination. In this model, there are two types of workers, black workers and white workers. The total number of white workers is a Poisson-distributed random variable $\tilde{Z}$ with mean Z, and the total number of black workers is a Poisson-distributed random variable $\tilde{Y}$ with mean Y, where both Z and Y are assumed to be large. The number of firms, N, is also assumed to be large. This quasicompetitive structure has the advantage of yielding compact closed-form solutions, which, in turn, permit straightforward comparative statics.[4]

We assume that, as in the previous section, the productivity of white workers is given by v, but black workers may be slightly less productive than white workers. Let $\delta \geq 0$ represent the productivity difference between white and black workers expressed as a fraction of white productivity. Whether this represents a difference in physical productivity or in output net of a distaste parameter is inconsequential for the model. We refer to δ as measuring a productivity difference but this need not be viewed as a difference in physical productivity. Indeed we prefer the taste interpretation. However, the productivity interpretation makes the presentation easier since we can refer to "profits" rather than "profits net of the disutility of employing black workers." The parameter δ is assumed to be small or zero.

As before, the first stage of the wage-posting game is the wage-setting stage. Firms simultaneously announce their wage offers, and we let $W \equiv \langle w_i \rangle$ denote their wage profile. Each firm is committed to its posted wage and cannot hold up applicants by reducing the wage offer latter. As explained in the introduction, the wage offer cannot be conditioned on the type of worker.

The second stage is the job-application stage. Workers observe W and apply to firms. As before, workers adopt mixed strategies of the form $q \equiv \langle q_i \rangle$, consistent with the anonymity of firms: if $w_i = w_j$, then $q_i = q_j$.

The discriminatory wage-posting game has a third stage that we call the hiring stage. In the nondiscriminatory game, the third stage was represented by a simple hiring rule: the employer chose randomly among all applicants. In this game, however, the employer can choose his hiring

[3] For example, for some parameter values there may be an equilibrium in which one worker follows the strategy described above except that if he observes a single offer of 5.134, he applies for that job with probability 1. The other workers follow the strategy described above except that if they see exactly one offer of 5.134, they never apply for that job. One firm offers 5.134. All other firms follow the strategy described above. In this way, the one firm and one worker are able to circumvent the anonymity of the market. We find this level of implicit coordination implausible.

[4] However, Rafael Repullo pointed out to us that both segregation and wage discrimination can be obtained by modeling a small number of firms and workers. Repullo established this fact in response to a presentation of this paper at CEMFI in Madrid. He used a game with two firms, two white workers and one black worker.

policy. For now we assume that aside from race there are no observable productivity-relevant differences between workers. Or equivalently, we could assume that the cost of screening workers would be greater than the potential productivity gain from identifying workers with high general or match-specific productivities.

If δ, the productivity-reduction parameter, is positive and if employment discrimination is not penalized, then discriminatory hiring policy in favor of whites would be the employer's best response. The employer would choose randomly among white applicants if he had white applicants; otherwise he would choose randomly among black applicants. If there were a sufficiently large penalty for employment discrimination, then a nondiscriminatory hiring policy would be the best response. Here, he would choose randomly among all applicants. If δ is zero and if there is no discrimination penalty, then all hiring policies are best responses, including discrimination in favor of blacks, or applying different hiring probabilities to whites and blacks.

If firms do not discriminate at the hiring stage, then the first two stages of the game in this section are equivalent to the nondiscriminatory game of the previous section, and the equilibrium (with minor notation changes) will be identical to the one described in Proposition 2. Now we proceed to identify and analyze the equilibria that are obtained when the discriminatory hiring strategy is the adopted best response. We search for equilibria that are symmetric among workers of a given type; that is, all workers of the same type adopt the same mixed strategy. As before, our solution concept for the wage-setting stage will be the subgame-perfect competitive equilibrium.

In the context of our analysis, firms cannot make a credible commitment to hiring policy inconsistent with its best response (in the hiring stage). In particular, the structure of our model is based on the presumption that a firm's promise not to discriminate against blacks (or, stronger, to discriminate in their favor) would not be believed by black workers. This is consistent with the economic environment we are modeling, one in which firms are anonymous and thus lack reputations. In this sort of environment, contractual or other legal enforcement of specific antidiscrimination hiring policies is difficult, and without reputations at stake, firms would have little reason to keep promises.[5] This being said, we would like to point out that the inability of firms to make credible commitments is not essential to our argument. We can extend our results to the case in which all workers believe that there is a fixed positive probability that a firm's claim of nondiscrimination would be truthful. We can show that the basic structure of the equilibrium (segregation, lower wages at firms that attract blacks) would extend to such a model and that all firms seeking to attract blacks would announce that they do not discriminate.

[5] Shimer (2001) considers the case of race-contingent wage, ruled out in this model. Surprisingly, with race-contingent wages, if the productivity difference between the two groups is not too large, the equilibrium involves both groups applying for the same jobs but the *less* productive workers being offered *higher* wages conditional on being hired. Of course, since they are both less costly and more productive, the more productive workers are always hired in preference to the other workers. The counterfactual prediction that blacks would receive higher wage offers than whites seems to us to be very problematic.

3.1 The Workers' Equilibrium Strategy in the Discriminatory Regime

In this section, we assume that in the hiring stage, which is the third and final stage of the game, all firms will apply the discriminatory strategy, which is the unique best-response when $\delta > 0$ (and is *a* best response if $\delta = 0$). As in Section 2, workers observe wage offers $W = \langle w_i \rangle$. We search for equilibria in which all workers of the same type (black or white) adopt the same mixed strategy.

First let us consider the situation of white workers. Given that wage offers have been set and that all firms will use the discriminatory strategy, white workers can consider black workers to be invisible, because blacks can have no effect on the probability that a white will be hired. Therefore, the equilibrium response of whites to W is identical to that of workers in the nondiscriminatory regime, described by Propositions 1 and **??** of the previous section. In an equilibrium of the subgame, the expected income of white workers, here denoted by $H^*(W)$, is the same at all firms to which they apply with positive probability, and no greater at firms to which they do not apply. The expected number of white applicants z_i to firm i is the continuous function defined by

$$z_i = \begin{cases} 0 & \text{for} \quad w_i \leq H^*(W) \\ \begin{array}{l}\text{the solution of:} \\ w_i f(z) = H^*(W)\end{array} & \text{for} \quad w_i > H^*(W) \end{cases} \tag{13}$$

The function z_i determines a unique equilibrium strategy $q^*(W)$ for white workers.

The situation for black workers is more complicated. As with whites, they will not apply to firms that offer wages that are too low. Moreover, given the discriminatory hiring strategy of firms, blacks will not apply to a firm that sets its wage offer too high, because high wages induce whites to apply with a high probability. Let y denote the expected number of black applicants to a designated firm. Black applicants will be hired only if no white applicants apply, an event that occurs with probability e^{-z}. However, with respect to other black workers, the situation of blacks is parallel to that of the whites. The probability that an additional black applicant would be hired is given by

$$g(y, z) \ \equiv e^{-z} f(y), \tag{14}$$

so that his expected income would be $wg(y, z)$.

Suppose now that the worker-application subgame has an equilibrium strategy profile in which all black workers adopt the same mixed strategy (we already know that whites adopt $q^*(W)$). Set $J^*(W) \equiv \max_i\{w_i g(y_i, z_i)\}$, the maximum expected income available to blacks in that equilibrium. Note that $J^*(W)$ must be less than $H^*(W)$, the maximum expected income available to whites, because an additional white applicant always has a better chance of being hired than an additional black applicant does.[6] Blacks will apply to firms with positive probability if and only if they

[6] We have more formally that whenever $y > 0$, $g(y, z) < f(z)$ for all values of y and z. We need show only that for $z \geq 0$, $e^{-z} \leq f(z) \equiv (1 - e^{-z})/z$, and this is equivalent to $(1 + z)e^{-z} \leq 1$. But $(1 + z)e^{-z} = 1$ when $z = 0$ and is decreasing in z.

can attain the maximum expected income $J^*(W)$ there. This is not possible for $w_i \leq J^*(W)$. Furthermore, for wage offers beyond a certain threshold, denoted here by $\hat{w}(W)$, the expected number of white applicants will be sufficiently high to force the expected wage for blacks below $J^*(W)$ again (see Proposition A4 in the Appendix). The expected number of black applicants will be positive for wage offers between these two limits and exactly sufficient to equalize expected incomes at $J^*(W)$. More formally, we can write that y_i is the continuous function of w_i defined by

$$y_i = \begin{cases} 0 & \text{for} \quad w_i \leq J^*(W) \\ \begin{array}{l}\text{the solution of:} \\ w_i g(y, z_i) = J^*(W)\end{array} & \text{for} \quad J^*(W) < w_i < \hat{w}(W) \\ 0 & \text{for} \quad w_i \geq \hat{w}(W) \end{cases} . \tag{15}$$

This leads to the following proposition, proved in the Appendix.

Proposition 3 *For any wage-profile W, there are mixed strategies $q^*(W)$ and $s^*(W)$ for white and black workers that form a unique symmetric equilibrium of the job-application subgame.*

Consequently, $\langle q^*(W)\,,\ s^*(W)\rangle$ is the only strategy profile for workers that is consistent with a perfect competitive equilibrium of the two-stage wage-posting game for white and black workers. The equilibrium expected incomes, $H^*(W)$ for white workers and $J^*(W)$ for black workers, depend only on W, are the same across all firms that receive applications from the respective types and satisfy $J^*(W) < H^*(W)$.

As in the case with homogeneous workers, the equilibrium of the job-application game is an extension of the Harris-Todaro model. Workers of each type distribute themselves so that the expected income is the same at all jobs to which they apply. Low-wage jobs receive no applicants. Jobs that are very likely to attract white applicants do not attract black applicants.

3.2 The Firms' Equilibrium Strategy in a Discriminatory Setting

We now search for a perfect competitive equilibrium of the three-stage game. As in Section **??**, in equilibrium, all firms will offer wages that have a positive probability of attracting applicants. Furthermore, Proposition A6 (see the Appendix) states that for all sufficiently small δ, profit-maximizing wage offers have an upper bound (denoted by $\tilde{w}$) that is less than both black and white productivity levels. Therefore, in the relevant range, a firm's expected operating profits are given by

$$\pi_i = (1 - e^{-z_i})(v - w_i) + e^{-z_i}(1 - e^{-y_i})((1 - \delta)v - w_i), \tag{16}$$

where z_i now represents the number of white applicants and y_i, the expected number of black applicants.

We proceed to eliminate several categories of possible equilibria. First we show that in equilibrium, there are no firms that attract both white and black applicants. The intuition of the demonstration is straightforward: if a firm offered a wage that attracted both white and black workers, and gradually lowered that wage, the expected number of white applicants would fall, but the expected number of black applicants would rise at an even faster rate—blacks are strongly discouraged by competition with white applicants. Because blacks and white have almost the same productivity, firms that are attracting both black and white applicants gain in two ways by lowering wages: both their labor costs and their probability of having a job vacancy fall. We prove the following proposition more formally in the appendix:

Proposition 4 *In any subgame-perfect competitive equilibrium, some firms will offer wages that attract only white applicants and the remaining firms will offer wages that attract only black applicants.*

Thus, like earlier taste-based discrimination models, our model implies complete racial segregation. This is true even for $\delta = 0$, provided that when productivities are the same, employers choose to hire whites in preference to blacks.

We proceed to further narrow the possibilities for subgame perfect competitive equilibria. Let N_z and N_y be the numbers of firms with only white and only black applicants, where $N_z + N_y = N$, the total number of firms, and let $r_z \equiv Z/N_z$ and $r_y \equiv Y/N_y$ denote the mean number of applicants to each firm with applicants of the given type. The following propositions, proved in the appendix, present closed-form solutions for wage-offers, expected incomes and profits at firms that hire white and black workers. But these quantities are functions of r_z and r_y, which are themselves endogenous variables.

Proposition 5 *Let W^* be an equilibrium wage-offer profile, and suppose that w_k^* is an element of W^* that attracts only white applicants. Then, in equilibrium we have*

$$w_k^* = \frac{v r_z}{e^{r_z} - 1}. \tag{17}$$

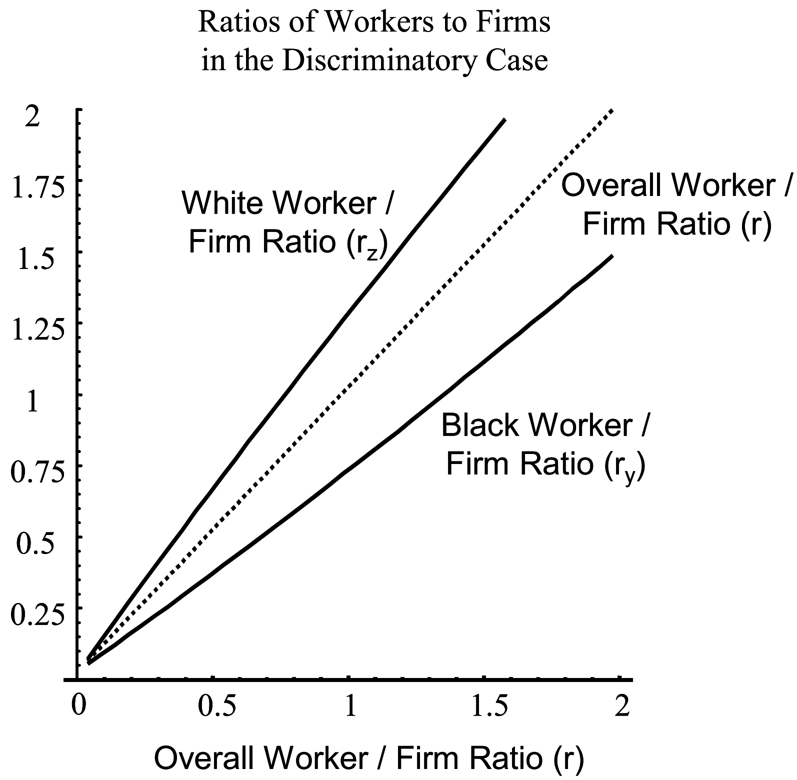
The expected income $H^*(W^*)$ of white workers is

$$H^*(W^*) = v e^{-r_z}, \tag{18}$$

and the operating profits π_k^* for white firms are

$$\pi_k^* = [1 - (1 + r_z) e^{-r_z}] v. \tag{19}$$

Figure 1:



Proposition 6 *Let W^* be an equilibrium wage-offer profile, and suppose that w_j^* is an element of W^* that attracts only black applicants. Then, for δ sufficiently small, we have in equilibrium*

$$w_j^* = H^*(W^*). \tag{20}$$

The expected income $J^*(W^*)$ of black workers is

$$J^*(W^*) = \frac{1 - e^{-r_y}}{r_y} H^*(W^*), \tag{21}$$

and the operating profits π_j^* of black firms are

$$\pi_j^* = (1 - e^{-r_y})[(1 - \delta)v - H^*(W^*)]. \tag{22}$$

Black workers are strictly worse off than white workers.

It remains to characterize the equilibrium values r_y and r_z, the ratios of black workers to black firms and of white workers to white firms. As in Section 2, let $r \equiv \frac{Z+Y}{N}$ denote the ratio of all workers to all firms, and let $\alpha \equiv Y/(Z + Y)$ denote the ratio of the expected number of black workers to the expected total number of workers. Then we have:

Proposition 7 *Only one pair of values of r_z and r_y, defined by the simultaneous solution of equations (A28) and (A29) in the appendix, is consistent with a subgame-perfect competitive equilibrium of the discriminatory game. In equilibrium, we have $r_y < r < r_z$, and both r_z and r_y are increasing in r and α.*[7]

In Figure 1, we graph the values of r_z and r_y as functions of the parameter r, with $\delta = .01$ and $\alpha = .5$. The graphs do not begin at the origin, because δ must be small as compared to r in order for these functions to be defined.

We have thus established that an equilibrium must take the following form: Some firms offer high wages and attract only white applicants. Other firms offer low wages and attract only black applicants. In equilibrium, firms offering the low wage must make the same expected profit as firms offering the high wage, so that the vacancy rate must be higher at low-wage firms ($r_y < r_z$). Conversely, blacks must have a lower unemployment rate than whites, a counterfactual implication to which we return in the discussion at the end of the paper. Despite their lower rate of unemployment, blacks are worse off than whites in this model, as Proposition 6 demonstrates. The proposition also demonstrates the seemingly paradoxical fact that as the proportion α of blacks is parametrically increased, the ratio of workers to firms within each worker group increases, even though the overall ratio of workers to firms is held constant. This will have interesting consequences, which are discussed below.

We sum up the results of this section as follows:

Proposition 8 *Let W^* be a vector of wage offers composed of $N_z = Z/r_z$ elements with value w_k as defined in (17) and $N_y = Y/r_y$ elements with value w_j as defined in (20). Let $q^*(W)$ and $s^*(W)$ denote the strategies for white and black workers defined in Proposition 3. Then the strategy profile $\langle W^*, q^*(W), s^*(W)\rangle$ is a unique*[8] *subgame-perfect competitive equilibrium of the discriminatory wage-posting game.*

The proof of uniqueness follows from the preceding propositions; the proof that W^*constitutes an equilibrium is analogous to that of Proposition 2, with the additional argument that no white firm or black firm would have an incentive to deviate to the other camp, because profits of the firms in the two categories are equated.

[7] As is common in the literature, we are ignoring the integer constraint on N_z and N_y, which are assumed to be large numbers. Constraining N_z and N_y to be integers would change the equilibrium in only minor ways.

[8] The term 'unique' in this context implies that the number of white firms and the number of black firms and the strategies of white firms, black firms and workers are all uniquely identified. However, uniqueness does not extend to the identity of the white and black firms. In other words, the equilibrium is unique to the extent permitted by the anonymity of firms.

4 The Effect of Discrimination on Wages, Profits and Output

In the previous section we established that for $\delta \geq 0$ an equilibrium exists in which all firms discriminate at the hiring stage. If $\delta = 0$ there is an additional equilibrium in which no firm discriminates at the hiring stage, an equilibrium isomorphic to that with homogeneous workers. While we have established that in the discriminatory equilibrium black workers are worse off than white workers, we have not determined who, if anyone, benefits from the discrimination and who suffers from it. With the nondiscriminatory equilibrium as a benchmark, we demonstrate that discrimination lowers the wages of all workers, reduces national income and increases profits.[9]

For a fixed number of firms and small δ, output is decreasing in the total number of job vacancies. But the probability that a firm has a job vacancy decreases at a decreasing rate as the number of workers per firm increases. Because of this, expected total vacancies are minimized when the proportion of workers to firms is equated across worker types, which implies:

Proposition 9 *With the number of firms held fixed and δ sufficiently small, output in the nondiscriminatory equilibrium is strictly greater than output in the discriminatory equilibrium.*

In contrast with most models of discrimination, we show in this model that discrimination hurts white as well as black workers. Furthermore, discrimination increases firms' profits.

Proposition 10 *With δ sufficiently small, the wages and expected incomes of both white and black workers are less and operating profits are more in the discriminatory equilibrium than in the nondiscriminatory equilibrium.*

This proposition, proved in the appendix, follows from the definition of these quantities for white workers in equations (8), (9), and (10) and in (17), (18), and (19) and from the fact that in the discriminatory equilibrium black workers have a lower wage and expected income than do whites.

In Figure 2, the expected incomes of black workers and white workers in the discriminatory equilibrium are compared to expected incomes in the nondiscriminatory equilibrium. Likewise, wage offers in the two equilibria are compared in Figure 3, and profits, in Figure 4. For these graphs, we set $\delta = 0$. The changes in the graphs are modest for positive values of δ less than .05, though as can be seen in Figure 1, the functions will not be defined very close to the origin. In all of these figures, α, the ratio of black to white workers is set at .5. If this ratio were to be parametrically increased, then the wage curves, anchored on their left, would rotate down, and the

[9] In a wage-posting model with a different structure from our own, Moen (1997) proves that in his context wage-posting leads to an efficient outcome when all workers of the same productivity apply to jobs that offer the same wages. Moen's efficiency condition is violated in our model, and we obtain the results his model would suggest.

Figure 2:

Ratio of Expected Incomes with Discrimination to Expected Incomes without Discrimination

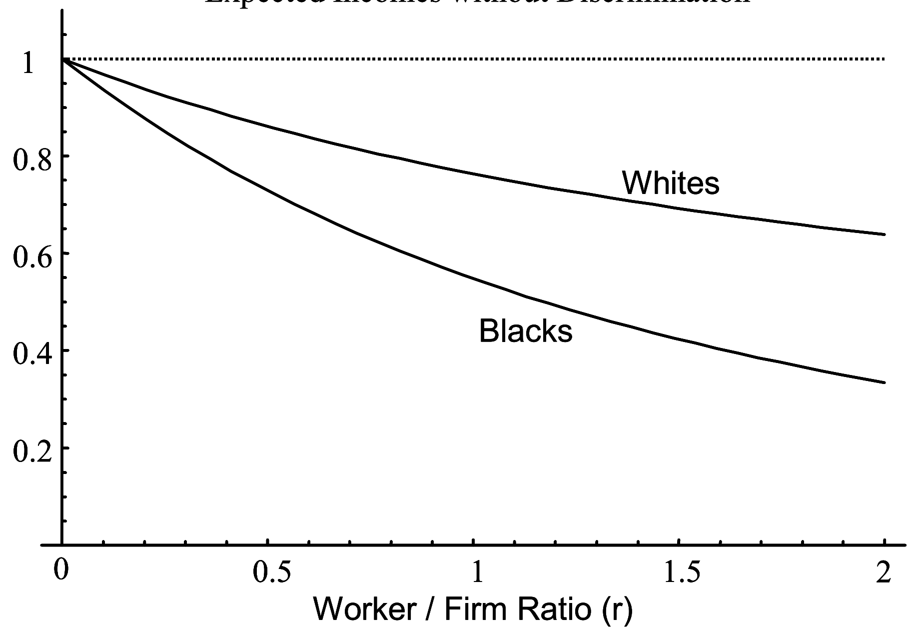
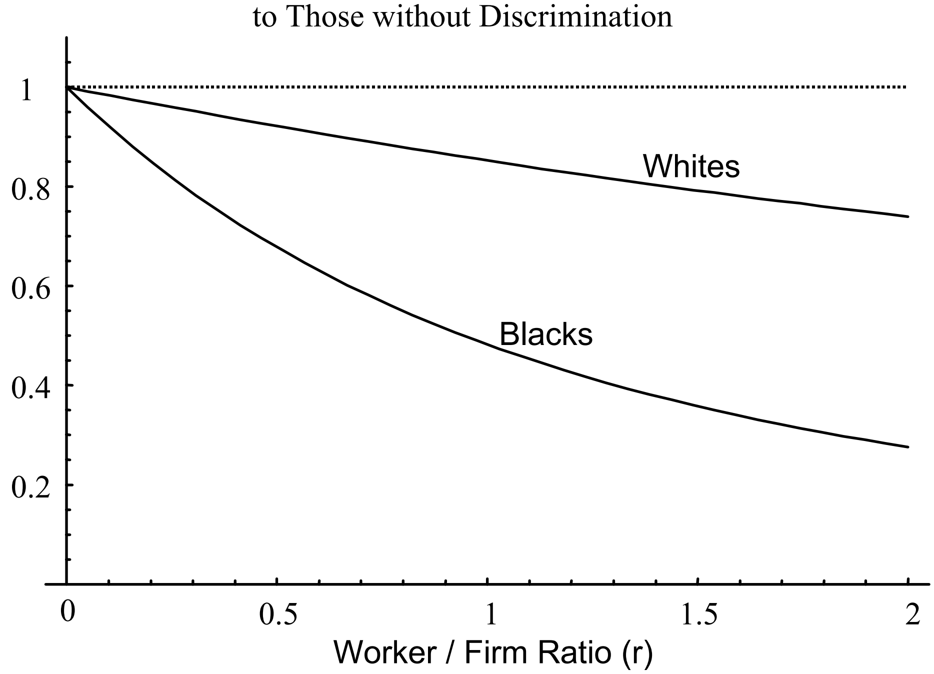


Figure 3:

Ratio of Wage Offers with Discrimination to Those without Discrimination



profit curve, anchored on the right, would rotate up. That is, the larger is the discriminated-against group, the more discrimination hurts workers and helps firms.

It is not surprising that discrimination hurts black workers, but it may be less clear why it also affects white workers adversely. The reason is that by lowering wages in the "black" sector, discrimination increases the profitability of hiring blacks. This, in turn, induces more firms to set wages that attract only black workers, which reduces the demand for white workers and thus their wages.

These results are similar to a recurring theme in Marxist labor economics – that capitalists use various devices to create false distinctions and thus disunity among workers (Bowles, 1985; Roemer, 1979). By generating a hierarchy within the ranks of the working class, capitalists prevent workers from recognizing their common interests. In addition, if the "favored" workers recognize that they are being exploited, they may nevertheless be reluctant to challenge employers or the distinctions out of fear of losing their favored status. One such distinction discussed in the Marxist literature is race (Reich, 1981).

The major difficulty with arguments of this type is that it is often difficult to demonstrate that the division hurts workers or helps capitalists. If firms "buy off" workers from organizing, it is probable that they are helping some workers and hurting others. The workers who are "bought off" presumably believe that they will be better off than they would be if they resisted capitalist exploitation. Upward-sloping wage profiles, for example, may be a mechanism for ensuring the cooperation of senior workers (Stone, 1975), though such profiles may be injurious to individual workers over their lifetimes.

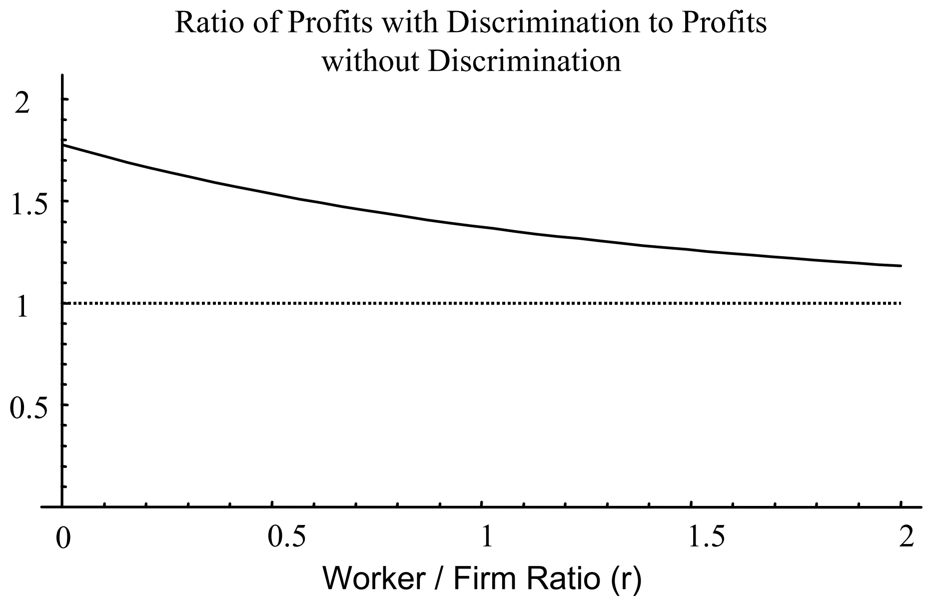
Furthermore, in most models of discrimination it is difficult to see how capitalists benefit from the discrimination except by weakening the political power of workers as a group. In the model developed here, however, discrimination is directly advantageous to capitalists as a class. By dividing previously homogeneous workers such as blacks and whites, capitalists hurt both types of workers in the short run while making themselves better off. Thus dividing the work force can be advantageous, even in the absence of a natural tendency towards worker unity in opposition to capitalists.

Although discrimination on the basis of arbitrary distinctions among workers increases profits, we do not wish to suggest that discrimination therefore must reflect a "capitalist conspiracy." On the contrary, an important point of this paper is that discrimination can arise as an equilibrium in the absence of either cooperative (conspiratorial) behavior or significant discriminatory tastes.

5 Extensions and Empirical Relevance

Of necessity the model is highly stylized. Some of our assumptions may seem very strong, and the reader may appropriately question whether our results are robust to changes in model char-

Figure 4:



acteristics. In this section we briefly address extensions that generalize and add realism to the model. We then build on these extensions to show that it is possible both to fit the model to a set of stylized facts regarding exit hazards from unemployment and to generate significant wage differentials numerically.

Free-Entry Equilibrium: We add to our model a preliminary stage in which firms decide whether or not to enter the market. If firm i enters, it must pay an entry cost (or fixed cost) in the amount c_i. The $\bar{n}$ potential entrants are ordered by their entry costs, so that $c_1 < c_2 < \ \dots \ < c_{\bar{n}}$ with some but not all of these entry costs less than productivity v. Expected profits for a firm in business are defined as the operating profits less entry costs; expected profits for firms not in business are zero.

As we demonstrated in the previous section, there is a unique equilibrium associated with any fixed number of entrants n, so that entry will continue until a marginal entrant cannot earn positive expected profits. The free-entry game has a unique equilibrium with symmetric strategies among workers of a given type. Relative to the nondiscriminatory free-entry equilibrium, the discriminatory free-entry equilibrium has the following properties: aside from the marginal entrant, every firm makes greater profit; all white workers have lower wages and lower expected incomes; all black workers have lower wages and expected incomes, and net output (output less entry costs) is lower. In the discriminatory equilibrium, blacks have lower expected wages than do whites while no such difference can arise in the equilibrium without discrimination.

Our analysis, with appropriate modifications, would also apply if entering firms were required to purchase capital and capital were sold in a market with an upward-sloping supply. Switching from a nondiscriminatory regime to a discriminatory one would yield a capital gain to the holders of capital. Only if marginal firms were identical in their costs and there were an infinitely elastic supply of capital would profits by unaffected by moving from a nondiscriminatory regime.

Match-Specific Productivity: In keeping with the model so far, we assume that white and black workers may differ but that within each race workers are *ex ante* homogeneous. However, after workers apply for a job but before the firm decides which worker to hire, an observable match-specific component of productivity is revealed, independently drawn from the random variable ε_H for white workers and ε_J for black workers, where $E[\varepsilon_H] = E[\varepsilon_J] = 0$. Thus net worker productivity is given by

$$\tilde{v} = \begin{cases} v + \varepsilon_H & \text{for} \quad \text{white workers} \\ v(1-\delta) + \varepsilon_J & \text{for} \quad \text{black workers} \end{cases} .$$

Because of the match-specific component of productivity, some blacks may be seen to be more productive than some whites at a given job. Therefore, even if some whites apply for a job, a black worker might turn out to be the most productive applicant and be hired.

Without match-specific productivity, our model yields a discriminatory equilibrium even when productivity for blacks and whites is the same. But if match-specific productivity is incorporated into the model, a discriminatory equilibrium cannot be supported when the productivity distributions satisfy all of the following conditions:[10]

i. whites and blacks have the same average level of productivity ($\delta = 0$);

ii. the distributions of match-specific productivity are without mass points; and

iii. match-specific productivity is distributed identically for the two groups.

However, when any of these conditions is dropped, the discriminatory equilibrium re-emerges for some distributions of match-specific productivity.

Suppose we relax the first condition, so that $\delta > 0$. The necessary conditions for a firm to have a positive number of both white and black applicants in the equilibrium of the worker-application subgame are similar to (**??**) and (**??**), but now f is decreasing rather than constant in y_i, because the probability that any given black will be hired in preference to a white is positive. For the same reason, g must now decrease more slowly in z_i. Because the match-specific productivity of workers varies, the expected productivity of the firm's best applicant will be increasing in both z_i and y_i and the firm's profit function must be generalized to take this into account.

If the variation of ε_H and ε_J is not too large compared with δv, the probability that a black will be hired in preference to a white, if positive, will be small. When it is small enough, the derivatives

[10] The authors would like to thank an anonymous referee for pointing this out.

of the modified functions f and g will be sufficiently close to those of the original that the the proof of Proposition 4 remains valid. This leads to the conclusion that a discriminatory equilibrium can be supported under these circumstances, even when the probability that a black will be hired in preference to a white is positive.

If $\delta = 0$ and blacks and white have the same distributions of match-specific productivities, one would expect blacks to be hired in preference to whites a substantial fraction of the time. But even then, if we drop the second condition and allow the productivity distributions of blacks and whites to have mass points at the same productivity levels, a discriminatory equilibrium may be supported. The mass points create a nonzero probability of a productivity tie between a black and a white applicant. If firms choose whites over blacks of equal productivity, then the probability of being hired will be lower in the presence of white applicants than of black applicants and this adverse effect will be greater for blacks. Therefore blacks will avoid firms that attract whites and the segregation result will go through, although the wage differential will be smaller than in the absence of match-specific productivity.

The obvious example is a case with just two points in the distribution each with probability .5. If two blacks or two whites apply, each has a probability of .5 of being selected. If a white and a black apply, the white is chosen with probability .75. Replacing a white co-applicant with a black co-applicant raises a white's probability of employment by fifty percent but doubles a black's probability of employment. Therefore a black would be willing to give up fifty percent of the wage in order to move from a firm with one white applicant to one with one black applicant (besides himself), but a white would give up only one-third of his wage.

To see what happens when the third condition is relaxed so that ε_H and ε_J may have different distributions, consider the extreme case where employers think "all blacks look alike" but can distinguish between whites who are good and bad matches (each with probability .5). If there are three black applicants, each will have a one-third chance of being chosen. If there are two white applicants and one black applicant, the black will be chosen only if both whites are bad matches which happens with probability one-fourth. Thus blacks prefer competing with other blacks. The authors have constructed an example of a segregated equilibrium along these lines in which blacks have lower wages even though, on average, blacks are more productive than are whites.

Moen (2003) also generates segregation in a directed search model with match-specific productivity. However, in contrast with our model, the presence of the low types drives up the wages of the high types, making them inefficiently high. The critical difference between our two models does not lie in the effect of match-specific productivity. Unlike us, Moen considers large firms that are committed to hire all applicants at their posted wage, provided only that the applicants surpass a specified productivity threshold. This means that at the time they apply for jobs, Moen's black and white workers are not directly competing with one another. It is the direct competition, which, we think reflects a real-world phenomenon, that drives the model in this paper. Moen's segregation result is striking, but it derives from other considerations.

Valuable Unemployment: If unemployment is valued at u, then labor market equilibrium requires workers to set expected income plus expected unemployment value to its maximum (H for whites and J for blacks) at every firm to which they apply. Profits in this case are given by the expression

$$\pi = (1 - e^{-z})(v - w) + e^{-z}(1 - e^{-y})(v(1 - \delta) - w), \tag{23}$$

which is maximized with respect to w, z, and y, subject to the workers' equilibrium conditions:

$$(w - u)\frac{1 - e^{-z}}{z} = H - u$$

when $z > 0$ and

$$(w - u)e^{-z}\frac{1 - e^{-y}}{y} = J - u \tag{24}$$

when $y > 0$. With an appropriate change of variables this problem becomes mathematically isomorphic to (11) and our proofs go through.

Multiple Periods: If we extend the model to multiple periods but prohibit on-the-job search, very little of substance changes. The model becomes similar (but not identical) to the model in which unemployment is valuable. It is relatively straightforward to show that the equilibrium must involve separation. The more important question is whether sizable wage differentials can persist when unsuccessful searchers can apply elsewhere in the next period. We believe the answer to this question to be "yes." However, the counterfactual prediction that blacks should experience less unemployment would persist. Therefore, we turn to a somewhat more realistic example that allows simultaneously for multiple periods and heterogeneity in workers' discount rates within both black and white groups.

A Numerical Exercise:[11] If workers are homogeneous within race, the model implies that blacks exit unemployment more rapidly than do comparable whites. This is inconsistent with the data. Van den Berg and van Ours (1996) estimate separate unemployment duration models for black men, black women, white men and white women. Their approach allows them to estimate an underlying baseline hazard for escaping unemployment for each group and to measure the extent of heterogeneity around that average. Unfortunately, it does not allow us to ascertain to what extent either the mean differences or the differences in heterogeneity are attributable to known characteristics. They find that blacks exhibit considerably more heterogeneity in their probability of exit from unemployment than do whites. Combining the differences in heterogeneity and exit hazards suggests that relative to whites there is a substantial group of blacks with high exit hazards and another with low exit hazards. We will show that such an outcome is consistent with our model provided that we introduce some heterogeneity into the personal characteristics of workers.

The high exit rate of blacks from unemployment in our model reflects the fact that they apply to low-wage jobs and never compete directly with whites. If, for some reason, some blacks chose to apply to jobs for which whites also apply, then blacks applying to these jobs would tend to have a

[11] Details of the calculations are available from the authors upon request.

relatively low exit hazard and the whites a relatively high one. We generate this type of behavior in the framework of our model by introducing heterogeneity in the rate at which workers discount future income (but keeping the distribution of discount rates the same for both types). Whites with high discount rates ought to accept relatively low wages in return for relative certainty of employment, while blacks with low discount rates ought to accept a relatively high risk of continued unemployment in return for the possibility of a high future wage. Therefore, we postulated that examples could be constructed in which low-discount-rate blacks and high-discount-rate whites would apply for the same jobs. The following numerical example confirms this intuition.

Suppose that there are patient workers with $\rho = .015$ and impatient workers with $\rho = .09$. The proportion of patient workers is the same for blacks and whites. We normalize the lifetime value of output (v) to 1 and set the flow cost of a vacancy to .25. We assume that the number of whites with $\rho = .09$ is sufficiently high relative to the number of blacks with $\rho = .015$ to be consistent with the requirements of our equilibrium. With these parameters, the equilibrium has four wages: a high wage to which only patient whites apply; a somewhat lower wage to which only some impatient whites apply; a yet lower wage to which some impatient whites and all patient blacks apply and a low wage equal to the expected wage of impatient white workers to which only impatient black workers apply.

With these parameters, impatient black workers have the fastest exit rate from unemployment and patient black workers have the slowest. If 10 percent of workers are black and 20 percent of each group is patient, then, on average, the black-white wage differential is approximately 8 percent, a reasonable value for the skill-adjusted wage differential. Moreover, assuming that all groups enter unemployment at the same rate, the steady-state unemployment rate among blacks is approximately 20 percent higher than among whites, a plausible number for a skill-adjusted unemployment differential.

This exercise was not designed to calibrate the model, which is too stylized to justify such an attempt. However, the numerical example suggests that the model is capable of generating empirically significant wage and unemployment differentials that are consistent with empirical regularities.

6 Discussion

The most general game we modeled in this paper has four stages: firm entry, wage posting, worker applications, and hiring. Once the hiring stage is reached, firms will be almost indifferent to the race of the workers they hire—their hiring choices have at most a negligible effect on profits. Yet the hiring choice of firms, however capriciously it may be made, is the tail that wags the dog. If firms discriminate against blacks in the hiring stage (or are expected to do so), the discriminatory equilibrium described above will prevail. If firms choose workers without regard to race (or are expected to do so), then the nondiscriminatory equilibrium will prevail. We believe that the first is the more natural equilibrium for a number of reasons.

First, if firms have even a very slight preference for white over black workers, they will use the discriminatory strategy. In particular, if firms maximize profits and prefer white to black workers given equal profits, then in the case of equal productivities only the discriminatory equilibrium remains. Although we do not wish to suggest that most employers are racists, anecdotal evidence suggests that many employers have at least very mild discriminatory preferences.

Second, the discriminatory equilibrium yields higher profits to every firm than does the nondiscriminatory equilibrium. If firms as a group create the expectation that they will discriminate in the hiring stage, they stand to make more money. Moreover, they would lose nothing by fulfilling such expectations. An ethos of discrimination among firm owners would be consistent with their economic interests.

Finally, the discriminatory equilibrium outcome does not require that firms would actually use discriminatory strategies, but only that black workers believe that they do. A belief in discrimination is sufficient to induce blacks to apply to low-wage firms where they would not be in competition with whites. Some firms would then choose a low-wage strategy designed to attract blacks just as in the equilibrium in which the strategies of firms entail discrimination. Since blacks do not apply to high-wage firms, their beliefs are not contradicted in equilibrium. This constitutes what Fudenberg and Levine (1993) term a self-confirming equilibrium.

Widespread and ongoing litigation over employment discrimination suggests that a substantial number of people believe that some firms discriminate. Even public enforcement of antidiscrimination laws has proved insufficient to dispel the belief that many so-called Equal Opportunity Employers do not live up to their announced policy. In the context of our model, we have assumed that it is impossible for firms to make credible promises not to discriminate. However, at the cost of some simplicity, we could substitute the much weaker assumption that in the eyes of potential employees, a firm's announced policy of nondiscrimination reduces the probability of discrimination but does not drive it to zero. This would be sufficient to obtain labor market segregation, and the broad outlines of our results would continue to hold: jobs would be segregated and firms attracting black workers would offer lower wages. All firms seeking to attract blacks would announce a policy of nondiscrimination.

Consequently, we believe that the equilibrium in which firms discriminate, or at least blacks believe that they do, is the natural equilibrium.[12] That being so, our model provides useful insights on the role of antidiscrimination policy. Such policies can be justified on pure efficiency grounds, but the distributional impacts are also significant. Although effective antidiscrimination measures would have the greatest impact on black workers, they would increase the incomes of white workers as well. Only owners of capital would be affected adversely.

The model also highlights the importance of targeting employment discrimination rather than

[12] Note that separating equilibria with segregation can be derived for directed search models in a wide range of settings. See Moen (2003) for a model rather different from our own that nevertheless yields a separating equilibrium.

wage discrimination alone. No firm practices wage discrimination in our model, but employment discrimination results in significantly lower wages for black workers anyway. Moreover, a significant discriminatory outcome can result from even very mild preferences for white workers. Employers do not have to be overt racists to collectively create a situation in which blacks are dramatically disadvantaged. The required preference for discrimination is sufficiently minimal that employers may not even be aware of it. This constitutes a justification for affirmative action designed to offset even mild discriminatory tendencies. The costs of combating such tendencies need not be high.

A Appendix

Proposition A2 *The function $f(z) \equiv (1 - e^{-z})/z$ is the unconditional probability that a designated worker will be hired when the number of other applicants is Poisson with mean $z > 0$.*

Proof. This follows from:

$$\sum_{n=0}^{\infty} \frac{1}{n+1} \frac{e^{-z} z^n}{n!} \equiv \frac{1}{z} \sum_{n=0}^{\infty} \frac{e^{-z} z^{n+1}}{(n+1)!} \equiv \frac{1}{z} \sum_{n=1}^{\infty} \frac{e^{-z} z^n}{n!} \equiv \frac{1}{z} \left(\sum_{n=0}^{\infty} \frac{e^{-z} z^n}{n!} - e^{-z} \right) = (1 - e^{-z})/z,$$

where the last equality derives from the fact that the Poisson probabilities sum to 1. ■

Proposition A3 *In any symmetric equilibrium of the worker application subgame, K_i is given by:*

$$K_i = \begin{cases} K & \text{for} \quad w_i \geq K \\ w_i & \text{for} \quad w_i < K \end{cases}, \tag{A1}$$

z_i satisfies

$$\begin{cases} z_i > 0 & \text{for} \quad w_i > K \\ z_i = 0 & \text{for} \quad w_i \leq K \end{cases}, \tag{A2}$$

and

$$z_i = f^{-1}(\tfrac{K}{w_i}) \quad \text{for} \quad w_i \geq K \ . \tag{A3}$$

Proof. For $w_i \geq K$ (4) implies that $K_i \geq K$ when $z_i = 0$. Furthermore, from the definition of K we know that $K_i \leq K$ in equilibrium (which rules out $K_i > K$), and $z_i > 0$ can be a best response only if $K_i = K$ (which rules out $K_i < K$), and the first part of (5) follows. Equation (4) yields $K_i \leq w_i$, so that $w_i < K$ implies that $K_i < K$, which yields $z_i = 0$ and $K_i = w_i$, and the second part of (5) is also demonstrated. If $w_i > K$, then $z_i = 0$ would imply that $K_i = w_i > K$, a contradiction that proves the first statement of (6). The combination $w_i \leq K$ and $z_i > 0$ would imply $K_i < w_i \leq K$, which in equilibrium requires that $z_i = 0$, a contradiction that proves the second statement of (6). Equation (7) follows from (4), (5), and the invertability of f. ■

Proposition ?? *For any wage-profile W, there is a unique worker strategy $q^*(W)$ that forms a symmetric equilibrium of the worker application subgame. In that equilibrium, workers' expected incomes, $K^*(W)$, are the same at all firms to which they apply with positive probability, and no greater at firms to which they do not apply.*

Proof. Given (4) and the definition of K, it follows from (5) and (6) that best-response worker strategies must be such that z_i satisfies the following condition :

$$f(z_i) = \begin{cases} \frac{K}{w_i} & \text{for} \quad w_i > K \\ 1 & \text{for} \quad w_i \leq K \end{cases}. \tag{A4}$$

Note that $f(z)$ is strictly decreasing and thus invertible for $z \geq 0$, so that we can define the continuous function

$$z(w, K) = f^{-1}(\frac{K}{w}). \tag{A5}$$

Then from (A4) we know that in the subgame equilibrium, we have

$$z_i = \begin{cases} z(w_i, K) & \text{for} \quad w_i > K \\ 0 & \text{for} \quad w_i \leq K \end{cases}. \tag{A6}$$

Furthermore, the expected number of applicants to all firms, denoted here by $Z(W, K) \equiv \quad {}_i\, z_i$, must equal the expected number of workers, i.e.

$$Z(W, K) = Z. \tag{A7}$$

From (2) we know that $f(z)$ is continuous and strictly decreasing, and it follows that with W held constant, f^{-1}, z and $Z(W, K)$ are also continuous and strictly decreasing in K. Furthermore, $Z(W, K)$ maps onto the interval $(0, \infty)$. Therefore (A7) determines a unique value of K as a function only of the wage profile W, and we write $K = K^*(W)$. Then, by substituting $K^*(W)$ back into (A6), we see that in the subgame equilibrium z_i is given by

$$z_i^*(W) = \begin{array}{lll} z(w_i, K^*(W)) & \text{for} & w_i > K^*(W) \\ & & \\ 0 & \text{for} & w_i \leq K^*(W) \end{array} . \tag{A8}$$

Thus, W determines a unique vector $Z^*(W) \equiv \langle z_i^*(W) \rangle$ that is consistent with a best response from the workers. From (3), it follows immediately that the only possible symmetric equilibrium of the subgame is composed of the worker's mixed strategy given by

$$q^*(W) \equiv \langle \frac{z_i^*(W)}{Z} \rangle. \tag{A9}$$

We now confirm that $q^*(W)$ constitutes an equilibrium. By (A9) and (5), workers' expected earnings at firm i are $K_i = K^*(W)$ whenever $q_i^* > 0$. From (A9), we see that $q_i^* = 0$ if and only if $z_i(W) = 0$, and this can occur only when $w_i \leq K^*(W)$. This means that for $q_i^* = 0$, we have $K_i = w_i \leq K^*(W)$. Consequently, no worker can increase his expected payoff by deviating. ■

Proposition 2 *The game between firms and workers has a subgame-perfect competitive equilibrium* $\{W^*, q^*\}$ *that is unique among those in which all workers adopt the same mixed strategy. In this equilibrium, workers adopt the strategy* q^* *as defined in the preceding section. Firms adopt the strategy profile* W^**, where for all firms* i*, the wage offer is given by*

$$w_i^* = \frac{vr}{e^r - 1};$$

the expected income of workers, by

$$K^*(W^*) = ve^{-r};$$

and operating profits, by

$$\pi_i^* = [1 - (1 + r)e^{-r}]v.$$

The values of w_i^* *and* $K^*(W^*)$ *go from* v *to* 0 *and* π_i *goes from* 0 *to* v *as* r *goes from* 0 *to* ∞.

Proof. Suppose firms adopt a strategy profile $W \leq v$. For $w_i \geq K^*(W)$, we know from Proposition 1 that z_i must satisfy $w_i = K^*(W)/f(z_i)$ in the equilibrium of the workers' subgame. Substitution into (1) then yields that

$$\pi_i = (1 - e^{-z_i})v - z_i K^*(W). \tag{A10}$$

For $w_i < K^*(W)$, $z_i = 0$ and $\pi_i = 0$, so the above equation is valid in this range as well.

Recall that for a subgame-perfect competitive equilibria, $K^*(W)$ must be assumed invariant with respect to a deviation by a single firm. Let $z_i^*(W)$ represent the profit-maximizing value of z_i. Holding $K^*(W)$ constant, we have $d\pi_i/dz_i = K^*(W) - ve^{-z_i}$, which is positive for $z_i = 0$. Therefore $z_i^*(W) > 0$, and the first-order condition yields

$$z_i^*(W) = \log \frac{v}{K^*(W)}. \tag{A11}$$

Because π_i is strictly concave in z_i for $z_i > 0$, we can conclude that z_i^* defines firm i's unique best response to the strategy profile (W_{-i}, q^*).

Suppose now that W^* is an equilibrium profile of wages. Then, it follows immediately from (12) that $z_i^*(W^*)$ is the same for all i. Since each worker applies to exactly one firm, it must be true that in any equilibrium $z_i^* = Z/N = r$ for all i, so that (9) follows from (12). Substitution into (7) and (11) and the definition of f then yield (8) and (10). Because (8) uniquely specifies the firms' equilibrium strategy profile W^*, the subgame-perfect competitive equilibrium must be unique if it exists.

Finally, we prove existence. Let W^* be a candidate equilibrium strategy profile defined by (8). Since all wages in W^* are identical, each firm must have $z_i^* \equiv Z/N \equiv r$ expected applicants, and from (4) and (2) it follows that $K_i^* = w_i^* f(r) \equiv e^{-r}v \equiv K^*(W^*)$. It is straightforward to confirm that $w_i^* \in (K^*(W^*), v)$, and this and (12) imply that w_i^* is the best response to W_{-i}^* and q^*. ■

Proposition A4 *Given W, there exists a wage rate $\hat{w}(W) > H^*(W)$ such that, in the equilibrium of the worker-application subgame, $y > 0$ when $H^*(W) < w < \hat{w}(W)$ and $y = 0$ when $w \geq \hat{w}(W)$.*

Proof. We know that in the subgame equilibrium, $y > 0$ if and only if $wg(0, z) > J$, that is, if and only if $we^{-z} > J$. Furthermore, we know that $z = 0$ when $w = H^*(W)$, so that $we^{-z} = H^*(W)$ when $w = H^*(W)$. For $w > H^*(W)$, we have $wf(z) = H^*(W)$, and standard calculations yield that we^{-z} decreases monotonically from $H^*(W)$ to 0 as w increases from $H^*(W)$. (To see this, incorporate the definition of f into $wf(z) \equiv H^*(W)$, and observe that we^{-z} goes to zero as w increases. To prove monotonicity, use implicit differentiation to solve for $z'(w)$. Differentiate we^{-z} with respect to w, apply the definition of $z'(w)$ previously obtained, and note that the sign the result is negative.) Because we^{-z} is continuous in w and $J < H^*(W)$, the intermediate-value theorem implies that the equation

$$we^{-z} = J \tag{A12}$$

has a unique solution in the range $w > H^*(W)$. Define $\hat{w}(W)$ to be that solution, and the proposition follows. ■

Proposition A5 *In any symmetric equilibrium of the discriminatory worker-application subgame, the expected income, J_i, of black applicants to firm i, is given by:*

$$J_i = \begin{array}{lll} w_i \exp[-f^{-1}(\frac{H^*(W)}{w_i})] < J & \text{for} & w_i > \hat{w}(W) \\ J & \text{for} & \hat{w}(W) \geq w_i \geq J \\ w_i & \text{for} & w_i < J \end{array}, \tag{A13}$$

and y_i, the expected number of black applicants to firm i, is a continuous function of w_i that satisfies

$$\begin{array}{lll} y_i = 0 & \text{for} & w_i \geq \hat{w}(W) \\ y_i > 0 & \text{for} & \hat{w}(W) > w_i > J \\ y_i = 0 & \text{for} & w_i \leq J \end{array}, \tag{A14}$$

where the value of y_i is determined by (15) and (13) in the case that $y_i > 0$. The function $y_i(w_i, J)$ is continuous in w_i.

Proof. The first two lines of (A14) follow immediately from Proposition A4; the third line, from the fact that when $y_i > 0$ and $w_i \leq J$, applying to firm i does not yield the maximum available expected income, J. The continuity of y_i as a function of w_i follows from the continuity of the left-hand side of (15) and from the continuity of z_i in w_i. If we recall that $w_i g(y_i, z_i)$ is the expected income of a black applicant to firm i, the first and third lines of (A13) follow from the value of z_i implicit in (13) and the fact that $y_i = 0$ in those ranges. The second line follows from (15). ■

Proposition 3 *For any wage-profile W, there are mixed strategies $q^*(W)$ and $s^*(W)$ for white and black workers that form a unique symmetric equilibrium of the job-application subgame.*

Proof. Given (A12) and the definition of J, it follows from (A13) that best-response worker strategies must be such that y_i satisfies the following condition :

$$g(y_i, z_i^*(W)) = \begin{array}{lll} e^{-z_i^*(W)} & \text{for} & w_i \geq \hat{w}(W) \\ \frac{J}{w_i} & \text{for} & \hat{w}(W) > w_i > J \\ 1 & \text{for} & w_i \leq J \end{array}. \tag{A15}$$

Note that with z held constant, $g(y,z)$ is strictly decreasing in y and thus invertible for $y \geq 0$, so that we can define the continuous function

$$y(w, z, J) = g^{-1}(\frac{J}{w}, z). \tag{A16}$$

Then from (A15) we know that in the subgame equilibrium, we have

$$y_i = \begin{array}{lll} 0 & \text{for} & w_i \geq \hat{w}(W) \\ y(w_i, z_i^*(W), J) & \text{for} & \hat{w}(W) > w_i > J \\ 0 & \text{for} & w_i \leq J \end{array}. \tag{A17}$$

Furthermore, the sum of the expected number of applicants at each firm must equal the expected number of workers, i.e.

$$Y(W, J) \equiv \sum_i y_i = Y. \tag{A18}$$

Equation (A16) and the definition of g imply that $Y(W, J)$ is continuous and strictly decreasing in J, so that (A18) uniquely determines the value of J as a function only of the wage profile W, and we write $J = J^*(W)$. Then, by substituting $J^*(W)$ back into (A17), we see that in the subgame equilibrium y_i is given by

$$y_i^*(W) = \begin{array}{ll} 0 & \text{for} \quad w_i \geq \hat{w}(W) \\ y(w_i, z_i^*(W), J^*(W)) & \text{for} \quad \hat{w}(W) > w_i > J \\ 0 & \text{for} \quad w_i \leq J \end{array} . \tag{A19}$$

Thus, W determines a unique vector $Y^*(W) \equiv \langle y_i^*(W) \rangle$ that is consistent with a best response from black workers. >From (3), it follows immediately that the only possible symmetric equilibrium of the subgame is composed of the workers' mixed strategies given by

$$q^*(W) \equiv \langle \frac{z_i^*(W)}{Z} \rangle \tag{A20}$$

for white workers and by

$$s^*(W) \equiv \langle \frac{y_i^*(W)}{Y} \rangle \tag{A21}$$

for black workers.

We now confirm that $\langle q^*(W),\ s^*(W) \rangle$ constitutes an equilibrium. By (A20), (A21), (5) and (A13), white workers' expected earnings at firm i are $H_i = H^*(W)$ whenever $q_i^* > 0$,and black workers' expected earnings at firm i are $J_i = J^*(W)$ whenever $s_i^* > 0$. From (A9), we see that $q_i^* = 0$ if and only if $z_i(W) = 0$, which can occur only when $w_i \leq H^*(W)$. This means that for $q_i^* = 0$, we have $H_i = w_i \leq H^*(W)$. Consequently, no white worker can increase his expected payoff by deviating. The demonstration for black workers is analogous.

That $J^*(W) < H^*(W)$ was proved in footnote 6. ∎

Proposition A6 *There exists $\tilde{w} < v$ and $\tilde{z} > 0$ such that for any vector of wage offers $W \leq v$ and any firm i, the profit-maximizing values of w_i and z_i (with workers at equilibrium) satisfy $J^*(W) < w_i \leq \tilde{w}$ and $z_i \geq \tilde{z}$.*

Proof. By assumption, a firm that sets $w = v$ would be offering the highest wage in W. In the equilibrium of the worker application subgame, workers apply to the highest wage firms with positive probability, so that z would be positive at $w = v$. However, z is a continuous function of w, so that by decreasing w a sufficiently small amount, the firm could maintain a positive value of z and by (1) earn positive expected profits. This shows that $w = v$ and $w \leq J^*(W)$ are not profit-maximizing, because each yields zero profits, even though positive profits are available. Define $\tilde{w} < v$ and $\tilde{z} > 0$ to be the profit-maximizing w_i and the corresponding z_i for a given firm when only white workers are available and when all other firms offer the wage $w = v$. Consider now any vector of wage offers $W \leq v$. If $\hat{w}(W) \leq \tilde{w}$, then the proposition follows from the fact that when only one type of worker is available, the profit-maximizing w_i and the corresponding z_i are monotonically increasing and decreasing, respectively, in $H^*(W)$ (see equations (12) and (4)) and thus in the wage offers of other firms. If $\hat{w}(W) > \tilde{w}$,then the profit-maximizing w_i would be smaller than it would be if only white workers are available, which proves the stated result. ∎

Proposition 4 *In any subgame-perfect competitive equilibrium, some firms will offer wages that attract only white applicants and the remaining firms will offer wages that attract only black applicants.*

This is an immediate consequence of the following:

Lemma A7 *Let $W \leq \tilde{w}$ be a wage-offer profile, let w be the wage of a designated firm, and assume that this firm considers the market-wide expected wages, $H^*(W)$ and $J^*(W)$, to be independent of its own w. Then, for sufficiently small δ, if w attracts both black and white applicants with positive probability, it cannot be profit maximizing.*

Proof. >From Propositions 1, **??** and A4, we know that a posted wage w will attract both black and white applicants, i.e. $y, z > 0$, if and only if w is in the open interval $(H^*(W), \hat{w}(W))$. But in this case z and y must satisfy (13) and (15). To see whether or not such a wage can be profit maximizing, we differentiate the profit function (16) with respect to w (with $H^*(W)$ and $J^*(W)$ assumed constant, as is required by the concept of subgame-perfect competitive equilibrium). We have

$$\frac{d\pi}{dw} = -(1 - e^{-y-z}) + e^{-y-z}((1-\delta)v - w)(y' + z') + e^{-z} z' \delta v \tag{A22}$$

for $w \in (H^*(W), \hat{w}(W))$, where z' and y' are the total derivatives of z and y with respect to w. We show that for sufficiently small δ, $d\pi/dw < 0$ on the interval $(H^*(W), \hat{w}(W))$, and the first assertion of the proposition follows immediately.

First we demonstrate that $y' + z' < 0$ for all $z, y > 0$, from which it follows that the second term of $d\pi/dw$ is negative. By implicitly differentiating (13) and (15) with respect to w, solving for y' and z', and grouping terms, we get

$$z' = \frac{2}{w} q(z) \tag{A23}$$

and

$$y' = -\frac{2}{w} q(z) q(y) s(z), \tag{A24}$$

where the functions $q(x)$ and $s(x)$ are given by

$$q(x) = \frac{x(e^x - 1)}{2(e^x - x - 1)} \tag{A25}$$

and

$$s(x) = \frac{2(xe^x - e^x + 1)}{x(e^x - 1)}. \tag{A26}$$

The numerators and denominators of $q(x)$ and $s(x)$ satisfy the following inequalities:

$$2(xe^x - e^x + 1) > x(e^x - 1) > 2(e^x - x - 1) > 0.$$

This is because $2(xe^x - e^x + 1) - x(e^x - 1)$ and its first derivative equal 0 for $x = 0$, and the second derivative is positive for $x > 0$, which proves the first inequality. The same is true for both $x(e^x - 1) - 2(e^x - x - 1)$ and $2(e^x - x - 1)$, which proves the second and third inequalities. From this we know that $q(x)$, $s(x) > 1$ for all $x > 0$. We can now write

$$z' + y' = -\frac{2}{w} q(z)[q(y)s(z) - 1], \tag{A27}$$

which is clearly negative for all $y, z > 0$.

Next, having shown that $z' + y' < 0$ for $w \in (H^*(W), \hat{w}(W))$ and knowing that $y = 0$ and $z' > 0$ for $w > \hat{w}(W)$, we can conclude from the continuity of z and y that $z + y$ takes a minimum at $w = \hat{w}(W)$. This means that $-(1 - e^{-y-z})$, the first term of $d\pi/dw$, which is negative for positive z, y, takes a negative maximum value at $w = \hat{w}(W)$. Therefore $-(1 - e^{-y-z})$ is negative and bounded away from zero on $(H^*(W), \hat{w}(W))$.

Finally, from (A23) and (A25), we know that $e^{-z}z'$, the coefficient of δv, is bounded on $(H^*(W), \hat{w}(W))$. Therefore, because the second term of $d\pi/dw$ is negative, and because the first term is both negative and bounded away from zero, we can now conclude from (A22) that for sufficiently small δ, $d\pi/dw < 0$ in that region. This implies that when a firm is attracting both black and white applicants, decreasing its wage will always net the firm higher expected profits, and the first statement of the proposition is proved. The second statement follows immediately from the argument that if all firms offer wages designed to attract only one type of applicant, it would be in the interest of the other type of applicant to apply for some of those jobs as well, a contradiction. ■

Proposition 5 *Let W^* be given, and suppose that w_k^* is an element of W^* that attracts only white applicants. Then, in equilibrium we have*

$$w_k^* = \frac{v r_z}{e^{r_z} - 1}.$$

The expected income $H^*(W^*)$ of white workers is

$$H^*(W^*) = v e^{-r_z},$$

and the operating profits π_k^* for white firms are

$$\pi_k^* = [1 - (1 + r_z)e^{-r_z}]v.$$

Proof. In the proof of Proposition 4, we demonstrate that for w in the interval $(H^*(W), \hat{w}(W))$, profits increase as w declines. Since profits as function of w are continuous at $w = \hat{w}(W)$, we can say that lower values of w are more profitable than $\hat{w}(W)$, so that $\hat{w}(W)$ will not be offered in equilibrium. Thus, any equilibrium wage w_k^* that attracts only white applicants must be strictly greater than $\hat{w}(W)$. Inasmuch as black applicants never appear at firms with wages in that range, and since profits as a function of w are maximized in the interior of the interval $[\hat{w}(W), \infty)$, Proposition 2 applies to this situation. The current proposition then follows from (8), (10) and (9), with r_z substituted for r and v_z substituted for v. ■

Proposition 6 *Let W^* be given, and suppose that w_j^* is an element of W^* that attracts only black applicants. Then, for δ sufficiently small, we have in equilibrium*

$$w_j^* = H^*(W^*). \tag{20}$$

The expected income $J^*(W^*)$ of black workers is

$$J^*(W^*) = \frac{1 - e^{-r_y}}{r_y} H^*(W^*), \tag{21}$$

and the operating profits π_j^* of black firms are

$$\pi_j^* = (1 - e^{-r_y})[(1 - \delta)v - H^*(W^*)]. \tag{22}$$

Black workers are strictly worse off than white workers.

Proof. An argument analogous to the proof of Proposition A6 shows that in equilibrium each firm will offer a wage that yields positive expected profits. The means that for every firm i, $J^*(W) < w_i < v$, and $z_i + y_i > 0$.

We prove (20) by a contradiction produced as δ becomes small parametrically. So suppose that (20) were false. This supposition along with Proposition 4 and the previous paragraph would restrict the value of w_j^* to $J^*(W^*) < w_j^* < H^*(W^*)$. As in the previous section, we would be searching for an interior optimum for one type of worker, so that Proposition 2 would apply, and we would have:

$$w_j^* = \frac{(1 - \delta)vr_y}{e^{r_y} - 1},$$

and

$$\pi_j^* = [1 - (1 + r_y)e^{-r_y}](1 - \delta)v.$$

Furthermore, in equilibrium operating profits must be identical for all wages in W^*; otherwise, firms earning less operating profits would deviate from their current wage. Thus we would have $\pi_j^* = \pi_k^*$, which would imply that r_y goes to r_z as δ becomes small. This, in turn, would imply that w_j^* converges to w_k^*. But since w_j^* was assumed to be less than $H^*(W^*)$ and w_k^*, which is independent of δ, is fixed at a value greater than $H^*(W^*)$ by construction, this is a contradiction. Thus (20) is proved. The probability that a black firm will have no applicants is e^{-r_y}, and (22) follows immediately.

Because all firms that attract black applicants in equilibrium have the same wage, our assumption that strategies are symmetric among workers of a given race implies that the expected number of applicants to each of these "black firms" is also the same. Thus $y_j^* = Y/N_y \equiv r_y$, so that we have $J^*(W^*) = w_j^* f(r_y)$ and (21) is demonstrated. That blacks are worse off than whites follows from the fact that $f(r_y) < 1$ for any $r_y > 0$. ■

Proposition 7 *Only one pair of values of r_z and r_y is consistent with a subgame-perfect competitive equilibrium of the discriminatory game. In equilibrium, we have $r_y < r < r_z$, and both r_z and r_y are increasing in r and α.*

Note: The statement of this proposition ignores the integer constraint on N_z and N_y, which are assumed to be large numbers. Were we to constrain N_z and N_y to be integers, their equilibrium values would change by at most 1. Equilibrium profits of white and black firms would then differ by a small amount, but no firm would deviate to join the higher-profit group, because the increased competition engendered by the deviation would reduce the firm's post-deviation profits to a value below the its current profit level.

Proof. First we eliminate three variables, Y, Z, and N_y, from the equations that define α, r, r_y and r_z. (Because of the relation between these quantities, a fourth variable, N_z, drops out.) From this, we obtain

$$r_y = \frac{\alpha r r_z}{(1 + \alpha)r_z - r}. \tag{A28}$$

Inasmuch as black and white firms must have equal profits in equilibrium, we have $\pi_k^* = \pi_j^*$, and applying (19), (22) and (18), yields

$$r_y = \ln \frac{1 - \delta - e^{-r_z}}{e^{-r_z} r_z - \delta}. \tag{A29}$$

Let R_y denote the argument of the logarithm. Because $w_j^* \equiv H^*(W) \equiv ve^{-r_z} < \tilde{w} < (1 - \delta)v$, the numerator of R_y is positive. The expression $e^{-r_z} r_z$ in the denominator, is monotonically increasing for $r_z < 1$ and monotonically decreasing for $r_z > 1$. Thus $e^{-r_z} r_z = \delta$ has exactly two solutions, one which goes to 0 as δ goes to zero, and another that goes to ∞ as δ goes to 0. R_y is positive on the interval between these values, so that $\ln R_y$ is defined. In addition, the derivative of R_y with respect to r_z is positive on an interval whose left-hand boundary goes to zero and right-hand boundary goes to infinity as δ goes to zero, which means that the function defined by (A29) is increasing on that interval.

The function defined by (A28) is continuous, positive, and downward sloping on the interval $((1-\alpha)r, \infty)$, and becomes arbitrarily large as $r_z \to (1-\alpha)r$ from the right. The function defined by (A29) is continuous and increasing on an interval that, for δ sufficiently small, begins to the left of $(1-\alpha)r$ and becomes arbitrarily large to the right Thus the graphs of these functions must intersect once and only once on $((1-\alpha)r, \infty)$. Because the first function is either negative or undefined on $[0, (1-\alpha)r]$, the existence of a unique solution of (A28) and (A29) is proved.

To see that $r_y < r < r_z$, first note that (A29) implies that as δ becomes small,

$$r_y - r_z = \ln \frac{1-e^{-r_z}}{r_z} = \ln f(r_z) < 0,$$

so it follows that $r_y < r_z$ for small δ. Because $r_y = \frac{Y}{N_y}$ and $r_z = \frac{Z}{N_z}$, we know that $r_y < \frac{Y+Z}{N_y+N_z} < r_z$, and the middle term equals r, by definition. To show that r_z and r_y are increasing in α, we differentiate the right-hand side of (A28), note that the derivative is positive, which demonstrates that the downward sloping curve is shifting out, so that the intersection of (A28) and (A29) is moving outward and upward. ■

Proposition 9 *With the number of firms held fixed and δ sufficiently small, output in the nondiscriminatory equilibrium is strictly greater than output in the discriminatory equilibrium.*

Proof. In the limit as $\delta \to 0$, output is inversely proportional to the fraction of firms with job vacancies. In the discriminatory equilibrium, N_z/N andN_y/N represent the fractions of white and black firms, and the respective vacancy rates for those firms are e^{-Z/N_z} and e^{-Y/N_y}. Consider now the function $V(\lambda)$ given by

$$V(\lambda) \equiv \lambda e^{-\frac{Z}{\lambda N}} + (1-\lambda)e^{-\frac{Y}{(1-\lambda)N}}.$$

Define $\hat{\lambda} \equiv N_z/N$ and $\lambda^* \equiv Z/(Z+Y)$. Then $V(\hat{\lambda})$ yields the average vacancy rate for all firms in the discriminatory equilibrium, but $V(\lambda^*)$ reduces to $e^{-\frac{Z+Y}{N}}$, the vacancy rate in the nondiscriminatory or homogeneous-worker case. Because $V(\lambda)$ is a strictly convex function that takes its unique minimum at $\lambda^* \equiv Z/(Z+Y)$, it is straightforward to show that for sufficiently small δ, the difference between the vacancy rates $V(\hat{\lambda})$ and $V(\lambda^*)$ is large enough to dominate the small productivity difference between the races. ■

Proposition 10 *With δ sufficiently small, the wages and expected incomes of both white and black workers are less and operating profits are more in the discriminatory equilibrium than in the nondiscriminatory equilibrium.*

Proof. We need to demonstrate this proposition for white workers and white firms only, because we have already shown that in the discriminatory equilibrium black workers have a lower wage and expected income than do whites and that all firms earn the same profits whatever the color of their workers. To do this, we first note that the equations that define these quantities in the nondiscriminatory equilibrium, (8), (9), and (10), and in the discriminatory equilibrium, (17), (18), and (19), are identical except for the values of the parameter r and r_z, respectively. We define a dummy variable $\hat{r}$, substitute it for r in the right hand sides of these equations, and think of the resulting expressions as functions of $\hat{r}$. When $\hat{r} = r$, these functions yield wages offers and expected incomes for each worker and profits for each firm in the non-discriminatory equilibrium. But from Proposition 5 we know that at $\hat{r} = r_z$ the same functions describe wages offers, expected income, and profits for each white worker and white firm in the discriminatory equilibrium. Furthermore, it is easy to show that the functions for wages and expected income are decreasing in $\hat{r}$, whereas the function for profits is increasing in $\hat{r}$. Since we have already shown that $r_z > r$, the proposition follows. ■

Proposition ?? *The free-entry game has a unique equilibrium with symmetric strategies among workers of a given type, both in the nondiscriminatory regime (with $\delta = 0$) and in the discriminatory regimes.*

Note: In using the term 'unique,' we are not distinguishing between equilibria that are identical except for the identities of the firms in business. Nor are we distinguishing between equilibria which are identical aside from the entry choice of a marginal firm with zero profits (a nongeneric event).

Proof. We begin with the nondiscriminatory regime, denoted by the subscript k. Suppose n firms enter. Then, by Proposition 2, there is a unique equilibrium of the wage-posting subgame. From (10), expected profits $\tilde{\pi}_k(n)$ for the marginal entrant (the nth firm) in the nondiscriminatory equilibrium is

$$\tilde{\pi}_k(n) = [1 - (1 + \frac{Z}{n})e^{-Z/n}]v - c_n. \tag{A30}$$

Conditions (**??**) now imply that $\pi_k(1)$ is positive, that $\tilde{\pi}_k(n)$ is strictly decreasing in n, and that $\pi_k(\bar{n})$ is negative.

Let N_k^* be the largest value of n for which $\tilde{\pi}_k(n)$ is positive. Then the entry of N_k^* firms along with the equilibrium of the continuation wage-posting game, constitutes a generically unique equilibrium of the free-entry game. No firm that enters would deviate by choosing not to enter, because its positive expected profits would go to zero; no firm not in business would choose to enter, because its zero profits could not become positive.

The proof for the discriminatory equilibrium is analogous and is not repeated. ■

Proposition ?? *Relative to the nondiscriminatory free-entry equilibrium, the discriminatory free-entry equilibrium has the following properties:*

i. *aside from the marginal entrant, every firm makes greater profit*

ii. *all white workers have lower wages and lower expected incomes,*

iii. *all black workers have lower wages and expected incomes, and*

iv. *net output (output less entry costs) is lower.*

Proof. Suppose N_k^* and N_h^* are the numbers of firms in the free-entry nondiscriminatory and discriminatory equilibria. From Proposition 10, we know that operating profits in the nondiscriminatory regime with N_k^* firms will be less than operating profits in the discriminatory regime with N_k^* firms; i.e., $\pi_h(N_k^*) > \pi_k(N_k^*)$, so that $\tilde{\pi}_h(N_k^*) > \tilde{\pi}_k(N_k^*) > 0$. To surmount the integer constraint on the number of entering firms, we need to assume that the differences between the entry costs of successive firms are small compared with the difference in operating profits between discriminatory and nondiscriminatory regimes ($N_h^* >> N_k^*$), and large compared with the decrease in operating profits that occurs when one firm enters ($\pi_k(N_k^*+1) - \pi_k(N_k^*) < c_{N_h^*} - c_{N_k^*+1}$). (These assumptions are a natural consequence of the requirement that the number of firms and workers in equilibrium be large.) By the definition of the free-entry equilibrium $\tilde{\pi}_k(N_k^*+1) \leq 0$, and it follows from the assumptions above that $\pi_k(N_k^*+1) \leq c_{N_k^*+1} < c_{N_h^*} < \pi_h(N_h^*)$ and $\pi_k(N_k^*) < \pi_h(N_h^*)$.

We have shown that firms' operating profits in the discriminatory regime are greater than in the nondiscriminatory regime, and Point 1) follows immediately. This along with (10) and (19) implies that the white-worker/white-firm ratio in the discriminatory regime is greater than the worker/firm ratio in the nondiscriminatory regime ($r_z > r$), and Point 2) follows. Since blacks are worse off than whites, we have Point 3).

To establish point 4), we first note that by Proposition 9, output in the nondiscriminatory equilibrium for N_h^* firms is greater than that of the discriminatory equilibrium for N_h^* firms, so the same must be true for net output (total entry costs are the same when the number of firms is the same). It remains to show only that in the nondiscriminatory regime net output with N_k^* firms is greater than with N_h^* firms. The expected total output with n firms in business, is given by $Q(n) = (1 - e^{-Z/n})vn$, which is the probability that each firm will fill its job vacancy, times the output per firm, times the number of firms. The derivative $Q'(n) \equiv [1 - (1 + \frac{Z}{n})e^{-Z/n}]v$ represents the nth firm's contribution to total output[13], so that $Q'(n) - c_n \equiv [1 - (1 + \frac{Z}{n})e^{-Z/n}]v - c_n$ is it's contribution to net output. But this expression is identical to the value of $\tilde{\pi}_k(n)$ given in (A30), so that we can conclude that $\tilde{\pi}_k(n)$ not only represents the expected profits of the nth firm, but represents the net contribution to total output as well. But as noted in the proof of Proposition **??**, $\tilde{\pi}_k(n)$ is strictly decreasing, and by the definition of a free-entry equilibrium we know that $\pi_k(N_k^*+1) \leq 0$. It follows that total net output is no greater with N_k^*+1 firms than with N_k^* firms, and it declines as n increases for all $n > N_k^*+1$, so that point 4) is proved. ■

Numerical Example: *Equilibrium given differing distributions of match-specific productivities between blacks and whites.*

Our strategy for constructing this example is as follows. We assume that high-wage firms attract only whites and that the incentive compatibility constraint is binding on white workers; that is, we assume that the equilibrium takes the same basic form as the equilibrium when there is no match-specific productivity. We then verify that the incentive compatibility constraint is not violated for blacks. Given the parameters from this equilibrium, we then check numerically that no wage that attracts both white and black workers is more profitable than the wages in our proposed equilibrium.

Let the probability that a white worker is a good match be p. Denote the productivity of whites as v_g and v_b for good and bad matches, and the productivity of blacks by v. Assume free entry with entry cost d. Then the expected

[13] More precisely, because of the discrete nature of the model, $Q'(n)$ lies between the marginal social product of the nth and $(n+1)$st firms. We will ignore this inconvenient fact, though it would not affect our conclusion.

profit of a firm that attracts only white workers is

$$E(\pi) = (1 - e^{-pz})v_g + e^{-pz}(1 - e^{-(1-p)z})v_b - (1 - e^{-z})w - d. \tag{A31}$$

Given the wage w_w paid in high-wage jobs (and thus received by white workers), workers set z to yield the competitive expected wage K, so that

$$\frac{(1 - e^{-z})}{z} w_w = K. \tag{A32}$$

Subject to this constraint, firms set w_w to maximize $E(\pi)$. The first-order condition is given by

$$pe^{-pz}(v_g - v_b) + e^{-z}v_b = K. \tag{A33}$$

The incentive compatibility constraint for white workers is given by

$$(p + (1 - p)e^{-y})w_b = K. \tag{A34}$$

where w_b is the wage paid in low-wage jobs (and thus received by black workers). Together with the equal-profit constraint for firms offering the low wage,

$$E(\pi) = (1 - e^{-y})(v - w_b) - d, \tag{A35}$$

this determines the arrival rate of job applicants and the wage at low-wage firms.

The expected wage of blacks applying to low-wage firms is given by

$$\frac{(1 - e^{-y})}{y} w_b = J. \tag{A36}$$

Equations (A32) - (A36) together with the requirement that expected profit equal zero, determine K, J, z, y, w and w_b.

To determine whether these values make up an equilibrium, we check that black workers be no better off if they apply to high-wage firms or

$$e^{-pz}w_w \leq J$$

and that no wage that attracts both black and white workers is profitable given the values of K and J in the proposed equilibrium.

If a wage attracts both black and white workers, then the firm's expected profit is given by

$$E(\pi) = (1 - e^{-pz})v_g + e^{-pz}(1 - e^{-y})v + e^{-pz-y}(1 - e^{-(1-p)z})v_b - (1 - e^{-z-y})w - d$$

which the firm maximizes subject to

$$\begin{aligned} K &= w(p\frac{1 - e^{-pz}}{pz} + (1 - p)e^{-pz-y}\frac{1 - e^{-(1-p)y}}{(1 - p)y}) \\ J &= we^{-pz}\frac{1 - e^{-y}}{y}. \end{aligned}$$

We use a grid search to check that no wage that attracts both white and black workers is more profitable than the wages in our proposed equilibrium.

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