EC237 Lecture Notes 5/2/06

set-up

$$\circledast \left\{ \begin{array}{lll} \theta \in [\underline{\theta}, \omega) & : F(\theta) & : \text{econ type} \\ \gamma \in \{h, l\} & : p_h, p_l & : \text{social type} \end{array} \right\} \underbrace{\tau = (\theta, \gamma)}_{\text{agent's type}}$$

unobservable

$\circledast$ $s \in [0,1]$: effort, observable

$\circledast$ $C(s,\theta)$: cost of effort : $\left[C(s,\theta) = \left(\frac{c-\theta}{2}\right) \cdot s^2 \right]$

$\circledast$ $W(s,\theta)$: productivity : $[W(s,\theta) = s * \theta]$

$\circledast$ $\alpha \in \{0,1\}$: social acceptance/rejection

utility

$$u = \underbrace{[W - C(s,\theta)]}_{\text{econ. payoff}} + \underbrace{[1 + \alpha\gamma] * (1-s)}_{\text{social payoff}} = u(w, s, \alpha; \tau)$$

Thus:

$$\alpha = 0 \Rightarrow u = w - [\underbrace{C(s,\theta) + s}_{\text{cost}|_{\alpha=0}}] + 1$$

$$\alpha = 1 \Rightarrow u = w - [\underbrace{C(s,\theta) + (1+\gamma)s}_{\text{cost}|_{\alpha=1}}] + (1+\gamma)$$

Equilibrium

social acceptance $\hat{\alpha}(s) = \left\{ \begin{array}{c} 1 \\ 0 \end{array} \right\}$ as $Pr[\gamma = h \mid s] \left\{ \begin{array}{c} \geq \\ < \end{array} \right\} p^* \in (0,1)$.

wage schedule $\hat{W}(s) = E\{ W(s,\theta) \mid \hat{\sigma}(\cdot) \}$

effort choice $\hat{\sigma}(\tau) = \arg\max_{s \in [0,1]} \{ u(\hat{w}(s), s, \hat{\alpha}(s); \tau) \}$

(θ, γ)

strategies $\langle \hat{\alpha}(s), \hat{W}(s), \hat{\sigma}(\tau) \rangle$ which satisfy above constitute a Signalling Equilibrium.

Benchmark: Assume γ obs., but θ unobs.

Then $\hat{\alpha}(\gamma) = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$ as $\gamma = \begin{Bmatrix} h \\ \ell \end{Bmatrix}$

$$\Rightarrow C_\gamma(s,\theta) = \begin{Bmatrix} c(s,\theta) + (1+h)s \\ c(s,\theta) + s \end{Bmatrix} \text{ as } \gamma = \begin{Bmatrix} h \\ \ell \end{Bmatrix}$$

Hence, separate signalling problems, given γ.

$$\hat{\sigma}_\gamma(\theta) \equiv \arg\max_s \{ W(s, \hat{\mu}_\gamma(s)) - C_\gamma(s,\theta) \}$$

$\boxed{\text{separating Equilibrium}}$ $\Rightarrow W_s(s, \hat{\mu}_\gamma(s)) + W_\theta(s, \hat{\mu}_\gamma(s)) \frac{d\hat{\mu}_\gamma}{ds} = \frac{d}{ds} C_\gamma(s, \hat{\mu}(s))$

Adopt Functional Forms

$$W(s,\theta) = s*\theta$$
$$c(s,\theta) = (c-\theta)\frac{s^2}{2}, \qquad \theta < c \equiv \text{constant}$$

~~Then FOC~~ $\Rightarrow$

$\gamma = h$: $\hat{\mu}_h(s) + s\frac{d}{ds}[\hat{\mu}_h(s)] = s[c - \hat{\mu}_h(s)] + (1+h)$

$\gamma = \ell$: $\hat{\mu}_\ell(s) + s\frac{d}{ds}[\hat{\mu}_\ell(s)] = s[c - \hat{\mu}_\ell(s)] + 1$

Consider Diff Eq: $\boxed{\mu(s) + s\mu'(s) = s*[c - \mu(s)] + d}$

Plus Boundary Condition:

$$\hat{\mu}_\gamma^{-1}(\theta_{Min}) = S_0 \equiv \arg\max_s \{ W(s,\theta_{Min}) - C_\gamma(s,\theta_{Min}) \}$$

(lowest economic ability type chose full-information 1^{st} best effort)

$\boxed{\text{Analysis}}$

let $g(s) \equiv s * [c - \mu(s)] + d \qquad \left(d = \begin{Bmatrix} 1 \\ \text{or} \\ 1+h \end{Bmatrix} \right)$

$= MC|_s$

Then $g'(s) = c - [\mu(s) + s\mu'(s)]$

so, diff eq becomes

$$\boxed{c - g'(s) = g(s)}$$

$$\Rightarrow g(s) = e^{-(s-s_0)} [g(s_0) + c * (e^{s-s_0} - 1)], \quad s \geq s_0$$

$$g(s_0) = s_0 [c - \mu(s_0)] + d$$

and $s_0 = \arg\max_s [W(s,\theta_{Min}) - C_\gamma(s,\theta_{Min})]$

so $[g(s) - g(s_0)] = [c - g(s_0)] * [1 - e^{-(s-s_0)}]$

and $\hat{\mu}(s) = c - \left(\frac{g(s) - d}{s} \right)$

$$= c - \left(\frac{1}{s}\right) * \{ g(s_0) e^{-(s-s_0)} + c(1 - e^{-(s-s_0)}) - d \}$$

and $S_0 = \arg\max_S \left\{ s * \theta_{Min} - \frac{s^2}{2}[c - \theta_{Min}] - d \cdot s \right\}$

$$\Rightarrow \quad S_0 = \frac{\theta_{Min} - d}{c - \theta_{Min}}$$

and

$$MC|_{S_0} \equiv g(S_0) = S_0 \, [c - \theta_{Min}] + d = \theta_{Min} = MB|_{S_0}$$

Hence:

$$\mu_\gamma(s) = c - \frac{1}{s}\left\{ \theta_{Min} e^{-(s-s_0)} + c\,[1 - e^{-(s-s_0)}] - d \right\}$$

$$\text{for } s \geq s_0 \equiv \frac{\theta_{Min} - d}{c - \theta_{Min}}$$

where $\left\{ \begin{array}{l} \gamma = h \Rightarrow d = 1+h \\ \gamma = \ell \Rightarrow d = 1 \end{array} \right\}$

[assuming $d < \theta_{Min} < C$].

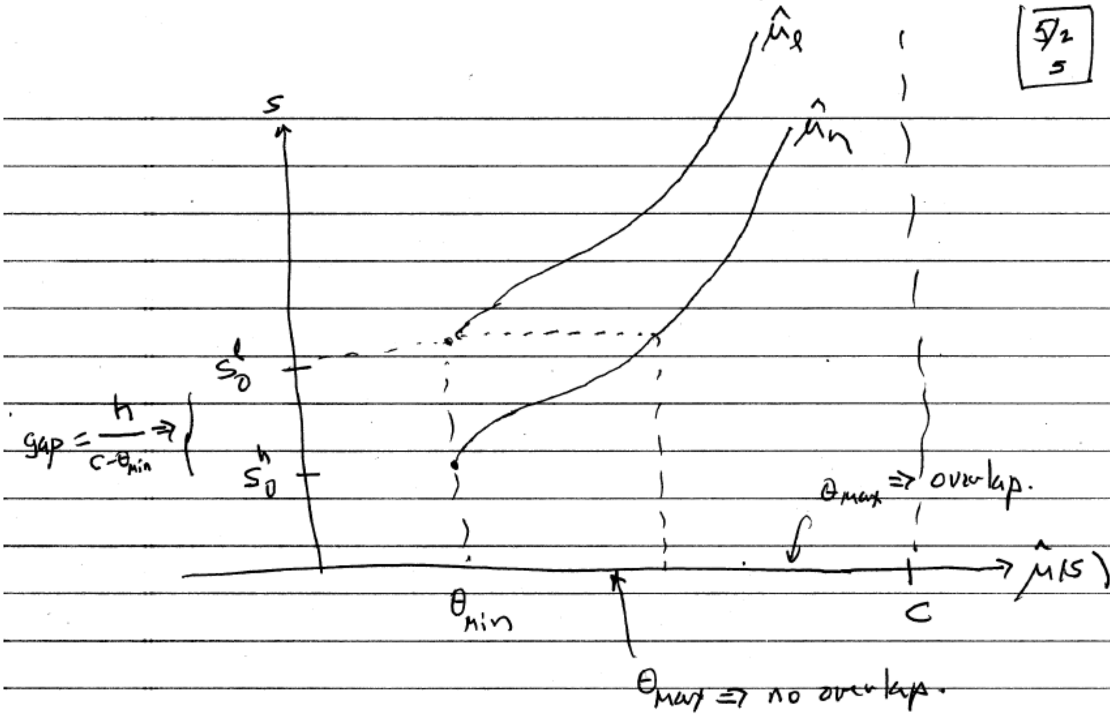
or,

$$[C - \hat{\mu}_\gamma(s)] = \frac{1}{s}\left\{ [c - \theta_{Min}] \cdot [1 - e^{-(s-s_0)}] + [\theta_{Min} - \underbrace{d}_{= 1+h \text{ or } = 1}] \right\}$$

$$s \geq s_0 \equiv \frac{\theta_{Min} - d}{c - \theta_{Min}}$$

Full solution for separating Eq.

and $\boxed{\hat{\sigma}_\gamma(\theta) = \hat{\mu}_\gamma^{-1}(\theta)}$



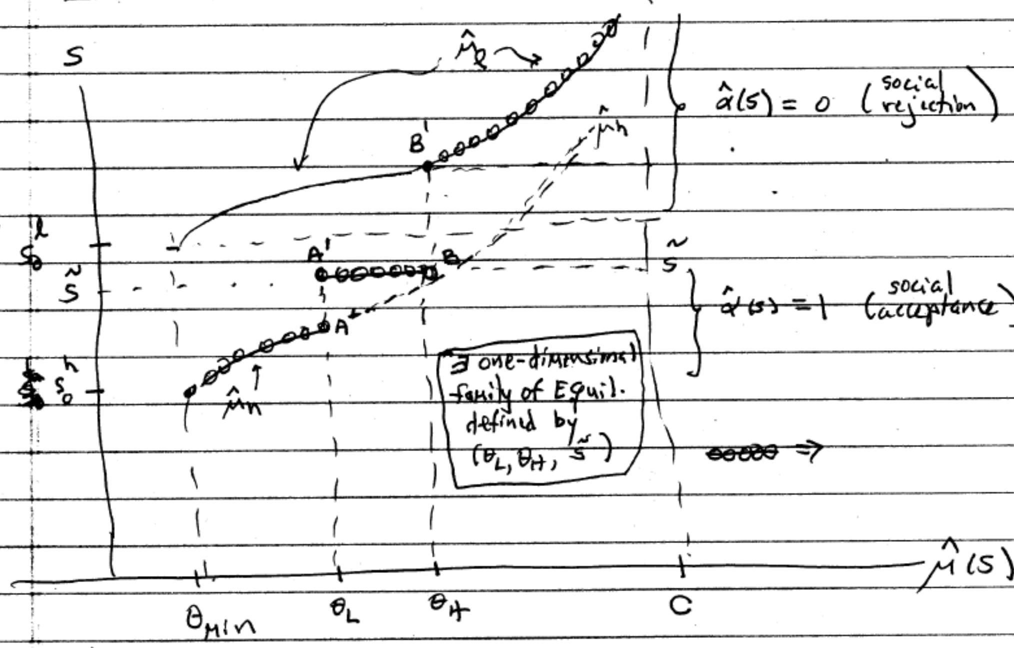
Notice:

① $d_h = 1+h > d_l = 1$, so low social types exert more effort ⇒ infer higher economic ability from given effort if observe high social ability.

② if range of θ is such that no overlap, then fully separating equilibrium exists.

③ otherwise, fully separating Eq. not possible.

What is Equilibrium when There is Overlap?



deviation $\in (B, B')$
$\Rightarrow$ $p_\ell = 1$

deviation $\in (A, A')$
$\Rightarrow$

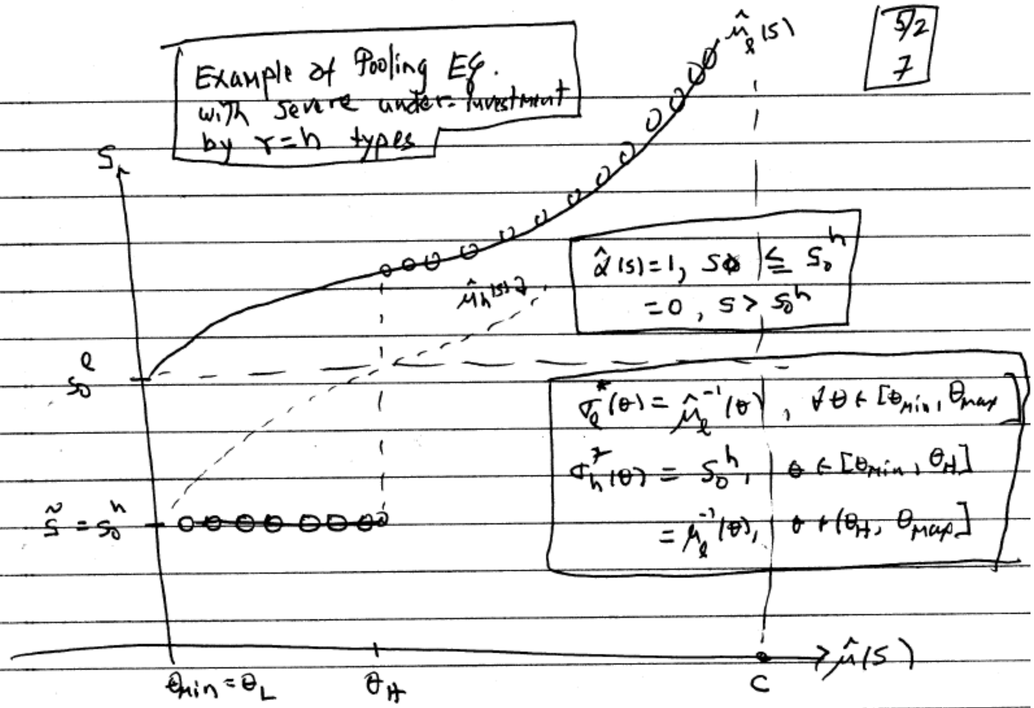
$\boxed{\gamma = \ell} \Rightarrow$ play separating strategy $[\hat{\mu}_\ell^{-1}(\theta) = \sigma_\ell]$ as before.

$\boxed{\gamma = h} \Rightarrow$ (1) play separating strategy $[\hat{\mu}_h^{-1}(\theta) = \sigma_h]$ for $\theta \in [\theta_{Min}, \theta_L)$

(2) pool on $\tilde{s} = \sigma_h(\theta)$, for $\theta \in [\theta_L, \theta_H]$, where $\tilde{s} \in (\sigma\mu_h^{-1}(\theta_L), \mu_\ell^{-1}(\theta_{Min}))$

(3) play separating strategy $[\hat{\mu}_h^{-1}(\theta) = \sigma_h]$ for $\theta \in [\theta_H, \theta_{Max}]$.

Such that (θ_L, h) types are indifferent between A & A', while (θ_H, h) types are indifferent between B & B'



where θ_H such that

$\tau = (\theta_H, h)$ type indifferent between

acceptance ⊕ $s = \tilde{s}$

and

rejection ⊕ $s = \sigma_\ell(\theta_H)$.