Memorandum

On the Dynamics of Group Inequality

To: EC 237 Students
From: Prof. Loury
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Segregation Group Inequality Dynamics

This note extends Loury's simple, intergenerational social mobility model (L) which has been presented in class, in order to generalize the analysis of persistent group inequality in the paper of Sam Bowles and Rajiv Sethi (BS).

Preliminaries

Let us begin by reviewing the BS model. As you will recall, BS posit a dynamic model of binary human capital choice with multiple social groups. Consider a society which exists over an infinite series of dates $t \in \{0, 1, 2, \ldots\}$, and with overlapping generations of agents. At each date a continuum of agents of unit measure enters society. Each agent lives for two periods, and each belongs to one of two social groups, B or W, where $\beta \in (0, 1)$ is the measure of group B agents who enter society at each date.

In the first period of life each agent makes a human capital investment which determines that agent's level of skill, $h \in \{0, 1\}$. (For now we assume that perfect markets for human capital loans exist, so there is no impediment to an agent acquiring a high level of skill if that is what maximizes the agent's anticipated net gain.) Agents work in the second period of life, receiving a gross return that depends on the level of skill. (For now, we take these returns to be exogenous. Later in this note we will relax these "perfect capital market" and "exogenous returns" assumptions.)

Without loss of generality, we take the *net* gain to an agent who acquires the low skill level to be zero, and we denote by $-c \in \Re$ the net gain to an agent from choosing the high skill level. Thus, a rational agent chooses to acquire a high (low) level of skill if $c < 0$ ($c > 0$). BS treat c as endogenous. Specifically they assume that, for each agent in a given generation, the net cost to that agent of acquiring the high skill level is a decreasing function of the rate of human capital investment among those from the immediately preceding generation who belong to that agent's social network.

Thus, denote by $\sigma \in [0, 1]$ the fraction of those with whom an agent is associated who have themselves acquired human capital. (I restrict attention in this note to the "homogeneous ability" case in BS.) Then, our assumption is that $c = c(\sigma)$, where $c(\bullet)$ is a strictly decreasing, differentiable function on $[0, 1]$ satisfying

$$c(0) > 0 > c(1). \qquad \#$$

We denote by σ^* the threshold quality of an agent's social network above which it becomes rational for that agent to acquire a high level of skill. That is, $c(\sigma^*) = 0$. Thus, if σ_t is the quality of a newborn agent's network at date t, then that agent acquires a high level of skill (i.e., chooses $h_t = 1$) if $c(\sigma_t) < 0$ (i.e., if $\sigma_t > \sigma^*$), while that agent acquires a low level of skill (i.e., chooses $h_t = 0$) if $c(\sigma_t) > 0$ (i.e., if $\sigma_t < \sigma^*$).

Social Structure in the BS Model

We now describe this model society's social structure. Assume that the composition of an agent's social network depends only on his group identity and his generation. Specifically, we envision that each agent entering society at date $t+1$ has a large number of social ties to agents who entered at date t, and that these associations are generated in the following manner: Each one of an agent's associates is with probability $\eta \in [0,1]$ chosen at random from among those who belong to that agent's social group, B or W, while with probability $1-\eta$ the associate is drawn at random from among the general population of agents. As a result, the probability that an associate of a group B agent also belongs to group B is $\eta + (1-\eta)\beta$; and, the probability that an associate of a group W agent also belongs to group W is $\eta + (1-\eta)(1-\beta)$. The parameter η represents the degree of in-group bias (or segregation) in the society's social networks.

Because of social complementarities in the acquisition of skills, and in light of (), multiple self-sustaining levels of human capital investment are possible within a network of affiliated agents: If enough people within an agent's social network are highly skilled ($\sigma > \sigma^*$), then it will pay for that agent to become highly skilled ($c(\sigma) < 0$). Conversely, if too few people in an agent's network are highly skilled ($\sigma < \sigma^*$), then it will not pay for that agent to become skilled ($c(\sigma) > 0$).

We now investigate the dynamics of this simple model of human capital acquisition with social externalities.

Dynamics of the BS Model

In any generation $t = 0,1,2,\ldots$ and for either social group $j \in \{B,W\}$ let x_t^j denote the fraction of agents from generation t belonging to social group j who become highly skilled. Then, the quality of an agent's social network at generation $t+1$ depends on that agent's social group, and on the human capital investment decisions of agents at generation t, and is given as follows:

$$\sigma_{t+1}^b = [\eta + (1-\eta)\beta]x_t^b + [(1-\eta)(1-\beta)]x_t^w \text{ and} \quad \#$$
$$\sigma_{t+1}^w = [(1-\eta)\beta]x_t^b + [\eta + (1-\eta)(1-\beta)]x_t^w$$

Notice that there are two circumstances under which agents from the two groups in generation t will have social networks of the same quality: either if $\eta = 0$ (meaning there is no group bias in the associational behavior of agents), or if $x_t^b = x_t^w$ (meaning that there is no group inequality in skill acquisition among the previous generation.) Otherwise, if one social group is less skilled than the other at generation $t-1$, then the quality of their social network is lower for generation t agents who belong to the less skilled group.

Furthermore, notice that under () we have the following equations of motion for $\{(x_t^b, x_t^w)\}$:

$$x_{t+1}^b = 1 \text{ if } [\eta + (1-\eta)\beta]x_t^b + [(1-\eta)(1-\beta)]x_t^w > \sigma^*, \text{and} \qquad \#$$
$$x_{t+1}^b = 0 \text{ if } [\eta + (1-\eta)\beta]x_t^b + [(1-\eta)(1-\beta)]x_t^w < \sigma^*$$

and

$$x_{t+1}^w = 1 \text{ if } [(1-\eta)\beta]x_t^b + [\eta + (1-\eta)(1-\beta)]x_t^w > \sigma^*, \text{and} \qquad \#$$
$$x_{t+1}^w = 0 \text{ if } [(1-\eta)\beta]x_t^b + [\eta + (1-\eta)(1-\beta)]x_t^w < \sigma^*$$

Given an historical legacy for this society, reflected in the level of skill acquisition for the two groups in some initial generation ($t = 0$), it is obvious from () and () that the society converges in one step to a steady state in which all agents in a given social group acquire the same level of skills.

Main Result in the BS Model

What interests me in the BS paper is that one can use this very simple coordination model of human capital investment to gain insight into how social structure, demography, and the economic fundamentals of tastes and technology operate in tandem to determine the stability of historical inequality between social groups. In particular, we have the following result:

Theorem: Given the fraction of the overall population belonging to an historically disadvantaged group, β*, there exists a minimal degree of within-group bias in the network of agent associations,* $\underline{\eta}(\beta) < 1$ *such that, whenever* $\eta > \underline{\eta}(\beta)$*, the initial condition* $(x_0^b, x_0^w) = (0,1)$ *is a stable steady state of the model.*

Proof: To see why this result is true notice that, in light of () and (), the historically given situation $(x_0^b, x_0^w) = (0,1)$ is a stable steady state if and only if: $\sigma_1^b < \sigma^* < \sigma_1^w$, which (given ()) amounts to the requirement that:

$$(1-\eta)(1-\beta) < \sigma^* < \eta + (1-\eta)(1-\beta)$$

or, equivalently, that:

$$\underline{\eta} > \eta(\beta) \equiv \text{Max}\{1 - \frac{\sigma^*}{1-\beta}; 1 - \frac{1-\sigma^*}{\beta}\}. \ \square \qquad \#$$

Interpretation of the Main Result: Demography vs. Occupational Structure

This result has the interpretation that, for sufficiently biased (segregated) social networks, a history of group inequality which left all Bs with a low level of skill and all Ws with a high level of skill could persist indefinitely, absent some policy intervention aimed at reducing the group disparity in investment rates. The first term in curly brackets above reflects the requirement that $\sigma_1^b < \sigma^*$ (which implies group B members of the next generation will choose $h = 0$), while the second term reflects the requirement that $\sigma^* < \sigma_1^w$ (implying group W members of the next generation choose $h = 1$.) So, if η exceeds both of he bracketed terms then an initial condition wherein all Bs acquire low skills and all Ws acquire high skills will persist indefinitely.

Notice that the first term on the RHS of () is a decreasing, concave function of β, and is negative for $1-\beta<\sigma^*$. So, if the fraction of the population belonging to the advantaged group ($1-\beta$) is small enough, then disadvantaged group members always acquire a low level of skill ($\sigma_1^b<\sigma^*$), no matter how small is the degree of in-group bias in the formation of social networks (i.e., even for $\eta\approx 0$). By contrast, as the proportion of Ws in the population rises above the threshold σ^*, the extent of social segregation must also increase, diluting human captial spillovers for Bs, in order for disadvantaged group members to continuing choosing $h=0$. Too much social integration de-stabilizes the unequal initial condition $(x_0^b,x_0^w)=(0,1)$ when the high skilled group is relatively large, because human captial spillovers to disadvantaged group members become so great that they find it cost effective to become highly skilled.

The second term on the RHS of () is an increasing function of β, and is negative for $1-\beta>\sigma^*$. So, starting at $(0,1)$, Ws continue to become highly skilled regardless of the degree of network integration so long as their group is large enough. However, as the size of the advantaged group falls below the thresold σ^*, the extent of social segregation must rise, hoarding the human captial spillovers within the group W, in order for its members to continuing choosing $h=1$. That is, when the high skilled group is relatively small, too much integration dilutes the human capital spillovers to the extent that becoming highly skilled is no longer cost effective for new members of the advantaged group [again, de-stabilizing the unequal initial condition, $(x_0^b,x_0^w)=(0,1)$].

Here, then, we can see how social structure (η), demographics (β) and tastes/technology (σ^*) interact (albeit, in a crude way) to determine whether historically generated group inequality will be a stable situation. It is worth noticing in this context that the range of social networks (parameterized by $\eta\in[0,1]$) for which inequality is stable grows wider, the closer is β to $1-\sigma^*$. (Indeed, under the knife-edge situation when $\beta=1-\sigma^*$ exactly, inequality is stable for all social networks no matter how integrated.) This can be interpreted to say that if group size matches-up with the “occupational structure” in just the right way, then no degree of social integration can by itself destablize historically generated group inequality. For if $1-\beta\approx\sigma^*$, then once the advantaged group obtains a monopoly on highly skilled positions, and regardless of the extent to which social networks are segregated or integrated, human capital spillovers will always be great enough to make investment in high skills cost-effective for Ws, but never sufficient to do so for Bs.

The parameter σ^*, which in the BS model represents the threshold level of network quality above which investment in human capital is cost effective, reflects “occupational structure” in the following (admittedly, crude) way: Define an *integrated hierarchical society (IHS)* as one where social networks are completely integrated ($\eta=0$), but initially one group has a monopoly on highly skilled positions. When, we ask, is it possible for there to exists a socially integrate but occupationally hierarchical society in a stationary equilibrium of the economy?

To answer this question, continue to let $1-\beta$ be the size of the advantaged group in an IHS. Then, since the society is fully *integrated*, $1-\beta$ is also the quality of everyone’s social network. Yet, since the society is hierarchical we must have that (dis)advantaged group members maximize their net payoff by (not) becoming highly skilled. It follows that an IHS is possible if and only if demography matches-up with technology in the following way: $1 - \beta = \sigma^*$. If the size of the advantaged group departs from this critical value then, as we have just seen, some social segregation is necessary to sustain occupational hierarchy. Thus, we can also think of σ^* as being that "aggregate skill intensity" in society's occupational structure that is maximally consistent with occupational hierarchy, in the sense that when the size of a group that would monopolize the skilled positions is near σ^*, then the amount of social segregation needed for that group to sustain its monopoly on high skill positions is small.

Now, in a more general model (see below) the gain from becoming highly skilled would depend on the relative supplies of high and low skilled labor. Intuitively, as the ratio of employed factors ($\frac{H}{L}$) rises, this gain falls because of diminishing returns. (In the presence of human capital spillovers the cost of becoming highly skilled could also decline as the skill intensity rises, though let us for now assume that this cost declines more slowly than does the gain, if at all.) Thus, there will exist a critical factor ratio in the context of a more general model at which, for a socially integrated society, these gains and costs are exactly in balance. Because all agents in an integrated society at this critical factor ratio are indifferent as between skill choices, we have great latitude to construct an IHS that is consistent with rational behavior of all agents.

Think of $\frac{\sigma^*}{1-\sigma^*}$ as being equal to this critical factor ratio in the more general model alluded to above. In light of the consequences of indifference just noted, were such a factor ratio to come about in equilibrium, a (knife-edged) IHS could be constructed by imagining that relatively few (many) members of the disadvantaged (advantaged) group choosing a high level of skill. But this might be an unstable situation in the sense that, because of their indifference, all agents' behaviors would shift discontinuously (relative to this knife-edged equilibrium supporting the IHS) in the presence of just the slightest degree of social segregation. (That is, for $\eta > 0$, no matter how small, advantaged (disadvantaged) group members would strictly prefer to choose a high (low) skill level.) And yet, if the relative size of the advantaged group in the IHS just happens to be equal to the relative number of skilled workers at this critical factor ratio, then the IHS would not be destablized by the presence of a small amount of segregation, since the discontinuous shift in the behaviors of group members would not imply a change in aggregate factor proportions.

For this reason, one can think of σ^* as a proxy for the "skill intensity of the society's occupational structure" that is maximally consistent with a socially integrated occupational hierarchy. Moreover, one can see that an IHS is more likely to be a stable arrangement whenever the size of a would-be advantaged group matches-up closely with this proxy for the society's occupational structure. But, if an IHS is stable, and if the advantaged group is of a size that matches closely the relative size of the highly skilled workforce, then group-based occupational hierarchy will also be stable when the society's social networks exhibit any positive degree of in-group bias: That is, the stability of group-based occupational hierarchy is guaranteed across the widest range of network structures (degrees of segregation) when the relative size of the high skilled occupational niche (a technology/preferences-based datum) matches-up closely with the relative size of a would-be advantaged group (a demographic datum)!

Networks, Mobility and Group Inequality

We now formalize the intuitive analysis above using the L model to introduce in class. In this model we explicitly allow for competitive factor markets, making the returns to human capital investments endogenous. And, we relaxing the assumption of perfect markets for human capital loans, thereby making the cost of human capital investment depend on the financial success of a child's parent, as well as on the quality of the child's social network.

Preliminaries

Imagine a society which, as before, exists over an infinite sequence of dates (generations), and which at any date consists of a continuum of workers belonging to one of two social groups. As before, the workers live for two periods, acquiring human capital in the first period of life and working for wages in the second period. The generations overlap, so that each young worker (i.e., the "child") is attached to an older worker (i.e., the young worker's "parent.") Parents decide on the human capital investment for their children in a manner specified below. We first describe the model in a homogeneous society (no social groups), and then we will introduce groups to the analysis.

There are two jobs, high skilled and low skilled, paying wages w_H and w_L, respectively. High and low skilled workers are combined to produce a homogeneous output by competitive firms who operate under conditions of constant returns to scale and diminishing returns to individual factors, selling their output into the world market at an exogenous price that is normalized to be one. Labor markets are perfectly competitive. Let x denote the fraction of the highly skilled among some generation of workers. Then, we can write wages as a function of relative factor intensity in the following manner: $w_H = w_H(x)$, with $w_H'(x) < 0$; $w_L = w_L(x)$ with $w_L'(x) > 0$; and $\Delta w(x) \equiv w_H(x) - w_L(x)$, with $\Delta w'(x) < 0$.

Parents in any generation derive utility from consumption and from the anticipated earnings of their child. (So, the anticipated wage difference for the next generation, $\Delta w(x_{t+1})$, will represent the incentive a parent in generation t faces to training a child to become a highly skilled worker.) Parental consumption is what's left over after making payment for a child's human capital. (In this way we capture the notion that human capital loans are not available, so a child's acquisition of skills must be financed out of parental earnings.) Thus, let $u(c) + w$ denote parent's utility when parent's consumption is c and child's wage is w. (Then, because of diminishing marginal utility of consumption, a parent with lower earnings will face a higher effective cost of human capital for the child.) We take $u(\cdot)$ to be a strictly increasing, strictly concave function.

We assume that all agents are qualified without further training for low skilled work, and we implicitly assume that such work is always preferrable to whatever outside option may be available to the agents. In order to be qualified in their second period of life for high skilled work, agents must have received training during their first period of life. (Note to students: The assumption I am making here regarding a very low outside option is *not* innocuous, as the analysis of “self-defeating flight” in Benabou’s 1993 QJE paper makes clear. It would be an instructive exercise for you to extent the analysis provided in this note to the case where workers opt out of the economy altogether in favor of some exogenous outside option, in the event that the low skilled wage falls below some threshold.)

We further assume that children vary in their natural ability, which can take three values – high, normal, or low. Parents know their child’s ability before deciding on human capital investment. It is costless for a parent to train a high ability child for skilled work, and the cost of training a low ability child for skilled work is so great that no parent would ever do so. In each generation, and for all parents regardless of *their* ability, the probability that a child is born with high ability equals the probability that a child is born with low ability. We denote the common value of these probabilities by $\theta \in (0, \frac{1}{2})$. Thus, $1 - 2\theta > 0$ is the likelihood that a child is normal. We assume that for a normal child the cost of training for highly skilled work is $K > 0$. (We have in mind the scenario where “most” children are normal, so we imagine that the parameter θ is “small.” Indeed, we will be particularly interested in the limit of the equilibria of this model as $\theta \downarrow 0$.)

Imagine for the moment that there were no human capital spillovers, and consider the utility cost (denoted by $\phi(w)$) to a parent with wage w of training a normal child for a high skilled job. Then: $\phi(w) = u(w) - u(w - K)$, where $\phi(\bullet)$ is a strictly decreasing function of w.

Time is measured in terms of generations, $t = 0, 1, 2, \ldots$, with x_t being the fraction of skilled workers in generation t. The fraction of skilled parents in the starting generation ($t = 0$) is denoted $x_0 \in (0, 1)$, and is assumed to have been given by history. In light of our assumptions about the random assignment of abilities, and without loss of generality, we assume that $x_0 \in [\theta, 1 - \theta]$. Our task, given x_0, is to find the path of the subsequent fractions of skilled workers $\{x_t : t \geq 1\}$ in the competitive equilibrium of this model economy.

In light of the foregoing discussion, a skilled parent at generation t receives wage $w_H(x_t)$ and an unskilled parent receives $w_L(x_t)$. Since skill acquisition is costly for the parents of normal children, it follows that in any competitive equilibrium where any normal child in generation t receives training, we must have that $\Delta w(x_t) > 0$. To avoid a trivial equilibrium (in which only those born with “high” ability become high skilled workers, or in which only those born with “low” ability become low skilled workers), we need to assume that:

$$\phi(w_H(\theta)) \; < \Delta w(\theta) \text{ and } \phi(w_L(1-\theta)) \; > \Delta w(1-\theta). \qquad \#$$

(This assumption is quite a weak one for small values of θ.)

Social Networks (revisited)

In order to incorporate the BS model into the framework being developed here, we adopt an analytically convenient (if not exactly compelling) specification for how human capital spillovers are propagated in this model society. Specifically, and as before, let $\sigma \in [0, 1]$ denote the fraction of those from the immediately previous generation with whom an agent is associated (or, and equivalently for our purposes, the fraction of those with whom that agent’s parents are associated) who have, themselves, acquired a high level skill. We think of σ as the quality of the child's (or, equivalently, of the parent's) social network. Then we will assume that, for some parameter $\gamma > 0$, the cost of training a normal child for skilled work – when the parent has wage w and social network of quality σ, is $\phi(w + \gamma \cdot \sigma)$. The parameter γ thus reflects the economic importance of human capital spillovers, and our specification amounts to assuming that high quality social networks reduce human capital investment costs by, in effect, increasing the economic resources available to the investing parent, thereby reducing the utility cost of the investment. So, in our model a better network is equivalent for investment purposes to the parent having a higher wage.

It will be instructive – before discussing social groups, but while maintaining the assumption that human capital spillovers operate as specified above – to consider what initial allocations of workers to high and low skilled jobs, x_0, could possibly be a stationary equilibrium in this model (i.e., an equilibrium in which $x_t = x_0, \forall t \geq 1$). Indeed, there is at most one such allocation, as demonstrated below:

Lemma: In a perfectly integrated society with random abilities ($\theta > 0$), if a stationary equilibrium allocation, x*, exists, and if all parents are playing pure strategies (i.e., investing or not with probability one), then $x^* = \frac{1}{2}$.

Proof: In any stationary equilibrium the wage must be greater in high than in low skilled work – otherwise, nobody would train a normal child for high skilled work, which in light of () is an impossibility in equilibrium. But, because training cost for normal children decreases with a parent's wage, it is the case that parents with high skilled jobs will want to train their children whenever parents with low skilled jobs do so. Likewise, parents with low skilled jobs will not want to train their children whenever parents with high skilled jobs do not want to do so. But, again invoking (), it could never be an equilibrium for all parents, or no parents, to train their normal children. Hence, stationarity equilibrium, if it exists, must involve skills being passed along within families in such a way that high skilled parents train, and low skilled parents do not train, their normal children for high skilled work.) But then, stationarity implies

$$x_0 \equiv x^* = x_1 = (1-\theta)x_0 + \theta(1-x_0), \qquad \#$$

which, in turn, implies $x_0 \equiv x^* = \frac{1}{2}$.□

But, when will this stationary equilibrium in pure strategies exist? Clearly, it must be the case that when exactly half the workforce is skilled and half unskilled, the wage premium to skilled work is high enough to induce the skilled parents of normal children to invest, but not so high that the unskilled parents of normal children are willing to invest. In other words, and allowing for human capital spillovers but continuing to assume an integrated society, stationarity with pure strategies is possible if and only if:

$$\phi(w_H(\frac{1}{2}) + \frac{\gamma}{2}) \leq \Delta w(\frac{1}{2}) \leq \phi(w_L(\frac{1}{2}) + \frac{\gamma}{2}) \qquad \#$$

Throughout this discussion we will assume that condition () hold. It follows from the argument for the Lemma that, when () fails, no stationary equilibrium with parents playing pure strategies is possible.

[Note to students: As an exercise you should extend this Lemma to show that:

(a) when one admits mixed strategies, stationary allocations always exists, whether or not () holds;

(b) when $\phi(w_H(\frac{1}{2}) + \frac{\gamma}{2}) > \Delta w(\frac{1}{2})$ (and, assuming $\frac{d}{dx}[w_H(x)] < -\gamma$), then $\hat{x}$ is a stationary equilibrium allocation (supported by high skilled parents playing a mixed strategy), where $\hat{x} < \frac{1}{2}$ solves

$$\phi(w_H(\hat{x}) + \gamma\hat{x}) = \Delta w(\hat{x}); \text{ and,}$$

(c) when $\phi(w_L(\frac{1}{2}) + \frac{\gamma}{2}) < \Delta w(\frac{1}{2})$ then x' is a stationary allocation (supported by low skilled parents playing a mixed strategy), where $x' > \frac{1}{2}$ solves

$$\phi(w_L(x') + \gamma x') = \Delta w(x').$$

Moreover, you should be able to convince yourselves based on our discussions in class that,

(d) with or without (), if a certain stability condition holds (which one?), then the economy always approaches *some* stationary allocation as $t \to \infty$, no matter where it starts.]

Now, one crucial difference between the current model and the one adopted in BS is that the presence of human capital spillovers does not imply the existence of multiple stationary equilibrium allocations here. True enough, if all normal parents have been trained for high skilled work and if the economic importance of spillovers (γ) is large, then the cost to any parent of training a normal child would be low. But, because of diminishing returns and in light of (), it would also be the case that the gross return to becoming skilled at such a putative stationary allocation (Δw) would be negative. Likewise, if no normal parents had been trained for high skilled work, then the cost to any parent of training a normal child would be high. But, again invoking diminishing returns and (), the gross return to becoming skilled at such a putative stationary allocation would be huge. So, although we have a model with complementarity, we do not have a model with multiple stationary allocations under complete social integration. Recall that, in the BS model, this multiplicity of equilibria in the integrated society was necessary for the existence of persistent group inequality in a stationary allocation for the segregated society. That is no longer this case here, as the following discussion shows.

Social Groups (Revisited)

Let us now, finally, introduce groups into this extended model of social mobility. We adopt here the same specification and notation used earlier to describe social groups and the segregation of their networks. Specifically, there are two groups, B and W, $\beta \in (0,1)$ is the fraction of any generation belonging to group B, x_{jt} is the fraction of agents from generation t belonging to social group j who become highly skilled, and σ_t^j is the quality of a group j agent's social network at generation t, $j = B, W$. Then x_t denotes the aggregate rate of high skilled employment in the economy at date t, and:

$$x_t \equiv \beta x_t^b + (1-\beta)x_t^w$$

The determination of network quality relevant to generation $t+1$ young agents is once again given by equations ().

We introduce the following notation to describe parental training behavior. Let $p(w, \Delta w, \sigma)$ denote the probability that a child is becomes qualified to do skilled work when the parent's wage is w, the anticipated wage gain for a child's being skilled is Δw, and the quality of the relevant social network is σ. Then if parents are maximizing their utility we must have that:

$$p(w, \Delta w, \sigma) \left\{ \begin{array}{c} = 1-\theta \\ \in [\theta, 1-\theta] \\ = \theta \end{array} \right\} \text{ as } \phi(w+\gamma\sigma) \left\{ \begin{array}{c} < \\ = \\ > \end{array} \right\} \Delta w \qquad \#$$

(In the case of equality above, parents are indifferent about training their children, and so they might be playing mixed strategies.)

The fundamental equation of motion defining intergenerational competitive equilibrium for this society is therefore given by:

$$x_{t+1}^j = p(w_H(x_t), \Delta w(x_{t+1}), \sigma_{t+1}^j)x_t^j + p(w_L(x_t), \Delta w(x_{t+1}), \sigma_{t+1}^j)(1-x_t^j) \qquad \#$$

for $j = B$ and W. By considering (), (), and () one can see that, given an initial condition (x_0^b, x_0^w), there is a unique equilibrium path consistent with labor market clearing and parental utility maximization. (Of course, utility maximizing parental behavior is not uniquely defined in the case of indifference. Nevertheless, the equilibrium allocation is uniquely defined by these equations, since parental indifference at date t only occurs for specific values of factor allocations at dates t and $t+1$.)

I can now state and prove the following proposition.

Proposition: Under condition () a stationary equilibrium allocation exists at which group inequality persists indefinitely if the following three conditions hold: (a) there is a sufficiently large degree of social segregation; (b) the disadvantaged group is not too big; and, (c) the economic significance of human capital spillovers is neither too weak nor too strong.

Proof: The remainder of this note is devoted to establishing this Proposition. The proof is constructive. That is I will, under the conditions hypothesized, exhibit an explicit allocation that has the asserted property. In this stationary allocation, high skilled B parents of normal children never train their progeny for highly skilled work, high skilled

W parents always do, and low skilled parents in both groups do not train their normal children. I denote the aggregate rate of skilled employment at this group-unequal stationary allocation by $\tilde{x}$. It follows from the foregoing discussion, then, that

$$\tilde{x} = \beta\theta + (1-\beta)(\frac{1}{2}). \quad \#$$

(It follows from () that equation () must hold at a stationary allocation, if no unskilled parents, all skilled W parents, and no skilled B parents are training their normal children.) Let $\tilde{\sigma}_j$ be the quality of social networks for members of group j at this putative stationary allocation, $j = B$ or W. Then () implies that:

$$\tilde{\sigma}_b = \eta\theta + (1-\eta)\tilde{x} \text{ and } \tilde{\sigma}_w = \eta(\frac{1}{2}) + (1-\eta)\tilde{x}$$

Hence (and obviously), given that half the Ws become highly skilled, and only the high ability Bs do so, the quality of the social network of Ws is greater than that of Bs (i.e., $\tilde{\sigma}_w > \tilde{\sigma}_b$) whenever $\eta > 0$. This disparity of network quality will be greater, the more segregated are the society's social networks.

Moreover, in light of (), the human capital investment behavior of parents that is needed to support this stationary allocation is consistent with parental utility maximization only if:

$$\phi(w_H(\tilde{x}) + \gamma \cdot \tilde{\sigma}_w) \leq \Delta w(\tilde{x}) \leq \phi(w_H(\tilde{x}) + \gamma \cdot \tilde{\sigma}_b) \quad \#$$

[It is also necessary here that $\Delta w(\tilde{x}) \leq \phi(w_L(\tilde{x}) + \gamma \cdot \tilde{\sigma}_w)$. (That is, low skilled Ws must want *not* to train their normal children.) I leave it as an exercise for students to derive a condition that assures this to be the case. The basic idea is that γ can't be too big relative to $\Delta w(\tilde{x})$.]

Now, $\phi(\cdot)$ is, under our assumptions, a strictly decreasing function. Therefore, it is invertible. Let $\phi^{-1}(\cdot)$ denote it's inverse. Then, () amounts to the requirement that:

$$\tilde{\sigma}_w \geq \frac{\phi^{-1}(\Delta w(\tilde{x})) - w_H(\tilde{x})}{\gamma} \geq \tilde{\sigma}_b \quad \#$$

So, one thing that must be true for this unequal stationary allocation to be consistent with equilibrium behavior is that the term in the middle above be positive. This will be the case as long as $\phi(w_H(\tilde{x})) < \Delta w(\tilde{x})$. But, we know from () and the continuity of the relevant functions, that the middle term will be positive if $\tilde{x}$ is close enough to $\frac{1}{2}$ which, in turn, will be the case if β is not too large. Moreover, using the definitions of $\tilde{\sigma}_b$, $\tilde{\sigma}_w$, and $\tilde{x}$, it is easy to see that $\tilde{\sigma}_w - \tilde{\sigma}_b = \eta[\frac{1}{2} - \theta]$. So (as was the case in the simpler BS model) the larger is η, other things equal, the larger is the gap between $\tilde{\sigma}_w$ and $\tilde{\sigma}_b$. Hence, given that the middle term above is positive, there will always be some intermediate range of values for γ allowing the inequalities () to hold. This completes my (sketch of a) proof for the Proposition. □

I will offer some further brief remarks on this material in class on 3/21. GL