Memorandum

To: EC 237 Students
From: Prof. Loury
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This note attempts to provide some intuition for the proof of Proposition 2 in Sethi and Somanathan. The proposition states that if the income distributions between blacks and whites are close enough ($\alpha \approx 1$), then when the preference for associating with one's own group is not too great ($\eta \approx 0$), there exists a stable integrated equilibrium in their model.

To prove this they first assert that for $\alpha \approx 1$ the allocation in which the top half of the income distribution, both black and white, go to one community (N2) and the bottom half go to N1 is, in fact, an equilibrium. This is the allocation where whites go to N2 if $z \geq z^*$ and blacks go there if $y \geq y_b(z^*)$, where $z^* = y_b(z^*) =$ the overall population's median income. Then, they argue that for $\eta \approx 0$ this equilibrium must be stable. Here I offer some intuition for this latter argument.

The equilibrium is stable if the black bid rent curve cuts the white one from above, so that at slightly higher levels of z the marginal white outbids the marginal black for access to N2 (causing z to fall as more whites enter N2), and likewise, at slightly lower levels of z the marginal black outbids the marginal white for access to N2s. In other words, one has to show that $\frac{d\rho_w}{dz}(z^*) > \frac{d\rho_b}{dz}(z^*)$. The proof offered in the paper uses calculus to show that this inequality must obtain for $\eta \approx 0$. To see why this must be so, consider a slight increase in the white threshold from $z = z^*$ to $z = z^* + dz$. A rise in z has two effects which I explain below: what I'll call an "*income effect*" and a "*demographic effect*." Basically, what's going on is that the assumption that $v(r)$ is quadratic with a peak at $r = 1/2$ – that is, the assumption that an individual's ideal neighborhood would be perfectly evenly integrated – allows us to ignore the demographic effect when checking for stability of the integrated equilibrium.

Starting at $z = z^*$, an increase in z of size dz alters the demographic composition of both communities: the poorest whites move out of N2 and they are replaced in equal numbers by the richest blacks from N1. As a result, β_2 rises and β_1 falls by an equal amount. Also, $\overline{y}_2$ falls and $\overline{y}_1$ rises by an equal amount. Thus , after the increase in z, we have that:

$$\beta_2(z^* + dz) > \beta_2(z^*) = \beta = \beta_1(z^*) > \beta_1(z^* + dz)$$

and

$$\mu^+ = \overline{y}_2(z^*) > y_2(z^* + dz) > y_1(z^* + dz) > \overline{y}_1(z^*) = \mu^-,$$

where μ^+ is population mean income conditional on being above the population median, and μ^- is mean income conditional on being below the median. By "*income effect*" I mean the consequences of the fact that a rise in z changes mean income in both communities AND changes the incomes of the marginal black and white bidders. By "*demographic effect*" I mean the consequences of the fact that a rise in z changes the racial composition in both communities. Now, with a rise in z from z* to z* + dz the income gap between the two communities narrows, since: $\overline{y}_2(z^*) - \overline{y}_1(z^*) > y_2(z^* + dz) - y_1(z^* + dz)$. So, to that extent, the amount any individual of given income bids for access to N2 goes down. But, the marginal white bidder – whose income has risen to $z^* + dz$, is now richer than the marginal black bidder – whose income has fallen to $z^* + y_b'(z^*)dz$ [recall $y_b'(z^*) < 0$]. Moreover, under the sorting assumption ($u_{12} > 0$), this means that access to the higher income neighborhood is now more important to the marginal white bidder than to the marginal black bidder. For this reason, if the only things that changed when z rises from z^* to $z^* + dz$ were the neighborhood incomes and the incomes of the marginal bidders, then the marginal white *would* actually outbid the marginal black for a place in the N2 neighborhood, and the equilibrium would be stable because it would be true that:

$$\rho_w(z^* + dz) > \rho_b(z * + dz)$$

But, or course, this is not the only thing that changes. The racial compositions of the two communities also change, and I refer to the

consequences of this shift as the "*demographic effect*." If its shifting racial composition were to make N2 relatively more attractive to a marginal bidder when compared to N1 then, other things equal, the result would be to raise his bid for access to N2. So, if the demographic change brought about by a rise in z were to raise the relative valuation of the N2 neighborhood for the marginal black bidder, and at the same time to not raise the relative valuation of N2 by much – or, even to lower it – for the marginal white bidder, then the stabilzing consequences of the "income effect" discussed above could be overturned by the de-stabilizing consequences of this "demographic effect." (Since, now the marginal black bidder just might out-bid the marginal white bidder for access to N2. The marginal white is richer and values access to the higher mean income community by more; but, this could be outweighted if the marginal black is willing to pay enough more than the marginal white for proximity to the greater number of blacks now residing in N2.)

What the assumption "$\eta \approx 0$" does (given the quadratic specification for $v(r)$) is assure that the "demographic effects" – on the relative valuations of neighborhood N2 by marginal black and white bidders' – are of the same sign, and are equal in magnitude. They thus cancel-out in the calculation of which marginal bidder's bid falls by more when z rises from z^* to $z^* + dz$. Let us analyze the demographic effect in order to prove this assertion. Notice that $\beta < 1/2 < 1 - \beta$, and that $1/2 - \beta = (1 - \beta) - 1/2$. [This is just observing that β and $1 - \beta$ are symmetrically placed relative to 1/2.] Initially, when $z = z^*$, we have that $\beta_2(z^*) = \beta_1(z^*) = \beta$. After z rises to $z^* + dz$ we have that :

$$\beta_2(z^* + dz) > \beta > \beta_1(z^* + dz) \text{ and } 1 - \beta_2(z^* + dz) < 1 - \beta < 1 - \beta_1(z^* + dz).$$

Now, perhaps not surprisingly, blacks' relative valuation of the racial composition of N2 rises with the rise in z. This is because β_2 rises, β_1 falls and $v'(\beta) > 0$, given $\beta < 1/2$. Moreover, and perhaps surpisingly, it is also the case that – as long as η is not too big – whites' relative valuation of the demography of N2 also rises with a rising z. This is due to the fact that $1 - \beta_2$ falls, $1 - \beta_1$ rises, and $v'(1 - \beta) < 0$ for $1 - \beta > 1/2 + \eta/2$. So, the shifting demography moves the relative valuation of the black and the white marginal bidder in the same direction.

Moreover, as η approaches zero, $v(r)$ comes to be perfectly symmetric about 1/2, in which case the changes in utility for marginal black and white bidders – associated with the fact that the racial composition of the two communities has shifted – come to be exactly equal in magnitude. GL