1. (a) We have,

\[ R = A_0 + A_1 T + A_2 T^2 + A_3 S + \epsilon \]
\[ S = B_0 T - B_1 T^2 \]

for \( B_0 > 0 \) and \( B_1 > 0 \).

Substituting the second into the first,

\[ R = A_0 + (A_1 + A_3 B_0)T + (A_2 - A_3 B_1)T^2 + \epsilon \]

If we estimate

\[ R = \hat{A}_0 + \hat{A}_1 T + \hat{A}_2 T^2 + \epsilon. \]

we’ll get

\[ \hat{A}_1 = (A_1 + A_3 B_0) \]
\[ \hat{A}_2 = (A_2 - A_3 B_1) \]

This means that we won’t be estimating the effect of climate on land rent, we’ll estimating the effect of climate and skill on land rent (though the sorting of skill across climate is itself caused by climate).

(b) Since \( B_0 > 0, B_1 > 0 \) the effect of the bias on our estimate of the effect of climate on land rent is ambiguous. If you differentiate the two regression equations, estimated and true, it is easy to show that if \( T \) is small, we will overestimate the effect of climate on land rent. If \( T \) is large, we underestimate the effect.

A little more specifically, if (for example) \( A_3 > 0 \), we’ll have \( \hat{A}_1 > A_1 \) and \( \hat{A}_2 > A_2 \).

2. Let \( T \) denote January temperature in Fahrenheit and \( R \) unit land rent. If we let every variable but \( T \) be zero and we want the mean land rent to be 1500, then from the first column of Tables 3 on Mendelsohn and Nordhaus, we have

\[ R = 1500 - 57T - 0.33T^2 \]

This is actually not exactly what we’re asked to do. The problem says that that the average land rent should be 1500. If we let \( \bar{T} \) denote the sample average temperature, then in the equation above, predicted average land rent is going to be

\[ R = 1500 - 57\bar{T} - 0.33\bar{T}^2 \]

which won’t generally be 1500. To make things easy, ignore this complication and work with the first equation.

If we plot this equation, we get,
Note that the axis in this figure actually runs from 20 to 50 degrees F, not 1 to 31 as labelled.

If we recall that the mean land rent is just $1500, this suggests that the effect of warming alone on agricultural productivity might be very quite large, if we don’t also get a lot more rain with it.