When you write up your answers, your goals should be to (1) be correct, and (2) convince your reader that your answer is correct. It is always helpful if your work is legible and if all steps are presented, possibly with a line of explanation.

In the case of empirical exercises, your goal should be to provide enough information to allow a reader to replicate your answer. This requires a description of data and data sources as well as a description of your analysis of the data.

Answers which do not achieve these goals will not be awarded full credit.

To assist us in complying with the University’s privacy policy, the first page of each problem set should be blank except for your name and the problem set number. This will allow us to write your score inside your problem set. Failure to include such a page will be understood as permission to write your score on the front of your problem set where others might accidentally see it.

Problems

1. Consider the ‘savings problem’ that we discussed in class. That is,

   \[
   \max_{s,c_1} \frac{c_1^{1-\eta}}{1-\eta} + \frac{1}{1+\rho} \frac{c_2^{1-\eta}}{1-\eta}
   \]

   s.t. \( W = c_1 + s \)

   \( c_2 = (1+r)s \)

where \( c_t \) is consumption in period \( t \), \( s \) is savings, \( W \) is initial income, \( \rho > 0 \) is the pure rate of time preference, \( 0 < \eta < 1 \) is inequality or risk aversion, and \( r > \rho \) is the rate of return to capital.

(a) Solve for \( c_1 \) and \( c_2 \) as functions of \( W, \rho, \eta \) and \( r \).

(b) Using your results above, explain what happens to \( c_1/c_2 \) as \( \eta \) varies between zero and one. Explain, briefly, why this is consistent with ‘inequality aversion’ increasing as \( \eta \to 1 \).

(c) Using your results above, show that \( c_1/c_2 \) increases in \( \rho \). Explain, briefly, why this is consistent with impatience increasing as \( \rho \) increases.

(d) Using your results above, show that \( c_1/c_2 \) decreases in \( r \). Briefly explain the intuition for this result.

2. Let \( Y_0 \) and \( E_0 \) denote world income and emissions of \( \text{CO}_2 \). Suppose that \( Y_0 = 1 \) and that the world consists of two countries, \( A \) and \( B \) that each account for half of emissions and income. Suppose that the relationship between income and mitigation is given by

   \[
   \Lambda_i = \frac{2}{3} \mu_i^3
   \]

where \( \mu_i E_i \) is the reduction in emissions in country \( i \) and \( \Lambda_i Y_i \) is the cost of this reduction.

We would like to accomplish a reduction of \( \alpha E_0 \) in world emissions, for \( 0 < \alpha < 1 \).

(a) Calculate the cost of this reduction if each country reduces its emissions by the same amount.
(b) Calculate the cost of this reduction if the entire reduction is accomplished by country A.

c) Calculate the ‘participation multiplier’, that is, the number by which we must multiply the cost in part 2a to get the cost in part 2b.

d) The Kyoto protocol required emissions reductions from only a fraction of the world. Briefly explain why this problem suggests that this is (or is not) a good idea.