Suppose \( n = 5 \). There are six candidate equilibria:

0, 1, 2, 3, 4, 5 people choose urban.

If 0 people choose urban, then everyone gets \( \bar{u} = 0 \) and unilateral deviations allow \( u_i(1) = -1 \), so \( u_a = 0 \) is Nash.

If \( n = 4 \), then \( u_i(4) = \bar{u} = 0 \). An urban dweller who deviates to non-urban gets 0. This is not a beneficial deviation. A rural dweller who deviates to urban gets \( u_i(5) = -1 \), also not a rational deviation.

All strategies s.t. \( n \in \{0, 4, 5\} \) is a Nash Equilibrium.

If \( n < 5 \), then \( u = -1 < \bar{u} \) so deviations to non-urban are rational.

If \( n = 2 \), then \( u = 2 \) and a deviation to urban.
Given pay-off $U(3) = 1 > \bar{U}$, so $N^* = 2$ is not Nash.

If $N = 3$, then deviation to urban gives $U(4) = 0 = \bar{U}$
and deviation from urban gives $\bar{U} = 0 < 1 = U(3)$.

Thus, strategies s.t. $N = 3$ are Nash.

Thus, strategies are Nash ($\Rightarrow$ $N \in \{0, 3, 4\}$).

For mixed strategies, each agent chooses $p \in [0, 1]$.

We can use this to write down the probability of each configuration.

<table>
<thead>
<tr>
<th>$N^*$</th>
<th>Prob</th>
<th>$U(U_N)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$(1-p)^5$</td>
<td>-4</td>
</tr>
<tr>
<td>1</td>
<td>$p(1-p)^4$</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
<td>$p^2(1-p)^3$</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>$p^3(1-p)^2$</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>$p^4(1-p)$</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>$p^5$</td>
<td>-1</td>
</tr>
</tbody>
</table>

For the purpose of calculating mixed equilibria, we need to know the effect on payoffs of changing $p$ for one agent, holding others fixed at $p'$. 
If all agents but 1 choose P, and 1 chooses P', then what we care about is the distribution of everyone else's move. Let n denote # of players, #1 choosing P. Then

\[
\begin{array}{c|c}
\text{#1} & \Pr(n_1) \\
0 & (1-p)^4 \\
1 & p(1-p)^3 \\
2 & p^2(1-p)^2 \\
3 & p^3(1-p) \\
4 & p^4 \\
\end{array}
\]

Then, if 1 chooses P',

\[
E(T_1) = \left(1 - p^4\right) \cdot 0 + p' \left[-1(1-p)^4 + 2(1-p)^3 p + 1(1-p)^2 p^2 + 0(1-p)p^3 - p^4\right] \quad (*)
\]

We need to find p' such that

\[
\frac{\partial E(T_1)}{\partial p'} = 0 \quad \Rightarrow \quad p' = p
\]

where the second condition is B/c we've restricted attention to symmetric equil.

By inspection of (*) we need the expression in brackets to be zero, so we will need to choose p so that it is a zero of this 4th order polynomial.

E) expect as many as 4 roots.
(c) THE PARETO OPTIMAL OUTCOMES ARE \( n \in \{2, 3\} \).

For either urban population, there is no way to make one person better without making another worse.

(d) We need \( n_1 + n_2 \leq 5 \).

1. \((n_1, n_2) \in \{(0, 0), (0, 4), (4, 0)\}\) are all Nash. In these equilibria each gets \( u \).

2. \((n_1, n_2) \in \{(2, 3), (3, 2)\}\)

Here the urban pay-off is positive, so no one moves to rural. No one wants to move from the small to the big city, this is strictly worse. Moving from small city to big does not change pay-off for an agent.

3. No outcome with \( n_1 \in \{1, 5\} \) is Nash.

Because one agent can deviate to rural for \( u > -1 \).

4. No outcome \( n_1 = 2 \) is an equilibrium unless \( n_1 + n_2 = 5 \). Otherwise rural agents will deviate.

5. \((n_1, n_2) \in \{(0, 3), (3, 0)\}\) are also equilibria. Urban residents put limit to move, and rural agents who move to the city are indifferent.
Summing up, we have:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>4</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td></td>
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</tr>
</tbody>
</table>

\[ n_1 + n_2 > 5 \]

\[ \Rightarrow \text{Nash} \]

\[ \Rightarrow \text{not Nash} \]

\[ \Rightarrow \text{T.O.} \]

Therefore, Nash Equilibrium is consistent with two cities of about optimal size and an empty countryside, or with one city larger than optimal with 1 or 2 still rural.

(c) Adding another city would not qualitatively change the equilibrium outcomes. It would just create indeterminacy about which cities are occupied. This seems special to the case \( n = 5 \).
\[(a) \quad \text{MAX} \left[ \sum_{s=1}^{M} \frac{\delta_s}{\delta_s} \alpha \left( \frac{s}{s-1} \right) \delta_s \right]^{-\frac{1}{2}} T^{1-m} \]

s.t. \[\sum_{s=1}^{M} \delta_s \alpha \left( \frac{s}{s-1} \right) \delta_s + \tau^T = 0 \]

First order condition is \( \forall \alpha = \tau P \delta \in [0, M], T > 0 \)

Set \( \tau = 1 \), W.O.L.G.

\[
\left[ \sum_{s=1}^{M} \delta_s \alpha \left( \frac{s}{s-1} \right) \delta_s \right]^{-\frac{1}{2}} \delta_s = \frac{1}{T} \]

\[\frac{M}{\delta_s} \left[ \sum_{s=1}^{M} \delta_s \alpha \left( \frac{s}{s-1} \right) \delta_s \right]^{-\frac{1}{2}} \delta_s \delta_s^{-\frac{1}{2}} T^{1-m} = \tau P(\delta), \quad \delta \in [0, M] \]

\[\Rightarrow \quad M \delta_s \left[ \sum_{s=1}^{M} \delta_s \alpha \left( \frac{s}{s-1} \right) \delta_s \right]^{-1} \delta_s \delta_s^{-\frac{1}{2}} T = (1-m) P(\delta), \quad \delta \in [0, M] \]

\[\Rightarrow \quad M \left[ \sum_{s=1}^{M} \delta_s \alpha \left( \frac{s}{s-1} \right) \delta_s \right]^{-1} \delta_s \delta_s^{-\frac{1}{2}} T = (1-m) P(\delta), \quad \delta \in [0, M] \]

\[\Rightarrow \quad \frac{P(\delta)}{P(\delta)} = \left[ \frac{\delta_s \delta_s^{-\frac{1}{2}}}{\delta_s} \right]^{-\frac{1}{2}} \quad \delta, \varepsilon \in [0, M] \]

\[\Rightarrow \quad \left( \frac{P(\delta)}{P(\delta)} \right)^{\delta_k} = \frac{\delta_\delta}{\delta_\delta} \quad \Rightarrow \quad \delta_\delta = \left( \frac{P(\delta)}{P(\delta)} \right)^{\delta_k} \quad \delta, \varepsilon \in [0, M] \]
Using (5) in (4):

\[ M \left[ \int_0^M \left( \frac{p(x)}{p(x') \sigma} \right)^{\frac{x-1}{\sigma}} dx' \right]^{-\frac{1}{2}} \sigma^{\frac{1}{2}} \int_0^M p(x')^{-\frac{1}{2}} dx' = (1-M) p(x) \]

\[ \sigma^{\frac{1}{2}} \int_0^M p(x')^{-\frac{1}{2}} dx' \left[ \int_0^M \left( \frac{p(x)}{p(x') \sigma} \right)^{\frac{x-1}{\sigma}} dx' \right]^{-\frac{1}{2}} = \frac{1-M}{MT} p(x) \]

\[ p(x)^{\frac{1}{2}} \left[ \int_0^M \left( \frac{p(x)}{p(x') \sigma} \right)^{\frac{x-1}{\sigma}} dx' \right]^{-\frac{1}{2}} = \frac{1-M}{MT} p(x) \]

\[ = \frac{1-M}{MT} p(x) \sigma^{\frac{1}{2}} \left[ \int_0^M p(x')^{-\frac{1}{2}} dx' \right] \frac{1-M}{MT} \]

\[ = \frac{1-M}{MT} p(x) \sigma^{\frac{1}{2}} \left[ \int_0^M p(x')^{-\frac{1}{2}} dx' \right] \frac{1-M}{MT} \]

\[ \Rightarrow p(x) = \frac{1-M}{MT} p(x) \sigma^{\frac{1}{2}} \left[ \int_0^M p(x')^{-\frac{1}{2}} dx' \right] \frac{1-M}{MT} \]

(6) ⇒ (2)

\[ \left( \int_0^M p(x)^{\frac{1}{2}} \sigma^{\frac{1}{2}} \left[ \int_0^M p(x')^{-\frac{1}{2}} dx' \right]^{-\frac{1}{2}} \frac{1-M}{MT} dx \right) + T = \omega \]

\[ \Rightarrow \frac{1-M}{1-M} + T = \omega \]

\[ \Rightarrow T = (1-M) \omega \]
\(2 \Rightarrow 6 \Rightarrow \phi^*(\gamma) = \frac{\rho(\gamma)^{-\alpha}}{\sum_{\delta} \rho(\gamma)^{-\alpha}} \quad \text{MW} \quad \delta \in [0, M] \)

\[\text{DEFINE} \quad P = \left[ \int_0^M p(\gamma) \, d\gamma \right]^{-\frac{1}{\alpha - 1}} \quad \text{"C.E.S. PRIOR INDEX"}
\]

\(\Rightarrow \quad P^{-(\alpha - 1)} = \left[ \sum_{\delta} p(\gamma)^{1-\alpha} \, d\gamma \right] \quad 6\)

\(\Rightarrow \quad \phi^*(\gamma) = \frac{\rho(\gamma)^{-\alpha}}{P^{-(\alpha - 1)}} \quad \text{MW} \)

\[\Rightarrow \quad \left\{ \begin{array}{l}
\phi^*(\gamma) = P^{\alpha - 1} \rho(\gamma)^{-\alpha} \quad \text{MW} \\
\text{(C.F. FUKTA + TASSIA, Eqn 9.4)}
\end{array} \right. \]

Strictly, if \( \gamma \in [0, M] \) measure zero and \( f: K \to IR \) then \( \phi^* + f \) also solves the consumer problem.
The profits for firm $j$ are

$$\Pi = P(g) \xi(g) - A - B g(g)$$

$$= (P(g) - B) \xi(g) - A$$

From (7)

$$\Pi = (P(g) - B) \frac{\mu w}{P^{1-\alpha}} P(g)^{1-\alpha} - A$$

(Recall, firms are monopolists)

$$= [P(g)^{1-\alpha} - B \frac{\mu w}{P^{1-\alpha}}] \frac{\mu w}{P^{1-\alpha}} - A$$

$$\frac{\partial \Pi}{\partial P(g)} = \frac{\mu w}{P^{1-\alpha}} \left[ \left(1-\alpha\right) P(g)^{-\alpha} + B \frac{\mu w}{P^{1-\alpha}} P(g)^{-\alpha-1} \right] = 0$$

$$\Rightarrow \left(1-\alpha\right) + B \frac{\mu w}{P^{1-\alpha}} = 0$$

$$\Rightarrow \left(1-\alpha\right) = -B \frac{\mu w}{P^{1-\alpha}}$$

$$\Rightarrow P^*(g) = \frac{-B \frac{\mu w}{1-\alpha}}{1-\alpha} \quad (\text{NEED } \alpha > 1)$$

Using (11) in (8)

$$g^*(g) = \frac{\left[ \frac{-B \frac{\mu w}{1-\alpha}}{1-\alpha} \right]^{-\alpha}}{M \left[ \frac{-B \frac{\mu w}{1-\alpha}}{1-\alpha} \right]^{1-\alpha}} = \frac{1-\alpha}{-MB \frac{\mu w}{1-\alpha}} = \frac{\alpha - 1}{MB \frac{\mu w}{1-\alpha}}$$
WITH FREE ENTRY OF FIRMS $T = 0$

$$\Rightarrow P_f - A - B \bar{g} = 0$$

Using (1) + (2)

$$\Rightarrow \left( \frac{-B \bar{g}}{1 - \sigma} \right) \left( \frac{\sigma - 1}{M B} \right) - A - 7B \left( \frac{\sigma - 1}{M B} \right) = 0$$

$$\Rightarrow \frac{1}{M B} - A = 0, \quad (13)$$

Here, unlike Fit on Krugman, wage is exogenous, so we only get Endogenous capital to hold if parameters happen to satisfy (13).

3. See from (11) above - This is constant mark-up beyond marginal cost (= 13)

4. You cannot verify this statement without endogenizing the wage,