

Pedro Dal Bó

# Social norms, cooperation and inequality

Received: 3 June 2004 / Accepted: 16 September 2005 / Published online: 13 November 2005  
© Springer-Verlag 2005

**Abstract** This paper analyzes the outcomes that can be supported by social norms in a society of infinitely lived and patient agents that are randomly matched in pairs every period to play a given game. I find that any mutually beneficial outcome can be supported by a self-enforcing social norm under both perfect information and a simple local information system. These Folk Theorem results explain not only how social norms can provide incentives that support cooperation in a community, providing support to the concepts of social capital and corporate culture, but also how they can support inequality and discrimination.

**Keywords** Repeated games · Folk theorem · Social norms · Social capital · Inequality · Discrimination

**JEL Classification Numbers** C72 · J7 · Z13

## 1 Introduction

Game theorists have long recognized that repeated playing and the possibility of future retaliation modifies current behavior and can encourage cooperation, for example see Luce and Raiffa (1957). In the case in which the same set of players plays the same game repeatedly, other studies have identified the conditions under which any feasible and strictly individually rational outcome can be supported in

---

I am grateful to David Levine for invaluable guidance and ideas. I also thank Anna Aizer, Hongbin Cai, Walter Cont, Ernesto Dal Bó, Jean-Laurent Rosenthal, Federico Weinschelbaun, William Zame, anonymous referees and seminar participants at Universidad de Buenos Aires, UCLA, Universidad T. Di Tella, Harvard Business School and Stockholm School of Economics for very useful comments and discussions.

---

P. Dal Bó  
Department of Economics, Brown University,  
Providence, RI 02912-9029, USA  
E-mail: pdalbo@brown.edu

equilibrium.<sup>1</sup> However, in many economically important cases the same players do not meet repeatedly but rather switch partners over time.<sup>2</sup> While the changing of partners might seem to make cooperation impossible by reducing the possibility of personal retaliation, that is not necessarily the case. As Kandori (1992) shows, social norms can create incentives for players to punish deviators even if the deviation occurred against another player, since failing to punish can itself be punishable. However, Kandori (1992) considers restricted environments that are not necessarily generalizable to many economic situations: a society divided in two groups and every period the players in one group are randomly matched with the players in the other group and all players in the same group have the same equilibrium payoff.<sup>3</sup> In this way, Kandori (1992) shows that any outcome that can be reached in the long-term relationship of two agents can also be reached in the long-term relationship of two groups.

This paper studies the set of equilibrium payoffs when we abandon those restrictions but only require the matching procedure to be independent of history and time. The main contribution of this paper is to show that community enforcement can support payoffs that can never be supported in the long-term relationship of two agents. In fact, there are highly unequal payoffs that can only be supported by community enforcement, as the next example shows.

Consider a society with four members that are matched every period to play the prisoner's dilemma game in Table 1.

Assume that each pair has the same probability of being matched and consider the following social norm: Player 1, who is called "king", always plays  $d$ , players 2, 3 and 4, who are called "serfs", play  $c$  if no one has deviated and play  $d$  if someone has ever deviated. Under this social norm, the king receives a payoff of 4 every period while the serfs receive an average payoff of  $\frac{1}{3}$  in equilibrium. It is easy to verify that this social norm is a subgame perfect equilibrium for discount factors greater than  $\frac{9}{10}$ .<sup>4</sup> It is interesting to note that in a repeated game with only one serf, the king would not be able to obtain a payoff of 4. For the serf to play  $c$ , the king will have to play  $c$  with a certain probability  $p_c$ , such that  $2p_c > 3(1 - p_c)$ . This gives  $p_c > \frac{3}{5}$  and an upper bound of  $\frac{14}{5}$  to the king's payoff. Therefore, community

<sup>1</sup> See Aumann and Shapley (1976) for the case without discounting, Friedman (1971) for the case of Nash threats, discounting and perfect information, Fudenberg and Maskin (1986) for the case of discounting with perfect information and Fudenberg, Levine, and Maskin (1992) for the case of discounting with imperfect public information.

<sup>2</sup> See, for example, the cases of Maghribi traders in the Mediterranean during the 11th century studied in Greif (1993) and the merchant guilds studied in Greif, Milgrom, and Weingast (1994)

<sup>3</sup> This characterization of the game has the appealing graphical property that we can represent an equilibrium outcome of the game in  $\mathbb{R}^2$ . Since all the members of a group receive the same payoff in equilibrium and there are only two groups, equilibrium payoffs can be written as the pair  $(v_1, v_2)$ , where  $v_1$  and  $v_2$  denote the utility received by each player in groups 1 and 2, respectively.

<sup>4</sup> The King clearly has no incentive to deviate. Seeing that the serfs do not have incentives to deviate requires some calculations. If no one has deviated, a serf facing the King would get an expected payoff of  $-3(1 - \delta) + \delta \frac{1}{3}$  for following the social norm, where  $\delta$  is the discount factor, and he would get 0 for deviating. Hence, if no one has deviated, the serfs will not deviate when playing against the King if  $\delta \geq \frac{9}{10}$ . Similar calculations show that for those discount factors a serf would not deviate when playing against another serf. Therefore, serfs have no incentives to deviate if no one has deviated before. If someone has deviated, they do not have incentives to deviate since the prescribed actions correspond with the stage game Nash equilibrium. Therefore, if  $\delta \geq \frac{9}{10}$ , no player has an incentive to deviate and, then, the social norm is a subgame perfect equilibrium.

**Table 1** Prisoner's dilemma game

	<i>c</i>	<i>d</i>
<i>c</i>	2, 2	-3, 4
<i>d</i>	4, -3	0, 0

enforcement allows the king to obtain a higher payoff than what he could ever obtain under a one-to-one long-term relationship.

For general stage games, I show that under perfect information any feasible and strictly individually rational outcome of the random matching game can be supported by a social norm if the players are patient enough. This includes payoffs that cannot be supported in one-to-one long-term relationship.

In the social norm used in this example all the players are punished for the deviation of one of the players, but this is subject to the criticism that schemes that only punish deviators are more realistic. Unfortunately, with social norms that only punish deviators, it is not always possible to reward punishers through long run continuation payoffs even under full dimensionality assumptions, as it is done for normal repeated games with more than two players by Fudenberg and Maskin (1986) and Abreu, Dutta, and Smith (1994). Nevertheless, I show that any feasible and strictly individually rational outcome can still be supported in a subgame perfect equilibrium when social norms only punish deviators under some restrictions on the stage game.

Since the assumption of perfect information is unrealistic when the size of the society is large, I also study the existence of Folk Theorem results under local information systems. In this case, in addition to knowledge gained from their own experience, players have access to information from a system that assigns status levels to players depending on their past behavior, allowing for rewards and punishments. Previous papers study social norms under local information systems under the assumption of two groups in which all the members of a group receive the same payoff in equilibrium. Okun-Fujiwara and Postlewaite (1995) prove a Folk Theorem for a weak (non-perfect) equilibrium concept (Norm equilibrium). Kandori (1992) proves a Folk Theorem for sequential equilibrium under certain assumptions of the stage game.

In this paper, instead, no restriction on the stage game is required for the (perfect) Folk Theorem under local information. This difference is not due to the different types of random matching procedures but to the fact that Kandori (1992) uses a social norm that is implicitly restricted to punish only deviators and therefore needs to provide short run incentives to players during the punishment stage. I show that if we are willing to allow non-deviators to be punished as well, there is no need to provide explicit short run incentives to punishers and, hence, no need for restrictions on the stage game. This is done without increasing either the number of status levels needed or the complexity of the social norm with respect to the one used by Kandori (1992). In fact, the social norm is simpler since only one action is used during the punishment stage.

The rest of the paper is laid as follows. In Section 2, I present the model. In Section 3, I provide Folk Theorem results under perfect information. I consider both the unrestricted case in which any player can be punished for a deviation

and the case in which only deviators can be punished. In Section 4, I provide Folk Theorem results under a local information system. Section 5 presents several examples explaining not only how social norms can provide incentives to forestall opportunistic behavior and support cooperation but also how they can support outcomes characterized by inequality.

## 2 The matching game

The society consists of  $N$  players, where  $N$  is an even number. In each period, every player is matched with another player to play the stage game  $\Gamma$ . I assume that the matching of players is independent of past actions or time: the probability that player  $i$  is matched with player  $j$  is  $\alpha_{ij}$ ,  $0 \leq \alpha_{ij} \leq 1$ , for every period and history. This definition allows for partitions of players in two groups as in Kandori (1992) and Okun-Fujiwara and Postlewaite (1995) but does not allow for matchings to depend on history.

The stage game  $\Gamma$  is a symmetric game played by two players, with a finite number of actions  $a \in A$  for both players and payoffs  $g: A^2 \rightarrow \mathbb{R}^2$ , with the property that  $g_{row}(a', a) = g_{col}(a, a')$ , from the symmetry of the game. To simplify notation I write  $g(a, a') = g_{row}(a, a') = g_{col}(a', a)$ . Therefore  $g(a, a')$  denotes the payoff for the player that is playing  $a$  when the other is playing  $a'$ .

To minimax the other player the prescribed strategy is

$$m = \arg \min_{a' \in A} \left( \max_{a \in A} g(a, a') \right).^5$$

I normalize the payoffs to have the minimax payoffs, not  $g(m, m)$ , equal to zero. If both players play  $m$ , each obtains a payoff of  $g(m, m)$ . Since  $m$  may not be the Best response to  $m$ , it follows that  $g(m, m) \leq 0$ . The maximum payoff that can be obtained in the stage game is  $\bar{g} = \max_{a, a' \in A} g(a, a')$  and the minimum payoff is  $\underline{g} = \min_{a, a' \in A} g(a, a')$ . I assume that players can condition their actions on public

randomization devices. Define  $\Delta(A^2)$  as the set of possible strategies in the stage game given the public randomization device. Abusing notation I denote an element of that set for player  $i$  and  $j$  as  $(a_{ij}, a_{ji}) \in \Delta(A^2)$ .

Now I proceed to define the set of feasible payoffs of the random matching game. Denote as  $P$  the set of all unordered possible pairs. I define next the “play” of the stage game: the play of the stage game describes what profile of actions would be played by each possible matching of players in each period, that is a play  $f$  is a function from  $P \times \{0, 1, 2, \dots\}$  to  $\Delta(A^2)$ . The play indicates the actions of each possible pair in each period, for example  $a_{ij}^t$  denotes what  $i$  would play if matched with  $j$  in period  $t$ . Therefore, the expected stage payoff for  $i$  in period  $t$  is  $\sum_{j \neq i} \alpha_{ij} g(a_{ij}^t, a_{ji}^t)$ . If  $\{a_{ij}^t\}$  is the “play” for every period and  $\delta$  is the discount factor,

the average expected payoff of player  $i$  is  $v_i = (1 - \delta) \sum_{t=0}^{\infty} \delta^t \sum_{j \neq i} \alpha_{ij} g(a_{ij}^t, a_{ji}^t)$ . Each play defines an expected payoff for every player  $v = (v_1, v_2, \dots, v_N)$ . Therefore the set of feasible payoffs are defined by the payoffs that result from every possible

<sup>5</sup> I assume for convenience that  $m$  is a pure action or an observable mixed action.

play. Denote this set as  $V \subset \mathfrak{R}^N$ . Note three characteristics of  $V$ . First, the set of feasible payoffs  $V$  is different from the set that arises in a two-player repeated game or in a repeated random matching game with two groups with all the members of each group receiving the same payoff as in Kandori (1992) and Okun-Fujiwara and Postlewaite (1995). In fact, the set  $V$  has a different dimension. Second,  $V$  depends on  $\alpha$ . Different matching probabilities result in different sets of feasible payoffs. For example, the payoffs of the king and serfs in the example of the introduction are not feasible if the pairs are the same in every period. The assumption of fixed  $\alpha$  is made for simplicity so that the set of feasible payoffs does not change from period to period. Third, it can be easily shown that any point in  $V$  can be achieved by a stationary play (constant over time). Thus, for simplicity, from now on I will focus on stationary plays.

### 3 Perfect information

In this section, I consider societies in which it is possible for every member to observe every other's behavior. As a first step, I show that if the players are sufficiently patient, any feasible and strictly individually rational outcome ( $v \in V : v \gg 0$ ) can be supported by a social norm that is a subgame perfect equilibrium.

**Theorem 1 (Folk Theorem with perfect information).** *With perfect information any feasible and strictly individually rational payoff ( $v \in V : v \gg 0$ ) can be supported by a subgame perfect equilibrium for  $\delta$  large enough.*

*Proof* Consider the following social norm to support  $v$ : if no one has deviated in the last  $T$  periods follow the “play” that yields  $v$ , if someone has deviated in the last  $T$  periods, play  $m$ .

First consider the incentives to deviate if no one deviated in the last  $T$  periods. For player  $i$  the expected utility of conforming with the equilibrium is at least  $(1 - \delta)\underline{g} + \delta v_i$  while he would get at most  $(1 - \delta)\bar{g} + \delta v_i^p$  by deviating, where  $v_i^p = (1 - \delta^T)g(m, m) + \delta^T v_i$ . Let  $d$  be a number in  $(0, 1)$ . For each  $\delta$  choose  $T$  so that  $\lim_{\delta \rightarrow 1} \delta^{T(\delta)} = d$ . Since  $v_i > 0 \geq g(m, m)$ ,  $v_i > \lim_{\delta \rightarrow 1} v_i^p$  and  $(1 - \delta)\underline{g} + \delta v_i > (1 - \delta)\bar{g} + \delta v_i^p$  for  $\delta$  large enough.

Now I consider the incentives to deviate if someone has deviated in the last  $T$  periods and  $\tau \leq T$  periods of punishment remain. If player  $i$  plays  $m$ , as the strategy prescribes, he receives a payoff of  $(1 - \delta^\tau)g(m, m) + \delta^\tau v_i$ . If he deviates he receives at most  $\delta v_i^p$ . Choose  $d$  large enough for  $v_i^p$  to be positive. Since  $\delta < 1$ , it follows that  $(1 - \delta^\tau)g(m, m) + \delta^\tau v_i \geq v_i^p > \delta v_i^p$ .

Note that there is no contradiction in the requirements made on  $\delta$  and  $T$  in the two parts of the proof, as it is only required that  $d$  and  $\delta$  are both large enough.  $\square$

While the statement of the theorem and its proof are familiar, note that the set of equilibrium outcomes includes payoff that could never be supported in the long-term relationship of two agents or under community enforcement in the environments studied by Kandori (1992) and Okun-Fujiwara and Postlewaite (1995). In addition, Theorem 1 is not a consequence of Fudenberg and Maskin (1986) result for games with  $N$  players – note, for a start, that there is no dimensionality requirement here. While their results apply to games in which  $N$  players play the

same stage game in all periods, in this case players may change partners every period. In addition, given that Fudenberg and Maskin (1986) assumes that actions are perfectly observed, Theorem 1 cannot be derived from their results by considering the “plays” of the random matching games (functions from possible matches to actions for each player) as the actions in their model. In addition, Theorem 1 is not a consequence of Fudenberg, Levine, and Maskin (1992) Folk Theorem under imperfect public information since the public information generated by the random matching game is not informative enough for the conditions required by the Folk Theorem in Fudenberg, Levine, and Maskin (1992) (see Appendix)

Note that during the punishment stage in the proof of Theorem 1 all the players receive the same low payoff  $g(m, m)$  regardless of who has deviated. In this social norm, then, all the players are punished for the deviation of one of the players. While societies may use this kind of punishment schemes to reduce opportunistic behavior, punishment schemes that only punish deviators seem to be more appealing.

**Definition 1** A social norm displays *personal punishment*<sup>6</sup> if the prescribed actions for two players who are matched together and who have not deviated in the past cannot depend on the past actions of the rest of the players.

In this way, under personal punishment, only the deviators can be punished. Of course punishing may impose a cost to the player facing the deviator even under personal punishment.

But personal punishment introduces new problems to the design of social norms that support a Folk Theorem. In fact, the following simple adaptation of the social norm used in the proof of Theorem 1 to the case of personal punishment may not work: if neither you or your matched player have been the last player to deviate in the last  $T$  periods, then play the equilibrium ‘play;’ otherwise play  $m$ . Given the possible inequality of payoffs, with personal punishment some players may have an expected payoff lower than  $g(m, m)$  during the punishment stage, and therefore may have incentives not to do their part during the punishment stage, breaking in that way the credibility of the social norm.<sup>7</sup> It is clear then that it is necessary to provide incentives for punishers to carry out their duty, as it is the case in normal infinitely repeated games with  $N \geq 3$ . But contrary to what happens in that case, it is not enough to have full dimensionality of the set of feasible payoffs of the stage game or non-equivalent utilities, as in Fudenberg and Maskin (1986) and Abreu et al. (1994), respectively.

The prisoners’ dilemma game from the introduction, with  $N$  players and matching probabilities  $\alpha$ , shows that it may not be possible to provide incentives to punishers through long run continuation payoffs, as it is done in Fudenberg and Maskin

<sup>6</sup> Personal *punishment* should not be confused with personal *enforcement*. Personal enforcement means that cheaters are only punished by the player that was cheated, while personal punishment means that only cheaters are punished.

<sup>7</sup> For example consider the case of the kingdom presented in the introduction but with personal punishment instead: only the players that have deviated are punished. Imagine now that one of the serfs has deviated and has to be punished for  $\tau$  periods. In this case the obedient serfs will earn 0 each time they meet with the deviator, 2 every time they meet each other and  $-3$  each time that they meet with the king. Then, they would get an expected payoff of  $-\frac{1}{3}$  each period during the punishment stage. By deviating they can get a zero payoff during  $T$  periods and, hence, they may be willing to deviate just to avoid the negative payoff they earn if they do not deviate.

(1986) and Abreu et al. (1994). Consider the outcome in which player 1 receives a payoff of 4 each time she meets player 2 and a payoff of  $-3$  every time she meets any of the other players – of course, for this to be strictly individually rational it has to be the case that  $4\alpha_{12} - 3(1 - \alpha_{12}) > 0$ . If player 2 has deviated, given the personal punishment restriction, player 1 is supposed to keep cooperating with the other players even when they do not cooperate with her. But, can that be a best response for player 1? Given that we are restricting ourselves to personal punishment, her future interaction with players other than 2 cannot provide incentives for her to conform. In addition, her interactions with player 2 cannot provide the long run incentives needed for her to conform with the social norm during the punishment stage as done in Fudenberg and Maskin (1986) and Abreu et al. (1994). In those papers a feasible and strictly individually rational payoff that is dominated by the equilibrium payoff is used as a pivot to generate long run continuation payoffs that give incentives to punishers to conform to their strategies during the punishment stage. This cannot be done in this example, given that there is no feasible payoff of the stage game that is Pareto dominated by the payoff that players 1 and 2 obtain when they meet each other.

Therefore, with random matching and personal punishment, a Folk Theorem cannot rely on incentives to punishers provided through long run continuation payoffs. Instead, short run incentives to punishers have to be provided throughout the punishment stage. For this, I use social norms in which the punisher earns a sufficiently higher payoff than the deviator (so that the punisher does not want to become a deviator) while at the same time the deviator can not get more than zero if he avoids the punishment (so that the deviator does not want to avoid punishment). For this I require the following additional assumption:

**Assumption 1**  $\exists r \in A$  such that  $g(m, r) - g(r, m) \geq \bar{g}$ .

Under this assumption there is an action  $r$  that allows a deviator to “ask for forgiveness while the punisher plays  $m$  and the difference between the payoffs of “asking for forgiveness” and “being asked for forgiveness” is greater than the highest payoff of the game.<sup>8</sup> As such, it is possible to create punishment schemes in which the punisher earns a higher payoff than the deviator and in which the deviator by refusing to take the punishment can at most obtain a payoff of zero. This allows me to construct a social norm that ensures that players have incentives to follow it even when some of the other players have deviated. This assumption is satisfied, for example, by the prisoner’s dilemma game, in which  $m$  stands for  $d$  and  $r$  stands for  $c$ .

**Theorem 2 (Folk Theorem with perfect information and personal punishment).** *Under perfect information, Assumption 1 and personal punishment, any feasible and strictly individually rational payoff ( $v \in V : v \gg 0$ ) can be supported by a subgame perfect equilibrium for  $\delta$  large enough.*

*Proof* Consider the following social norm that yields  $v$  in equilibrium: if player  $i$  meets player  $j$  and neither has been the last player to deviate in the last  $T$  periods, they play  $(a_{ij}, a_{ji})$  (which yields  $v$  in equilibrium); the last player to have deviated in the last  $T$  periods (simultaneous deviations are ignored) plays  $r$  and his match plays  $m$ .

<sup>8</sup> I thank an anonymous referee for suggesting this assumption over an alternative one.

**Table 2** Stage game

	A	B
A	1, 1	0, -1
B	-1, 0	-2, -2

First, consider the incentives to deviate if no one has deviated in the last  $T$  periods. For player  $i$  the expected utility of conforming with the equilibrium is at least  $(1 - \delta)g + \delta v_i$  while he would get at most  $(1 - \delta)\bar{g} + \delta v_i^p$  by deviating, where  $v_i^p = (1 - \delta^T)g(r, m) + \delta^T v_i$ . Let  $d$  be a number in  $(0, 1)$ . For each  $\delta$  choose  $T$  so that  $\lim_{\delta \rightarrow 1} \delta^{T(\delta)} = d$ . Since  $v_i > 0 \geq g(r, m)$ ,  $v_i > \lim_{\delta \rightarrow 1} v_i^p$  and  $(1 - \delta)g + \delta v_i > (1 - \delta)\bar{g} + \delta v_i^p$  for  $\delta$  large enough.

Second, consider the incentives to deviate if player  $i$  has been the last player to deviate in the last  $T$  periods and  $\tau \leq T$  periods of punishment remain. If player  $i$  plays  $r$ , as the strategy prescribes, he receives a payoff of  $(1 - \delta^\tau)g(r, m) + \delta^\tau v_i$ . If he deviates he receives at most  $\delta v_i^p$ . Choose  $d$  large enough for  $v_i^p$  to be positive. Since  $\delta < 1$ , it follows that  $(1 - \delta^\tau)g(r, m) + \delta^\tau v_i \geq v_i^p > \delta v_i^p$ .

Third, consider the case in which player  $j$  has been the last player to deviate in the last  $T$  periods and there are  $\tau + 1$  punishment periods left. In this case, for player  $i$ , the expected utility of conforming with the equilibrium is at least  $(1 - \delta)g + \delta v_i^\tau$ , where  $v_i^\tau = (1 - \delta^\tau)(\alpha_{ij}g(m, r) + (1 - \alpha_{ij})g_{i-j}) + \delta^\tau v_i$  and  $g_{i-j}$  is player  $i$ 's expected payoff from playing with all players but  $j$ . He would get at most  $(1 - \delta)\bar{g} + \delta v_i^p$  by deviating, where  $v_i^p = (1 - \delta^T)g(r, m) + \delta^T v_i$  as before. It can be easily checked that  $\alpha_{ij}g(m, r) + (1 - \alpha_{ij})g_{i-j} > g(r, m)$  if  $v_i - \alpha_{ij}(g_{ij} - g(m, r)) > g(r, m)$  which is satisfied if Assumption 1 holds given that  $v_i > 0$ ,  $\alpha_{ij} \leq 1$  and  $g_{ij} \leq \bar{g}$ . By the previous choice of  $T$ , we have that  $\lim_{\delta \rightarrow 1} (v_i^\tau - v_i^p) > 0$  for any  $\tau$  (including  $\tau = T - 1$ ) and it follows that  $(1 - \delta)g + \delta v_i^\tau > (1 - \delta)\bar{g} + \delta v_i^p$  for  $\delta$  large enough.

Note that there is no contradiction in the requirements made on  $\delta$  and  $T$  in the different parts of the proof, as it is only required that  $d$  and  $\delta$  are both large enough.  $\square$

It is important to note that for some stage games that violate Assumption 1 it is still possible to support a Folk Theorem. Consider the stage game in Table 2.

In this case, for any payoffs other than  $(-2, -2)$  that players  $i$  and  $j$  are suppose to get in equilibrium when playing with each other, it is possible to use long run incentives for punishers by using interior continuation payoffs as in Fudenberg and Maskin (1986). This is not possible for  $(-2, -2)$ . But in this case long run incentives are not needed: if one of the players deviates, their payoff when playing with each other during the punishment stage can not be lower than on the equilibrium path. Then, no additional incentives to conform with the social norm are needed.

The social norms used in the proofs of the former theorems, in which players punish deviators since failing to do so is itself punished, resemble, in some ways, the enforcement of castes in India. When describing marriage customs in India, Akerlof (1976) says: "The caste rules dictate not only the code of behavior, but

also the punishment for infractions: violators will be outcasted; furthermore, those who fail to treat outcastes as dictated by caste code will themselves be outcasted.”

While perfect information may be plausible in a small community it will certainly be implausible in a large one: it would be difficult for each player to know what every other player has done in the past if the number of players is very large. Therefore, Theorems 1 and 2 would not apply to the study of social norms and their impact on cooperation and inequality in large communities. In the next section I study the outcomes that can be supported by social norms under fewer information requirements.

#### 4 Local information

Even though in particular cases it is possible to forestall opportunistic behavior without the players having more information than their own experience, as the “contagious equilibrium” in Kandori (1992) and Ellison (1994), in more general cases extra information is necessary to provide the needed structure of punishments. In this section I assume that, in addition to their own experience, players have access to a local information processing system that gives players some information regarding their opponent’s past behavior. Following Okun-Fujiwara and Postlewaite (1995) and Kandori (1992), the local information processing system has the following structure: (1) in period  $t$  player  $i$  has a “status” or “flag”  $z_i(t) \in Z_i$ , where  $Z_i$  is a finite set (without loss of generality I can assume that  $Z_i \subset N_+$ ); (2) if players  $i$  and  $j$  are matched in period  $t$  and play  $a_i(t)$  and  $a_j(t)$ , the update of status follows a transition mapping  $(z_i(t+1), z_j(t+1)) = \tau_{ij}(a_i(t), a_j(t), z_i(t), z_j(t))$ ; (3) if at time  $t$  player  $i$  is matched with player  $j$ , the former only knows his own history and  $(z_i(t), z_j(t))$ .

Based on the local information processing system, a social norm prescribes the behavior for each player as a function of his past history, his status and the status of the matched player.<sup>9</sup> I show that any feasible and strictly individually rational outcome can be supported by a social norm in equilibrium.

**Theorem 3 (Folk Theorem with local information).** *With local information any feasible and strictly individually rational payoff ( $v \in V : v \gg 0$ ) can be supported by a sequential equilibrium for  $\delta$  large enough.*

*Proof* Consider the following social norm:

$$s_i(t) = \begin{cases} a_{ij} & \text{if } z_i(t) = z_j(t) = 0, \\ m & \text{if } z_i(t) \neq 0 \text{ or } z_j(t) \neq 0. \end{cases}$$

<sup>9</sup> An example of the use of a status system to provide incentives can be found in surfing summer camps: “Every, night, they have a public forum and give awards for the day’s events.... The surfer who receives the ‘Wave Hog’ award for dropping in too many times will have the distinction of wearing a bright pink jersey with a big ol’ hog printed on it the next day. And house (or boat) rules state that anyone wearing the pink jersey is fair game for drop-ins.” Surfer, Vol. 42, No. 7, (2001). The pink jersey summarizes the information that allows surfers to punish a surfer who has been selfish catching waves.

where  $\{a_{ij}\}$  is the play that yields  $v_i$ ; and the following local information system:

$$z_i(t + 1) = \begin{cases} T & \text{if } a_i(t) \neq s_i(t) \text{ or } a_j(t) \neq s_j(t) \\ \max \{z_i(t), z_j(t)\} - 1 & \text{if } \max \{z_i(t), z_j(t)\} > 0, a_i(t) \\ & = s_i(t) \text{ and } a_j(t) = s_j(t) \\ 0 & \text{if } z_i(t) = z_j(t) = 0, a_i(t) = s_i(t) \text{ and } a_j(t) = s_j(t) \end{cases}$$

With respect to beliefs outside the equilibrium path, assume that when player  $i$  is matched with player  $j$  and  $\max \{z_i(t), z_j(t)\} = \tau$ , player  $i$ 's assigns probability zero to the event that  $\max_{k \in \{1, \dots, N\}} z_k(t) > \tau$ . That is, every player believes that there have been no more deviations after the ones he knows of. It is easy to see that these beliefs are consistent (as defined in Kreps and Wilson (1982)): if the probabilities of trembles are converging to zero the probability that other deviations happened also converges to zero.

Next I check that in every information node no player has incentives to deviate. First, consider the case in which player  $i$  is matched with player  $j$  and  $z_i = z_j = 0$ . Then, player  $i$  believes that all players have zero flags. In this situation, player  $i$  expects to earn at least  $(1 - \delta)\underline{g} + \delta v_i$  by conforming, and at most  $(1 - \delta)\bar{g} + \delta v_i^p$  by deviating, where  $v_i^p = (1 - \delta^T)g(m, m) + \delta^T v_i$ . Let  $d$  be a number in  $(0, 1)$ . For each  $\delta$  choose  $T$  so that  $\lim_{\delta \rightarrow 1} \delta^T(\delta) = d$ . Since  $v_i > g(m, m)$ ,  $v_i > \lim_{\delta \rightarrow 1} v_i^p$  and it follows that  $(1 - \delta)\underline{g} + \delta v_i > (1 - \delta)\bar{g} + \delta v_i^p$  for  $\delta$  large enough.

Second, consider the case in which player  $i$  is matched with player  $j$  and  $\max \{z_i, z_j\} = \tau \geq 1$ . Then, player  $i$  believes that in  $\tau$  periods all players will have zero flags and he will earn  $v_i$  every period. In this case  $i$  obtains  $(1 - \delta^\tau)g(m, m) + \delta^\tau v_i$  by conforming, and at most  $\delta v_i^p$  by deviating. Given  $v_i > 0$ , choose  $d$  large enough for  $v_i^p$  to be positive. Since  $\delta < 1$  and  $\tau \leq T$ , it follows that  $(1 - \delta^\tau)g(m, m) + \delta^\tau v_i \geq v_i^p > \delta v_i^p$ .

Note that there is no contradiction in the requirements made on  $\delta$  and  $T$  in the different parts of the proof, as it is only required that  $d$  and  $\delta$  are both large enough. □

In the social norm used in the previous proof,  $z_i > 0$  denotes that  $i$  has deviated in the last  $T$  periods or is aware that someone has deviated in the last  $T$  periods. These contagious flags allow me to construct a social norm that adapts to the local information case the social norm used in the proof of Theorem 1 (Folk Theorem with perfect information and *without* personal punishment). Note that any player that deviates, sees a deviation or knows that a deviation occurred, will be punished until the end of the punishment stage. In this way, when a player knows that there has been a deviation his incentives to enforce the social norm do not depend on whom or how many have deviated. Whoever has deviated, once out-of-the-path beliefs are properly specified, it is easy to check that every player will enforce the punishment. The lack of personal punishment in this social norm allows me to prove the Folk Theorem without restrictions on the stage game.

A similar social norm could be used to prove the Folk Theorem under local information system in the environment studied by Kandori (1992) without the restriction imposed on the stage game by Theorem 2 of that paper. Note that this can be done without increasing either the number of flags needed or the complexity of the social norm (in fact, the social norm would be simpler since only one

action would be used during the punishment stage). If we are willing to accept that non-deviators be punished as well, there is no need to provide explicit short run incentives to punishers and, hence, no need for restrictions on the stage game.

But the social norm in the proof of Theorem 3 may be criticized, precisely, because players other than the deviator are punished. The next result shows that with local information and Assumption 1 any feasible and strictly individually rational outcome can be supported in a sequential equilibrium with personal punishment.

**Theorem 4 (Folk Theorem with local information and personal punishment).**

*Under local information, Assumption 1 and personal punishment any feasible and strictly individually rational payoff ( $v \in V : v \gg 0$ ) can be supported by a sequential equilibrium for  $\delta$  large enough.*

*Proof* Consider the following social norm:

$$s_i(t) = \begin{cases} a_{ij} & \text{if } z_i(t) = z_j(t) = 0 \\ m & \text{if } z_j(t) \geq z_i(t) \\ r & \text{if } z_j(t) < z_i(t) \end{cases}$$

where  $\{a_{ij}\}$  is the profile of actions that yields  $v$ , and the following local information system:

$$z_i(t + 1) = \begin{cases} T & \text{if } a_i(t) \neq s_i(t) \\ z_i(t) - 1 & \text{if } z_i(t) > 0 \text{ and } a_i(t) = s_i(t). \\ 0 & \text{if } z_i(t) = 0 \text{ and } a_i(t) = s_i(t) \end{cases}$$

I show next that no player has incentives to deviate after any possible history if the rest of the players follow the social norm. This makes beliefs unimportant, since regardless of what has happened in the past no player has incentives to deviate.

First, I show that player  $i$  has no incentive to deviate if  $z_i = 0$ . Denote the set of players different than  $i$  with a positive (zero) flag at time  $t$  as  $N_t^+$  ( $N_t^0$ ) and denote  $g(a_{ij}, a_{ji})$  as  $g_{ij}$ . If player  $i$  conforms with the prescribed strategy he would get at

least  $(1 - \delta)\underline{g} + (1 - \delta) \sum_{t=1}^{T-1} \delta^t \left[ \sum_{j \in N_t^+} \alpha_{ij} g(m, r) + \sum_{j \in N_t^0} \alpha_{ij} g_{ij} \right] + \delta^T v_i$ . There exist

sets  $N_t^+$  and  $N_t^0$  that minimize the term in brackets. Denote the minimum value of the term in brackets as  $g^{\min}$ . Then, by conforming to the social norm, player  $i$  earns at least  $(1 - \delta)\underline{g} + \delta v_i^{\min}$ , where  $v_i^{\min} = (1 - \delta^{T-1}) g^{\min} + \delta^{T-1} v_i$ . By deviating he would get at most  $(1 - \delta)\bar{g} + \delta v_i^p$ , where  $v_i^p = (1 - \delta^T) g(r, m) + \delta^T v_i$ . As in the proof of Theorem 3, it can be shown that Assumption 1 implies that  $\sum_{j \in N_t^+} \alpha_{ij} g(m, r) + \sum_{j \in N_t^0} \alpha_{ij} g_{ij} > g(r, m)$ , for any  $N_t^+$  and  $N_t^0$ , and then  $g^{\min} >$

$g(r, m)$ . Let  $d$  be a number in  $(0, 1)$ . For each  $\delta$  choose  $T$  so that  $\lim_{\delta \rightarrow 1} \delta^{T(\delta)} = d$ .

Since  $v_i > 0 \geq g(r, m)$  and  $g^{\min} > g(r, m)$ ,  $\lim_{\delta \rightarrow 1} (v_i^{\min} - v_i^p) > 0$  and it follows that  $(1 - \delta)\underline{g} + \delta v_i^{\min} > (1 - \delta)\bar{g} + \delta v_i^p$  for  $\delta$  large enough.

Second, I show that  $i$  has no incentive to deviate if  $z_i = \tau > 0$ . Denote the set of players different than  $i$  and with  $z_j(t) > z_i(t)$ ,  $z_j(t) = z_i(t)$  and  $z_j(t) < z_i(t)$

as  $N_i^>$ ,  $N_i^=$  and  $N_i^<$  respectively. If player  $i$  conforms with the prescribed strategy he would get:

$$(1 - \delta) \left[ g(r, m) + \sum_{t=1}^{\tau-1} \delta^t \left( \sum_{j \in N_i^>} \alpha_{ij} g(m, r) + \sum_{j \in N_i^=} \alpha_{ij} g(m, m) + \sum_{j \in N_i^<} \alpha_{ij} g(r, m) \right) \right. \\ \left. + \sum_{t=\tau}^{T-1} \delta^t \left( \sum_{j \in N_i^>} \alpha_{ij} g(m, r) + \sum_{j \in N_i^=} \alpha_{ij} g_{ij} \right) \right] + \delta^T v_i.$$

By deviating he would get at most  $(1 - \delta) \left[ \sum_{t=1}^{\tau-1} \delta^t g(r, m) + \sum_{t=\tau}^{T-1} \delta^t g(r, m) \right. \\ \left. + \delta^T g(r, m) \right] + \delta^{T+1} v_i$

Since  $g(m, r) > g(r, m)$  and  $g(m, m) \geq g(r, m)$ , the gains from deviation are at most  $(1 - \delta) \left[ \sum_{t=\tau}^{T-1} \delta^t \left( g(r, m) - \left( \sum_{j \in N_i^>} \alpha_{ij} g(m, r) + \sum_{j \in N_i^=} \alpha_{ij} g_{ij} \right) \right) \right. \\ \left. - ((1 - \delta^T) g(r, m) + \delta^T v_i) \right]$

By Assumption 1, the first term inside the brackets is negative. In addition, the second term is positive for  $d$  large enough. Then, there are no gains from deviation.

Note that there is no contradiction in the requirements made on  $\delta$  and  $T$  in the two parts of the proof, it is only required that  $d$  and  $\delta$  are both large enough.  $\square$

The local information systems needed in the previous two proofs in this section are “simple”, in the sense that the number of flags needed is finite and does not increase with time or the number of deviations. Since the punishment stage consists of  $T$  periods an information mechanism with at least  $T + 1$  flags per player is needed: one for each period of punishment and one for when the player is not in the punishment stage.<sup>10</sup> It is interesting to note that the restriction to personal punishment does not result in a local information system with a larger set of flags.

In the equilibria in this section, the best response of any player in any situation depends only on his own and his match’s flags. Any other information that players may have is irrelevant for making decisions: the best response is to follow the social norm, which tells players what to do under every combination of flags. In this way, the flags are sufficient statistics for the players decision making since they summarize all the relevant information.<sup>11</sup>

The equilibria described in this paper present some characteristics that are worth mentioning. First, the long run behavior of the community is not affected by any finite sequence of deviations. In the equilibria of this paper, if there have been deviations, the prescribed actions revert to the original ones after  $T$  periods of punishment. Hence, the actions on the equilibrium path are globally stable: regardless

<sup>10</sup> While the number of flags needed in Theorems 3 and 4 is finite, it can be very large. A way to drastically reduce the number of needed flags is to allow for a random transition rule of flags as is done in Kandori (1992). To have punishments of a random length *à la* Abreu et al. (1994) under a local information system, we can have two types of flags per player: guilty and nice, and, every period, all the guilty players that have conformed with the punishment are forgiven with probability  $p \in (0, 1)$  and they become nice. In this way,  $p$  can be used to establish the severity of punishment, as  $T$  was doing before, with the need of only two flags. Random forgiveness eliminates the need of counting the number of periods of punishment.

<sup>11</sup> This property of equilibria is called “straightforward” in Kandori (1992).

**Table 3** Prisoner's dilemma game

	<i>c</i>	<i>d</i>
<i>c</i>	2, 2	-1, 4
<i>d</i>	4, -1	0, 0

of how many deviations have been up today, in the future the play of the game will return to the equilibrium play. This property is of special importance when studying societies with a large number of members. If a single deviation may take the community out of the equilibrium path forever, it would be difficult to observe the equilibrium behavior in a large community in which each member has a small probability of making mistakes.

Second, the equilibria described in this paper are robust to small perturbations of the payoffs matrix of the stage game. Given that in the proofs of this paper all the inequalities are strict, if a social norm is an equilibrium under a given payoff matrix, it will also be an equilibrium with a payoff matrix that is arbitrarily close to the original one. Therefore, the equilibria presented here do not depend on a precise characterization of the players payoffs.

## 5 Applications: corporate culture, social capital and inequality

I present next several outcomes that can be supported in equilibrium by social norms showing the richness of the equilibria analyzed in this paper. Consider a community of ten members that are matched uniformly to play the prisoner's dilemma in table 3.

I present first a society in which social norms support an equal and efficient outcome.

*Optimal egalitarian society:* In equilibrium all the players play *c* and receive a payoff of 2. This outcome is feasible and individually rational and, hence, can be supported by a self-enforcing social norm under either perfect information or local information. Therefore, a social norm with its promise of punishment to deviators can support an egalitarian outcome that Pareto dominates the inefficient egalitarian equilibrium of the one shot game.

We may think of the ten members of the previous example as workers in a firm who are randomly matched in teams to perform their duties and whose level of effort are not directly observed by the principal. This example can then help us understand how workers may follow reward and punishment schemes based on their repeated interaction to overcome opportunistic behavior. In this environment, the concept of social norm coincides with the concept of "corporate culture," that is, the unwritten codes of behavior that shape interaction among members of the firm affecting their behavior and the performance of the firm (see Baker, Gibbons, and Murphy (2002) and references therein). In this way, the results of this paper help us understand horizontal relationships in organizations.<sup>12</sup>

<sup>12</sup> With the exception of Crémer (1986) most applications of repeated games to the study of firms have focused on vertical relationships. See for example, Bull (1987), MacLeod and Malcolmson (1989), Baker, Gibbons, and Murphy (1994), and Baker, Gibbons, and Murphy (2002). Outside the repeated game literature, there is a literature that studies the effects of social norms that are followed because players derive direct utility from it, see Kandell and Lazear (1992), Huck et al. (2001), and Rob and Zemsky (2002).

In addition, the idea of social norms supports the concept of “social capital” in a community (Loury 1977). In the sociology literature (see for example Coleman 1990) social capital comprises the information channels, norms and trust that facilitate cooperation and coordination in a community. By studying self-enforcing social norms, this paper help us understand how norms and trust can be supported, even in a community of selfish people, providing game theoretical support to the widely invoked concept of social capital.

This paper also explains how social norms can support outcomes characterized by inequality as the example in the introduction and the next example illustrate.

*Caste (or Class) society:* Consider a society divided in three castes: one player belongs to the high caste and in equilibrium he always plays  $d$ ; three players belong to the middle caste and they play  $c$  when matched with a member of the same of higher cast and  $d$  otherwise; and the remaining six players belong to the lower cast and they always play  $c$  in equilibrium. Then, the high caste member receives 4, the middle cast members receive 3 and the low caste members receive  $\frac{2}{3}$ . This outcome is feasible and individually rational and then can be supported by a self-enforcing social norm under either perfect information or local information.

These examples show that social norms can support unequal outcomes even when all the members of the community are basically equal. While in these examples the division of members among the different groups is arbitrary, in reality it may correspond to differences in race, religion or gender. In this way, the results have implications for the study of inequality and discrimination. Most existent theories of inequality and discrimination are based on differences in tastes, skills or capital. These fundamental characteristics are considered either inherent to the person (differences in productivity as in the neoclassical theory or differences in taste as in Becker 1959) or acquired (as in models of statistical discrimination, Arrow 1973). This paper, instead, shows that social norms can credibly enforce inequality even when no such differences exist.

The connection between social norms and inequality has been studied before (see Akerlof 1976; Axelrod 1984). However, the punishments used in those studies are not credible since the equilibria they study are not perfect. In addition, the results in this paper are general to any stage game and not limited to particular examples as were previous studies.

## 6 Appendix:

### The connection with repeated games with imperfect public information

Fudenberg et al. (1992) provide a Folk Theorem for repeated games with imperfect public information. They show that any interior point of the set of feasible and individually rational payoffs can be obtained in equilibrium if players are patient enough and the public signal is sufficiently informative. In this appendix I show that, while random matching repeated games can be interpreted as repeated games with imperfect public information, Theorem 1 does not follow from Fudenberg et al. (1992) even if we restrict ourselves to interior points.

The model they study is as follows. In the stage game players move simultaneously and player  $i = 1, \dots, N$  chooses an action  $s_i$  from a finite set  $S_i$ . Denote as  $|S_i|$  the cardinality of  $S_i$ . A profile of actions  $s$  induces a probability distribution  $\pi(y | s)$  over a public signal  $y \in Y$ . Denote as  $|Y|$  the cardinality of  $Y$ . Each

player’s realized payoff depends only on her own action and the public signal. They assume that the set of feasible and individually rational payoffs has a non-empty interior. Players play the same stage game repeatedly and each player only observes her own actions and the public signal.

Their Folk Theorem result rests on the public signal being sufficiently informative. The required informativeness can be expressed as properties of the matrix  $\pi(y | s)$ . Let  $\sigma$  be a profile of actions (possibly mixed) and  $\Pi_i(\sigma_{-i}) = \pi(\cdot | \cdot, \sigma_{-i})$ . That is,  $\Pi_i(\sigma_{-i})$  is a  $|S_i| \times |Y|$  matrix whose rows correspond to player  $i$ ’s actions, and the columns to the signals. A profile  $\sigma$  has *individual full rank* if  $\text{rank}(\Pi_i(\sigma_{-i})) = |S_i|$  for all  $i$ .

Let  $\Pi_{ij}(\sigma) = \begin{pmatrix} \Pi_i(\sigma_{-i}) \\ \Pi_j(\sigma_{-j}) \end{pmatrix}$ . A profile  $\sigma$  has *pairwise full rank* if  $\text{rank}(\Pi_{ij}(\sigma)) = |S_i| + |S_j| - 1$  for all  $i, j$ .

Fudenberg et al. (1992) show that if every pure action profile has individual full rank and for every pair of players there exists a profile of actions that has pairwise full rank, then any point in the interior of the set of feasible and individually rational payoffs can be supported in equilibrium if players are sufficiently patient.

I show next that the random matching repeated games considered in this paper can be interpreted as infinitely repeated games with imperfect public information as in Fudenberg et al. (1992). Remember that a “play” in a random matching game defines what a player would do when matched with each of the other players. By defining the play for each player as an action in the model by Fudenberg et al. (1992), we can interpret the random matching game as a game with an imperfect public signal. The public signal now reveals who was matched with whom, and what they did. It does not reveal what players would have done if they had been matched with other players.

Since in the random matching model a player can take  $|A|$  possible actions and there are  $N - 1$  other players, the number of possible pure plays for a player is  $|S_i| = |A|^{N-1}$  for every  $i$ .<sup>13</sup> Since there are  $\frac{N!}{2^{\frac{N}{2}} \frac{N}{2}!}$  ways in which  $N$  players can be matched in pairs and there are  $|A|^N$  profiles of pure plays, the cardinality of the set of signals is  $|Y| = \frac{N!}{2^{\frac{N}{2}} \frac{N}{2}!} |A|^N$ . The probability distribution  $\pi(y | s)$  of the public signal can be constructed in a straightforward manner given the play in the random matching game and the matching probabilities.

As an example consider the game presented in the introduction: four players that are uniformly matched to play a prisoner’s dilemma game. In this case, the number of possible plays for a player is  $|S| = 8$ , there are three possible ways of matching the players (1-2, 3-4; 1-3, 2-4; and 1-4, 2-3) and the number of signals is  $|Y| = 48$  (1-2, 3-4, cccc; 1-2, 3-4, dccc; ...; 1-4, 2-3, dddd; where the first two pairs of numbers indicate the matching and the four letters the actions of the four players). The probability distribution over the signals follows from the play and matching probabilities. For example, if the play is to always cooperate, the probability of match 1-2 and 3-4 and everybody cooperating is  $\pi(1-2, 3-4, cccc | ccc, ccc, ccc, ccc) = \frac{1}{3}$ , and the probability of match 1-2 and 3-4 with player 1 defecting and everybody else cooperating is  $\pi(1-2, 3-4, dccc | ccc, ccc, ccc, ccc) = 0$ .

<sup>13</sup> For simplicity of notation I assume here that  $0 < \alpha_{ij} < 1$  for every pair of players  $i$  and  $j$ . The extension to the general case is straightforward but requires more notation.

**Table 4** Probability distribution over signals

	1-2,3-4 <i>cccc</i>	1-2,3-4 <i>dccc</i>	1-3,2-4 <i>cccc</i>	1-3,2-4 <i>dccc</i>	1-4,2-3 <i>cccc</i>	1-4,2-3 <i>dccc</i>	1-2,3-4 <i>cdcc</i>	...	1-4,2-3 <i>dddd</i>
<i>ccc</i>	$\frac{1}{3}$	0	$\frac{1}{3}$	0	$\frac{1}{3}$	0	0	...	0
<i>dcc</i>	0	$\frac{1}{3}$	$\frac{1}{3}$	0	$\frac{1}{3}$	0	0		0
<i>cdc</i>	$\frac{1}{3}$	0	0	$\frac{1}{3}$	$\frac{1}{3}$	0	0		0
<i>ccd</i>	$\frac{1}{3}$	0	$\frac{1}{3}$	0	0	$\frac{1}{3}$	0		0
<i>ddc</i>	0	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{1}{3}$	0	0		0
<i>dcd</i>	0	$\frac{1}{3}$	$\frac{1}{3}$	0	0	$\frac{1}{3}$	0		0
<i>cdd</i>	$\frac{1}{3}$	0	0	$\frac{1}{3}$	0	$\frac{1}{3}$	0		0
<i>ddd</i>	0	$\frac{1}{3}$	0	$\frac{1}{3}$	0	$\frac{1}{3}$	0	...	0

Since the number of signals is greater than the number of plays of a player and every play of a player results in a different probability distribution over signals, it may appear that every pure play will have individual full rank. But that is not the case as the next example shows. Consider again the game presented in the Introduction. If the play is for all players to cooperate,  $\Pi_1(s_{-1})$  is shown in Table 4, where the rows correspond to each of the eight different pure plays of player 1 (for example the second row corresponds to the play defect with player 2 and cooperate with players 3 and 4) and the columns correspond to each of the possible 48 signals (I have only included the six columns with positive numbers and two of the 42 columns that have only zeros).

Given that players 2, 3 and 4 always cooperate, the different pure plays of player 1 can only result in outcomes *cccc* or *dccc*. In addition, player 1 can face three different partners implying that the number of signals that can be reached with positive probability is 6.<sup>14</sup> That is, the number of columns with at least one positive entry is 6, which is lower than the number of player 1's pure plays, 8, and hence the play does not have individual full rank.<sup>15</sup>

In general, note that a player chooses which of  $|A|$  actions to take when being matched with  $N - 1$  possible partners, and then her  $|A|^{N-1}$  pure plays only generate  $(N - 1) |A|$  signals with positive probability (only  $(N - 1) |A|$  columns in the matrix  $\Pi_i(s_{-i})$  will have positive entries). In addition, note that  $|A|^{N-1} > (N - 1) |A|$  if  $N \geq 4$  and  $|A| \geq 2$ , that is, the number of pure plays of a player is greater than the number of signals that can be reached with positive probability with her plays. Therefore, for non-trivial random matching games, no pure play has individual full rank and, hence, the conditions for the Folk Theorem presented in Fudenberg et al. (1992) do not hold.

<sup>14</sup> It may seem that the number of signals that can be reached is equal to the number of pure plays for a player times the total number of possible matches. But this is not correct. Consider the case with  $N = 6$ . The columns in  $\Pi_1(s_{-1})$  corresponding to matches 1-2,3-4,5-6 and 1-2,3-5,4-6 are linearly dependent.

<sup>15</sup> Note that not all of these columns are linearly independent. In fact, in this example there are only four linearly independent columns.

## References

- Abreu, D., Dutta, P.K., Smith, L.: The folk theorem for repeated games: a NEU condition. *Econometrica* **62**, 939–948 (1994)
- Akerlof, G.: The economics of caste and of the rat race and other woeful tales. *Quart J Econ* **90**, 599–617 (1976)
- Arrow, K.: The theory of discrimination. In: Ashenfelter, O., Rees, A. (eds.) *Discrimination in labor markets*. Princeton: Princeton University Press 1973
- Aumann, R., Shapley, L.: Long term competition: a game theoretic analysis. Mimeo (1976)
- Axelrod, R.: *The evolution of cooperation*. New York: Basic Books 1984
- Baker, G., Gibbons, R., Murphy, K.J.: Subjective performance measures in optimal incentive contracts. *Quart J Econ* **109**, 1125–1156 (1994)
- Baker, G., Gibbons, R., Murphy, K.J.: Relational contracts and the theory of the firm. *Quart J Econ* **117**, 39–84 (2002)
- Becker, G.: *The economics of discrimination*. Chicago: University of Chicago Press 1959
- Bull, C.: The existence of self-enforcing implicit contracts. *Quart J Econ* **102**, 147–159 (1987)
- Coleman, J.: *Foundations of social theory*. Cambridge: Harvard University Press 1990
- Crémer, J.: Cooperation in ongoing organizations. *Quart J Econ* **101**, 33–49 (1986)
- Ellison, G.: Cooperation in the prisoner's dilemma with anonymous random matching. *Rev Econ Stud* **61**, 567–588 (1994)
- Friedman, J.W.: A non-cooperative equilibrium for supergames. *Rev Econ Stud* **38**, 1–12 (1971)
- Fudenberg, D., Levine, D.K., Maskin, E.: The folk theorem with imperfect information. *Econometrica* **62**, 997–1039 (1992)
- Fudenberg, D., Maskin, E.: The folk theorem in repeated games with discounting or with incomplete information. *Econometrica* **54**, 533–554 (1986)
- Greif, A.: Contract enforceability and economic institutions in early trade – the Maghribi traders coalition. *Am Econ Rev* **83**, 525–548 (1993)
- Greif, A., Milgrom, P., Weingast, B.: Coordination, Commitment, and Enforcement - the case of the merchant guild. *J Polit Econ* **102**, 745–776 (1994)
- Huck, S., Kübler, D., Weibull, J.W.: Social norms and optimal incentives in firms. Mimeo (2001)
- Kandel, E., Lazear, E.P.: Peer pressure and partnerships. *J Polit Econ* **100**, 801–817 (1992)
- Kandori, M.: Social norms and community enforcement. *Rev Econ Stud* **59**, 63–80 (1992)
- Kreps, D.M., Wilson, R.: Sequential equilibrium. *Econometrica* **50**, 863–894 (1982)
- Loury, G.: A dynamic theory of racial income differences. In: Wallace P.A., Le Mund A. (eds.) *Women, minorities, and employment discrimination*. Lexington, MA: Lexington Books 1977
- Luce, R.D., Raiffa, H.: *Games and decisions*. New York: Wiley 1957
- MacLeod, W.B., Malcomson, J.M.: Implicit contracts, incentive compatibility, and involuntary unemployment. *Econometrica* **57**, 447–80 (1989)
- Okuno-Fujiwara, M., Postlewaite, A.: Social norms and random matching games. *Games Econ Behav* **9**, 79–109 (1995)
- Rob, R., Zemsky, P.: Social capital, corporate culture, and incentive intensity. *RAND J Econ* **33**, 243–257 (2002)