Innovation as a search process: The Kortum model

Kortum (1997) developed a growth model in which innovation is portrayed as a search process. This section shows how our basic Schumpeterian model can be modified to incorporate Kortum's search-theoretic ideas.[1]

Assume that the Poisson arrival rate of new discoveries to each researcher is a constant, $\lambda > 0$. Each new discovery is a draw from a distribution of potential technologies, each technology having a different productivity parameter. Following Kortum,[2] assume that the productivity A_i of a discovery in sector i is distributed according to a Pareto distribution:

$$\text{Prob}\left\{A_i \leq A\right\} = F\left(A\right) = 1 - \left(\frac{X}{A}\right)^{\theta}; A \geq X \tag{1}$$

with the two parameters $\theta > 1$ and $X > 0$. The parameter X represents the lowest possible productivity of any discovery.

Suppose that the technology currently in use in sector i has a productivity $\overline{A}_i$. Then a newly discovered technology will represent a technological improvement only if $A_i > \overline{A}_i$; otherwise the discovery will be a false start instead of an innovation. So if there are n researchers active in sector i, the Poisson arrival rate μ_i of an innovation in the sector will be the arrival rate of discoveries to all n researchers times the probability that each new discovery has a productivity greater than $\overline{A}_i$:

$$\mu_i = n\lambda \cdot \left(1 - F\left(\overline{A}_i\right)\right) = n\lambda \cdot \left(\frac{X}{\overline{A}_i}\right)^{\theta} \tag{2}$$

[1] Kortum's model implies there can be no long-run economic growth without population growth. That is, unlike our basic Schumpterian model, his is one of "semi-endogenous" growth. We will discuss semi-endogenous growth models in chapter XX below.

[2] See also Bental and Peled (1996).

Now the Pareto distribution (1) has the desirable property that the expected size of each innovation relative to the incumbent's productivity ($A_i/\overline{A}_i - 1$) is the same no matter how advanced the incumbent's productivity (provided that $\overline{A}_i$ is no less than the lower bound X of A_i). That is:[3]

$$E\left(\left(\frac{A_i}{\overline{A}_i} - 1\right)|A_i > \overline{A}_i\right) = \frac{1}{\theta - 1}. \tag{3}$$

So if we could find a steady state in which the frequency of innovations was constant, then the expected growth rate (which is the frequency times the expected size of innovations) would also be constant.

However, the problem with this approach is that a steady state would also have a constant number of researchers in each industry, in which case equation (2) indicates that, as the incumbent's productivity $\overline{A}_i$ grows, the frequency of innovations will fall asymptotically to zero. Thus the expected growth rate will also fall to zero.

The source of this problem is that researchers are not keeping up with progress. They keep searching through the same old distribution of potential technologies even though the state of the art has improved. In reality, of course, researchers learn more as the state of

[3] The density of A_i is

$$f(A_i) = \theta\left(\frac{X^\theta}{A_i^{1+\theta}}\right)$$

and the conditional expectation is:

$$\begin{aligned} & E\left(\frac{A_i}{\overline{A}_i}|A_i > \overline{A}_i\right) - 1 \\ &= \frac{1}{\overline{A}_i}\frac{\int_{\overline{A}_i}^{\infty} xf(x)\,dx}{1 - F(\overline{A}_i)} - 1 \\ &= \frac{1}{\overline{A}_i}\frac{\int_{\overline{A}_i}^{\infty} \theta\left(\frac{X}{x}\right)^\theta dx}{\left(\frac{X}{\overline{A}_i}\right)^\theta} - 1 \\ &= \frac{1}{\theta - 1} \end{aligned}$$

the art improves; in particular, they learn to direct their search towards technologies that are more advanced than anything they were likely to think of before. Kortum's model takes this learning into account by assuming that the distribution F improves over time (that is, $F(A)$ gets smaller for any fixed A), at a rate that depends on how much research is being done.

We can do something similar by supposing that the parameter X of the distribution (1), which represents the minimum possible productivity parameter of any newly discovered technology, rises with each innovation. In particular, suppose that X always remains proportional to the incumbent's productivity, so we have:

$$X = \chi \overline{A}_i, \quad 0 < \chi \leq 1 \tag{4}$$

Substituting from (4) into (2), we see that, even with a constant number of researchers, the frequency of innovations will now be constant:

$$\mu_i = n\lambda\chi^\theta \tag{5}$$

so that in a steady state the expected growth rate will also be constant, equal to the product of the frequency (5) and the expected size (3):

$$Eg = \frac{n\lambda\chi^\theta}{\theta - 1}$$

The rest of the modified Schumpeterian model can be filled in exactly as before, but with the fixed size of innovation $\gamma - 1$ being replaced everywhere by the expected size $\frac{1}{\theta-1}$.

References

Kortum, Samuel S. "Research, Patenting, and Technological Change." *Econometrica* 65 (November 1997): 1389–1419.

Bental, Benjamin, and Dan Peled. "Endogenous Technical Progress and Growth: A Search Theoretic Approach." *International Economic Review* 37 (August 1996): 687–718.