Learning, Leverage and Stability[1]

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Most analyses of the 2008-09 financial crisis have stressed particular factors that are unique to modern financial systems, such as credit default swaps, collateralized debt obligations, liquidity puts, securitized mortgages, government sponsored enterprises, conduits, special investment vehicles, asset-backed commercial paper, and capital requirements dependent on private credit-rating agencies. But serious financial crises have been a recurrent feature of capitalist economies since long before any of these factors existed. This paper takes the view that although practical policy analysis must be based on the particularities of present-day systems, a deeper understanding of the origins of financial crises can best be achieved by abstracting from such details and focusing on those factors that seem common to all crises. In particular, we focus on two factors that seem particularly universal, namely leverage and expectations.

More specifically, I construct and analyze a simple general equilibrium model with an active credit market in which borrowers and lenders form expectations adaptively. Except for expectation formation the model is a small open economy variant of the standard Lucas (1978) tree model. To abstract from institutional detail there is just a representative investor/borrower and a representative lender (rest of the world), both of whom are learning from experience. The point is to show that even in this fairly standard model financial crises can occur, and to examine the conditions, with respect to policy, institutions and other aspects of the environment, that affect the likelihood of crises in the model.

Financial crises can occur in the model because leverage and expectations combine to form a positive feedback loop. That is, a high rate of return experienced by the investor will induce both him and the lender to become more optimistic and hence to extend leverage. With increased leverage, asset prices will be driven up even further, which reinforces optimism and results in even higher leverage. Likewise, a low experienced rate of return will result in cumulative pessimism and a cumulative fall in asset prices. A crisis can occur when a period of cumulative optimism is followed by a period of cumulative pessimism. The high debt left over from the former period, combined with the low asset prices produced by the latter, leaves the representative investor insolvent.

Adaptive formation of expectations is critical because the model contains a unique stable rational expectations equilibrium in which insolvency never occurs. Leverage is also critical, not just because without borrowing there cannot be insolvency but also because in cases where the world interest rate (lender's rate of time preference) is high enough that the investor also becomes a lender in equilibrium, the unique rational expectations equilibrium is well approximated by the outcome under adaptive learning.

The model is too stark to capture many features of real world financial markets. But prior to almost all crises it does exhibit the three phenomena that Reinhart and Rogoff

(2009) show characterize the years preceding almost all real world financial crises, namely an expansion of credit, an unusually rapid rate of increase in asset prices and a subsequent collapse of asset prices. It is also consistent with the observation by Perez (2002) to the effect that the credit expansion and asset price runup preceding a major crisis typically begin with a real technological breakthrough that would have raised the rate of return to investors even without the capital gain. However, the model does not exhibit the recession that Reinhart and Rogoff show typically follows a crisis; by construction, output remains unchanged throughout.

Although the model is also too stark to yield practical policy implications, it does suggest some general policy measures that could be used to reduce the frequency of crises. Clearly a limit on leverage would be one such measure. Less obviously, a limit on deleverage would also work. This is because the crisis is brought about by the attempt to reduce indebtedness. More specifically, a simple restriction on the amount by which the representative investor can reduce her indebtedness during any period can prevent a financial crisis from ever occurring. As will be explained below, this latter policy implication stems from a fundamental conflict between micro- and macro-prudential regulation, which exists even in this institution-free analysis.

1 Related literature

The analysis is related to the line of research on financial fragility started by Hyman Minsky (1982), and carried on by Perez (2002), Kindleberger and Aliber (2005) and Leijonhufvud (2009) among others. Minsky's thesis was that during the expansion phase of the business cycle people become overconfident, that the effects of this overconfidence on asset prices is amplified by the expansion of credit, which continues until key players in the system became so highly leveraged that a downturn in yields would drive some of them into bankruptcy. When this started to happen, confidence disappears, and the same process works in reverse, with the downward movement in asset prices amplified by contraction in credit. As Kindleberger and Aliber (2005, ch.2) point out, Minsky's analysis is very classical in nature, building on the concepts of "overtrading, revulsion and discredit" with which Smith and his contemporaries analyzed the credit cycle.

Kindleberger and Aliber provide an historical account of financial manias and crashes, from the early 17th Century to the end of the 20th, which they argue validates the Minsky model. They emphasize the unstable and procyclical behavior of credit as the key to asset price bubbles. The runup of asset prices is always accompanied by an extraordinary expansion of credit, and the crash by an extraordinary contraction of credit. They also emphasize the nonrational aspects of financial crises - euphoria during bubbles and panic during crashes. But their account is nevertheless based in fundamentals; indeed they provide evidence that during the early phase of the runup there is always some sound fundamental basis for the rising optimism; for example, the late 1920s bubble was fueled by the very real economic profits generated by the diffusion of the automobile, electrification, the telephone and radio, while the tech boom of the late 1990s was fueled by firms trying to emulate the very real success of pioneers such as Microsoft, Intel, Cisco and Dell.

Perez carries the analysis one step further than Minsky and Kindleberger, tying the analysis of bubbles and crashes into the analysis of major technological changes - what Freeman and Perez (1988) call "techno-economic paradigm shifts", a concept similar to but much broader than Bresnahan and Trajtenberg's (1995) notion of "general purpose technology." According to Perez, the world economy has gone through five major technological changes since the eighteenth century. Each of them involves not just a change in some basic enabling technology but also a major restructuring of economic and financial organizations; in her words, each technological era corresponds to a different "organizational common sense." The replacement of one common sense by another every half century or so involves a period of two or three turbulent decades before the economy enters a new "golden age" where the full benefits of the new technology are finally enjoyed. It is during the transitional period that the economy experiences a cycle of euphoria, financial crisis and depression.

One of the objectives of this paper is to bring Minsky's ideas closer to mainstream analysis and to strip the analysis down to its fundamentals. The literature surveyed in the preceding paragraphs is basically model-free, and makes appeals to irrational psychological factors such as manias and panics. In addition to casting the ideas in terms of a standard framework, the paper replaces irrational psychology with a simple learning behavior of the sort elaborated by Sargent (1993 and 1999) and Evans and Honkapohja (2001).[1]

In addition to borrowing its central idea from Minsky, the paper sheds light on his "financial fragility" hypothesis, to the effect that periods of financial stability are inherently self-limiting because people tend to become increasingly confident as memory of the previous crisis recedes. The model spelled out below exhibits this feature as a result of constant-gain learning, in which older evidence is discounted relative to newer evidence in the process of adjusting expectations.

Formal models of bubbles and crashes under adaptive behavior with heterogeneous agents have been presented in the literature on agent-based models by writers such as Arthur *et al.* (1997), Brock and Hommes (1998) and LeBaron (2001). Recent surveys of this literature are

[1] Until now this literature has not been directed at understanding financial crises; for example, Evans and Honkapohja (2001, pp.220-222) show that rational asset bubbles are typically not expectationally stable.

provided by Hommes (2006) and LeBaron (2006). But these models for the most part do not deal with the leverage dynamics at the heart of the Minsky thesis. One notable exception is Thurner, Farmer and Geanakoplos (2009) which studies the dynamics of asset prices under leverage, with collateral constraints. In their model asset prices crash from time to time because of the same Minsky mechanism; as prices start to decline, the attempt by investors to protect themselves by limiting leverage works to exacerbate the decline. Their paper is focused on explaining fat tails and clustered volatility in an institutionally rich partial equilibrium context. The focus of the present paper is instead on providing a parsimonious explanation of recurrent financial crises in a general equilibrium context without having to invoke heterogeneity or collateral constraints.

The paper is also related to the literature on learning in financial markets (recently surveyed by Pastor and Veronesi (2009a). This literature does not focus on leverage dynamics, but some of the papers in the literature relate asset bubbles to learning. Thus Scheinkman and Xiong (2003) show that a variation of the Harrison-Kreps model of heterogeneous beliefs, Bayesian updating and short-sale constraints can account for the dynamics of asset bubbles. Donaldson and Kamstra (1996) show that a model in which expectations are based on a neural net model can rationalize the apparent bubble in US asset prices in 1929. Pastor and Veronesi (2009b) provide an elegant account of stock-market dynamics at the time of technological revolutions, without any of the irrationality invoked in the above-cited contribution of Perez (2002). Nabar (2007) also provides a Bayesian learning argument for asset price runups during a technological revolution. None of these papers deals with debt dynamics and bankruptcy.

Another literature related to the present work is that on "sudden stops" of capital inflow to small open economies (see Calvo, 1998; Calvo et al., 2006). Indeed the most transparent interpretation of the model to be developed in this paper is that of a small open economy, in which the representative lender is the rest of the world. Under this interpretation the buildup of credit before the crash is a capital inflow. Moreover, as we shall see in the prototype described below, the model does produce a large drop, indeed reversal, of this inflow in the periods just prior to a crisis, although it does not appear to be particularly sudden. Perhaps the main difference between my model and the sudden stops that have characterized recent financial history in Latin America and South Asia is that in the model the contraction of credit results mainly from the dynamics of expectations formed by domestic investors, whereas in recent episodes these stops seem to have been forced by the rest of the world. Theoretical models of sudden stops have invoked various factors not needed below, such as multiple equilibria (Martin and Rey, 2006) and collateral constraints (Mendoza, 2010).

There is also a growing literature (for example, Christiano et al, 2014 or Jermann and

Quadrini, 2012) on volatility in closed economies based on collateral constraints that were originally modelled by Bernanke and Gertler (1989), Bernanke et al. (1999) and Kiyotaki and Moore (1997). The paper on leverage cycles by Fostel and Geanakoplos (2008) is also based on collateral constraints.

2 A simple model with no lender learning

There are two goods: an asset, whose fixed supply is normalized to $\overline{X} = 1$, produces a non-storeable consumption good. A risk-neutral representative lender, with cost of capital r_t at date $t-1$, has an unlimited potential supply of the consumption good to lend. (Alternatively we can regard this as a model of a small open economy facing a world interest rate r_t.) All loans have a term of one period.

There is also a representative investor/borrower, whose intertemporal utility depends logarithmically on consumption:

$$\sum_{0}^{\infty} (1+\rho)^{-t} u(C_t) = \sum_{0}^{\infty} (1+\rho)^{-t} \ln(C_t), \qquad \rho > 0$$

The lender cannot acquire the asset. (Either markets are segmented, as in Guvenen (2009), or foreigners cannot distinguish valid from counterfeit units of the asset.) The price P_t clears the asset market each period.

The representative investor's budget constraint each period is:

$$P_t X_t + C_t = W_t + D_t$$

where X_t is demand for the asset, D_t is demand for loans and W_t is the investor's net wealth, defined as

$$W_t = (P_t + R_t) X_{t-1} - (1 + r_t) D_{t-1} \tag{1}$$

where R_t is the real output per unit of the asset (the "dividend"), an *iid* random variable with a two-point distribution. Thus the state of the world s_t is either H or L, with

$$prob\{s_t = H\} = p, \quad constant,$$

and the random dividend is

$$R_t = \left\{ \begin{array}{ll} R_H^* & \text{if } s_t = H \\ R_L^* & \text{if } s_t = L \end{array} \right\}, R_H^* > R_L^*$$

The investor also faces a solvency constraint:

$$W_t \geq 0.$$

An investor who violates this constraint must sell all assets (for consumption goods) and surrender the proceeds to the lender.

Each period the investor starts with $X_{t-1} = 1$ and maximizes expected utility by choosing

$$C_t = \gamma W_t \tag{2}$$

where $\gamma = \rho / (1 + \rho)$. Thus the demands for assets and loans must satisfy

$$P_t X_t = (1 - \gamma) W_t + D_t \tag{3}$$

The portfolio allocation problem that determines the asset-loan mix must satisfy the standard arbitrage condition:

$$E_t \left\{ u' (C_{t+1}) \left(\frac{P_{t+1} + R_{t+1}}{P_t} - (1 + r_{t+1}) \right) \right\} = 0$$

where E_t denotes the expectation formed at date t. Using (1) and (2) we can rewrite this as

$$E_t \left\{ \left(\frac{1}{\gamma} \right) \left(\frac{P_{t+1} + R_{t+1} - (1 + r_{t+1}) P_t}{P_{t+1} + R_{t+1} - (1 + r_{t+1}) D_t} \right) \right\} = 0 \tag{4}$$

(Equation (4) shows that, as usual in the log utility case, the optimal choice of debt level maximizes the expected log of wealth next period, independently of future interest expectations concerning P, R or r.)

The equilibrium condition $X_t = 1$ together with (1) ~(3) imply that in equilibrium:

$$C_t = R_t + D_t - (1 + r_t) D_{t-1} \tag{5}$$

and

$$P_t = (1/\rho) C_t + D_t \tag{6}$$

Equation (5) is the current account identity in an open economy.

For the rest of this section I make the simplifying assumption that r_t is a constant $r > 0$.

2.1 Rational expectations

To simplify this basic model I assume that the investor much choose the debt level D_t before learning the state of the world s_t. Accordingly I focus on a rational expectations equilibrium (REE) defined as a constant level of debt D^* and equilibrium prices P_H^*, P_L^* for the two states of the world such that

$$P_s^* = (1/\rho)\left(R_s^* + (\rho - r)\, D^*\right), \;\; s = H, L \tag{7}$$

and

$$E\left\{\frac{P_{s'}^* + R_{s'}^* - (1+r)\, P_s^*}{P_{s'}^* + R_{s'}^* - (1+r)\, D^*}\right\} = 0 \tag{8}$$

where E is the unconditional expectation over the independent states s and $s^{'}$. Substituting from (7) into (8) we can redefine an REE as a debt level D^* that satisfies the reduced-form arbitrage condition:

$$E\left\{\frac{R_s^* - rD^*}{R_{s'}^* - rD^*}\right\} = \frac{1+\rho}{1+r} \tag{9}$$

Since the expected ratio of two positive[2] nondegenerate iid random variables is always greater than unity, a necessary condition for existence of an REE is that the world rate of interest be less than the rate of time preference:

$$r < \rho \tag{10}$$

It is straightforward to show that (10) is also sufficient for the existence of a unique REE. Moreover, letting σ denote the standard deviation of R_t, we have:

Proposition 1 *Given the existence condition (10), if σ is small enough, or if ρ or $\overline{R}$ is large enough, the unique REE has a positive debt level ($D^* > 0$).*

2.2 Expectational Stability

Now suppose that people are not endowed with rational expectations. Each investor's problem at t is to find a value of D_t that solves the arbitrage equation (4), in which (P_t, P_{t+1}) is governed by some unknown stochastic process. Suppose they act adaptively, using the procedure:

$$D_t - D_{t-1} = (\alpha_t/\gamma)\left\{\frac{P_{t-1} + R_{t-1} - (1+r)\, P_{t-2}}{P_{t-1} + R_{t-1} - (1+r)\, D_{t-2}}\right\} \tag{11}$$

[2] According to (5), $R_s^* - rD^*$ is equilibrium consumption in state s, and hence must be positive.

where $\{\alpha_t\}$ is a sequence of "gains". According to (11), people go further into debt when the last observed ex post marginal payoff from debt is positive, and reduce their debt when the last observed ex post marginal payoff is negative. I consider two alternatives: (a) the "decreasing gain" version specifies $\alpha_t = \alpha/t$ for some positive constant α, while (b) the "constant gain" version has $\alpha_t = \alpha$.

The algorithm specified by (11) is a special case of the Robbins-Munro algorithm which forms the basis of the engineering literature on stochastic approximation. Sargent (1993) shows that least squares learning and many other adaptive mechanisms employed in economics are also special cases of Robbins-Munro. The algorithm is also a more direct way for the agent to be estimating risk than the more usual procedure of approximating utility by a quadratic function and then separately forecasting mean return and variance, since all that matters to the investor is her forecast of the LHS of (4).

Using equations (5) and (6) we can re-express (11) as

$$D_t - D_{t-1} = \frac{\alpha_t}{\gamma}\left\{1 - \frac{1+r}{1+\rho}\frac{R_{t-2} + D_{t-2} - (1+r)\,D_{t-3}}{R_{t-1} + D_{t-1} - (1+r)\,D_{t-2}}\right\} \tag{12}$$

The asymptotic properties of the decreasing gain version of the stochastic difference equation (12) are determined by those of the corresponding ordinary differential equation whose right hand side is the asymptotic expectation of that of (12) for a constant value of D:

$$\dot{D} = \frac{\alpha}{\gamma}E\left\{1 - \frac{1+r}{1+\rho}\frac{R_{t-2} - rD}{R_{t-1} - rD}\right\} \tag{13}$$

Under the existence condition (10), there is a unique rest point to (13). Comparison of (13) with the equilibrium condition (9) shows that this rest point is the restricted perception equilibrium D^*. Moreover, the serial independence of R_t implies that the right hand side of (13) is strictly decreasing in D in a neighborhood of D^*, which implies that D^* is locally asymptotically stable under the ODE. Thus our adaptive scheme satisfies the usual definition of expectational stability.

More specifically, it can be shown using standard methods (see Evans and Honkapohja, 2001; Kushner and Yin, 2003; or Ljung and Soderstrom, 1983) that if (12) is augmented by a "projection facility" which returns D_t to a certain neighborhood of D^* whenever (12) would have have taken it out of that neighborhood, then, under the decreasing gain version of our procedure, the economy will converge to the unique REE in the long run: $D_t \to D^*$ with probability one as $t \to \infty$.

2.3 Real time dynamics and financial crises

In a rational expectations equilibrium, asset prices are iid and the representative investor is always solvent, because her net worth:

$$W_t = (1/\gamma)\,(R_t - rD^*)$$

is always strictly positive. Thus our expectational stability result implies that even without rational expectations the model will not exhibit bubbles or crashes in the long run, provided that people use the decreasing gain version of the adaptive algorithm and are endowed with a projection facility.

However as the literature on stochastic approximation makes clear, a constant gain algorithm is generally needed to avoid an overly sluggish response to unperceived structural changes in the unknown probabilities. Accordingly we now suppose that D_t is governed by the stochastic difference equation (12) with a constant α_t and without any projection facility.

Under these assumptions it is possible for the representative investor's net worth to become negative, because of the positive feedback involved in deleverage. More specifically, when the economy is not in an REE, her net worth is

$$W_t = P_t + R_t - (1+r)\,D_{t-1}$$

So whenever last period's debt D_{t-1} is greater than $R_t/\,(1+r)$, the representative investor is at risk of insolvency if the asset price P_t falls by enough. Moreover, if the most recently observed ex post marginal payoff to debt (the RHS of (11) is negative then the investor will choose $D_t < D_{t-1}$, and this will indeed drive down P_t, which according to (5) and (6) is given by

$$P_t = (1/\rho)\,(R_t - (1+r)\,D_{t-1}) + (1/\gamma)\,D_t$$

Specifically, according to the last two equations, each unit reduction in debt will reduce current net worth by more than one unit, since $\gamma < 1$. Thus a fall in yield which reduces the ex post marginal payoff to debt can start a cumulative process of deleverage that ends up bankrupting the representative investor. Each unit reduction in debt reduces current net worth Nor is there anything that the representative investor can do about this at time t, since each investor is assumed to take P_t as given.

In a financial crisis the representative lender is not just illiquid but insolvent. However the country as a whole is not necessarily insolvent, because the expected present value of future dividends ER/r may be greater than the debt level D_t. The previous two equations imply that bankruptcy will occur whenever the representative investor chooses D_t less than

$R_t - (1 + r) D_{t-1}$, which can be arbitrarily small. Indeed in the numerical results below D_t is never close to ER/r when a crisis occurs. Thus from a macroeconomic perspective a crisis is a liquidity problem rather than an insolvency problem.

This raises the question of why the solvency condition is enforced in cases where D_t is small. One tentative answer is that the condition arises from bankruptcy law, which requires an insolvent debtor to go bankrupt. Of course this is only a partial answer because in principle the lender could still provide debtor-in-possession financing, which I am implicitly ruling out. So a deeper answer must be that the investor would prefer to take advantage of bankruptcy protection rather than continuing to go further into debt. If bankruptcy law allowed the investor to escape all prior obligations by surrendering everything except some minimum basic consumption $C_{\min}$, then she would clearly prefer this option if $C_{\min}$ were large enough. Alternatively we could think of the representative investor/borrower as a financial institution subject to mark-to-market capital requirements which prohibit borrowing when equity falls too low.

2.4 Numerical results

Simulation of the model under constant gain shows that a financial crisis, resulting in $W_t < 0$ does indeed happen under various conditions. In this section I report results from simulations with $\rho = 0.08$, $(R_H^*, R_L^*) = (1.4, 0.6)$, $p = .05$ and $\alpha = 0.019$. For each interest rate r in Table 1 I ran 10,000 simulations. Each simulation started the economy off with a debt level $D_t = 0$ for $t = 1, 2, 3$, and then allowed D_t to evolve according to the difference equation (12) for 5,000 periods or until a financial crisis occurred.

Column (2) of Table 1 indicates the fraction (*freq*) of simulations in which a crisis occurs. It shows that crises are more likely when credit is easier. For low enough interest rates a crisis always occurs, whereas for high enough rates it never occurs. The interest rates for which a crisis never occurs are also the only ones in the table for which there is no domestic borrowing at all in the rational expectations equilibrium; that is, for which $D^* < 0$.

Almost all crises take place shortly after two consecutive periods of high returns ($R_t = R_H^*$), which is the closest we can get in this model to a "technology boom". Column (3) indicates the fraction (*boom*) of crises that occurred no more than 5 periods after a technology boom. As suggested by Perez (2002), this fraction was never less than .97. Given that technology booms occur on average only once every 400 periods ($p = .05$) this result would basically never occur if crises were unrelated to booms.

TABLE 1

Simulation results

(1) r	(2) $freq$	(3) $boom$	(4) $dP1$	(5) $dP2$	(6) $dD1$	(7) $dD2$
0.005	1.00	0.98	1.94	−3.84	1.09	−9.36
0.01	1.00	0.97	1.92	−3.63	1.07	−8.27
0.015	1.00	0.98	1.92	−3.56	1.07	−8.02
0.02	1.00	0.98	1.93	−3.50	1.06	−8.15
0.025	0.98	0.98	1.95	−3.45	1.07	−8.56
0.03	0.85	0.99	1.98	−3.44	1.09	−9.21
0.035	0.50	0.99	2.03	−3.48	1.12	−9.93
0.04	0.16	0.99	2.09	−3.59	1.12	−10.6
0.045	0.00	–	–	–	–	–
0.05	0.00	–	–	–	–	–

The simulations also show that crises are typically preceded by a runup and then a decline in asset prices. The runup is almost always over the same two periods as the technology boom. For each simulation in which a crisis took place within 5 years of a technology boom I computed the per period change in the asset price during the boom, minus the average per period change over the entire simulation, normalized by the standard deviation of the change over the entire simulation. Column (4) indicates the average ($dP1$) of these normalized price changes across all simulations. It shows that the per period price change was typically about two standard deviations above average during the two periods of the boom preceding the crisis. Column (5) indicates the average ($dP2$) normalized price change per period between the boom and the subsequent crisis. Typically asset prices fell at a rate that was three and a half standard deviations below normal in the periods immediately preceding the crisis. These results accord roughly with the above-mentioned stylized facts documented by Reinhart and Rogoff (2009).

The crisis was also typically preceded by a runup and subsequent decline in credit. Column (6) of Table 1 indicates the normalized change in credit during the two periods starting with the second period of the preceding technology boom. Typically, credit expanded at a rate that was about one standard deviation higher than average during this period. This again accords roughly with the Reinhart-Rogoff facts, although the credit expansion seems rather small, and comes a period after the asset price runup.

The last column of Table 1 shows that credit collapsed during the periods just prior to the crisis, starting with the end of the credit runup. It indicates that over these periods the per period change in credit was 8 to 11 standard deviations below average. Under the open economy interpretation of the model this means a dramatic "sudden stop;" that is, a huge reversal of capital inflows.

Regulatory debt ceilings can reduce or even eliminate the likelihood of crises. I redid the simulations for $r = 0.035$ with the added the restriction that the investor cannot borrow more than some fraction $mxFac$ of the rational-expectations equilibrium debt level D^*. Table 2 shows that the more severe this restriction the less likely was a crisis. If $mxFac$ is 0.3 or lower then crises never occur. Interestingly, setting the debt ceiling equal to the rational expectations equilibrium level actually *in*creased the frequency of crises. In all cases the ceiling was far below the conservative level R_L^*/r that would guarantee the country's solvency if all assets were freely tradeable. Imposing this conservative limit had no effect on the simulation results.

TABLE 2

Results when debt is restricted

$mxFac$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
freq	0.00	0.00	0.00	0.04	0.07	0.19	0.34	0.42	0.45	0.53

Even more effective than a ceiling on debt is a ceiling on debt-reduction. In particular, when we require that the investor always borrow enough that she can repay her pre-existing debt and have some minimal consumption even if she experiences a low return:

$$D_t \geq R_L^* - (1 + r)\, D_{t-1} + 0.01$$

then with $r = 0.035$ the frequency of crises was reduced from 0.50 to 0. Thus, as Calvo (1998) has argued, it is not deficits that cause crises; rather it is the sudden cessation of deficits. Controlling leverage can reduce the frequency of crises but only because that is an indirect way of controlling *de*leverage.

Another key factor is the gain parameter α. When $r = 0.035$,raising α above 0.025 results in a crisis every time whereas lowering α below 0.014 eliminates all crises. This suggests that more insight is to be gained from exploring the endogenous determination of α, an issue to be discussed below in the concluding section.

3 Learning by lenders

Until now I have supposed that lenders are willing to lend unlimited amounts at the exogenous interest rate r with no thought to the prospect of default. This section addresses the issue by allowing lenders to learn as well as borrowers.

As a starting point, suppose that in the event of default by all investors the government seizes all assets (which have no value to the lenders) and lenders are paid the current return R_t minus a small amount $C_{\min}$ which the representative investor is allowed to consume. Next period the government redistributes the assets to all investors on a lump-sum basis. The one period debt of last period is now cancelled and investors are free to resume their activities.

When this happens, the ex post return factor on loans issued the period before the crisis will be

$$\frac{R_t - C_{\min}}{D_{t-1}} < 1 + r_t$$

where we now allow that contractual interest rate r_t to vary with time. More generally, the ex post return will be $\lambda_t (1 + r_t)$ where λ_t is the fraction of interest plus principle $(1 + r_t) D_{t-1}$ that the investor repays, which equals unity whenever a crisis does not occur. Thus the lender's ex ante expected return factor is $\lambda_t^e (1 + r_t)$ where λ_t^e is his expectation of λ_t. In order for the risk-neutral lender to be in equilibrium this expected return factor must equal $(1 + r^*)$ where r^* is his rate of time preference. So the equilibrium interest rate must satisfy the equilibrium condition:

$$1 + r_t = \frac{1 + r^*}{\lambda_t^e}$$

Suppose that in each non-crisis period lenders also form their expectations adaptively, using a constant gain estimator:

$$\lambda_t^e = \beta \lambda_{t-1} + (1 - \beta) \left(\lambda_{t-1}^e - \lambda_{t-1} \right)$$

where $0 < \beta < 1$, but when a crisis occurs they act as if their gain parameter β is unity, by setting $\lambda_t^e = \frac{R_t - C_{\min}}{(1+r) D_{t-1}}$. Then whenever a crisis occurs the interest rate will shoot up, and will gradually return towards r^* as memory of the crisis recedes, in accordance with Minsky's financial fragility hypothesis.

I simulated this revised model under the simplifying assumption that investors cannot lend (which by itself makes little difference to the preceding results) and assuming that lenders start off believing that the fraction of debt to be repaid is unity. Instead of ending each simulation with a crisis, the simulation this time continues, but with expectations and debt levels that have been altered by the crisis. Generally speaking the results were similar to before, although as expected the extra caution by lenders reduces the frequency of crises. Figures 1 and 2 below shows what happens in a typical simulation of 50,000 periods, during which there were 6 crises. Figure 1 shows that the interest rate spiked immediately following each crisis and then gradually returned to normal. Figure 2 shows that debt builds up only slowly after each crisis, before collapsing with the next crisis.

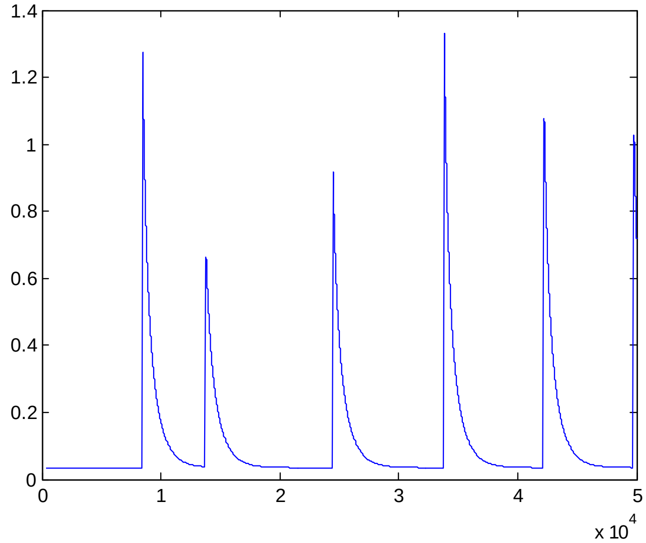

FIGURE 1
Behavior of interest rate with lender learning

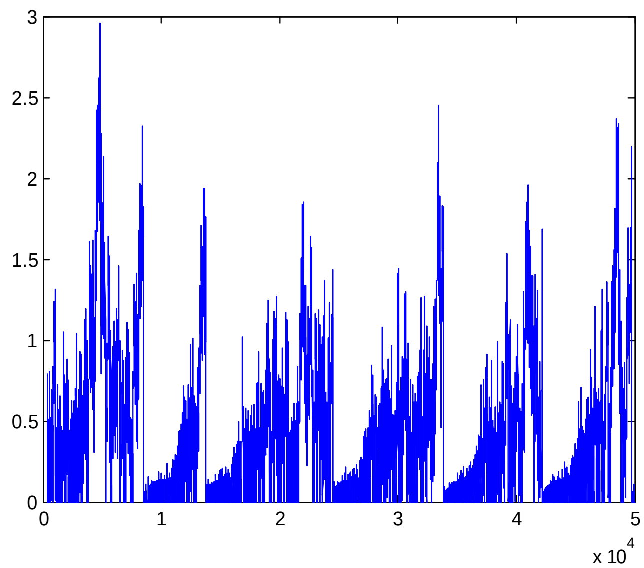

FIGURE 2
Behavior of debt with lender learning

4 Conclusion

The work of Reinhart, Rogoff and others has shown that financial crises emerge from a process that involves first a period of rising asset prices, increasingly optimistic expectations, easy credit and a buildup of leverage, and subsequently a period of falling asset prices, disappointed expectations and rapid deleveraging. The model used above, although quite special, captures these common features in an otherwise standard framework that has been kept as simple as possible. Specifically, the model is a small-open-economy version of a Lucas tree model, in which people learn to optimize their debt levels using the same Robbins-Munro algorithm that underlies most contributions to the recent literature on macroeconomic learning.

The model supports Minsky's financial fragility hypothesis; increased complacency by lenders makes crises more likely to occur the longer it has been since the last crisis. It also supports Perez's view to the effect that the initial force that starts in motion the process leading to a crisis is a technological shift that justifies the initial rise in asset prices and the initial rise in borrowing; specifically, almost none of the crises produced by the model occur more than 5 years after the most recent "technology boom," where the latter is defined in such a way that it occurs on average only once in 400 years.

Finally, the model also supports Calvo's view to the effect that crises are precipitated by sudden stops of credit inflows. Indeed, one policy implication suggested by the model is that crises are averted more effectively by restrictions on deleveraging than restrictions on leveraging.

More work is needed before the model can be used for real world policy analysis. The solvency condition facing the representative investor needs to be derived from a clear micro-foundation. The model needs to allow a crisis to result in reduced output, as indeed happens in reality. The gain parameter governing the critical speed of adaptation of expectations needs to be endogenized. Generality also needs to be added by allowing a broader range of preferences and a richer dividend process. All of these issues are the subject of ongoing research. I conclude with a few brief remarks about the direction of this research.

To deal more systematically with the insolvency constraint, and to allow output to be affected by a crisis, I replace the current assumption concerning non-tradeability of the asset with an assumption that only investors know how to make the asset produce its dividend, so that if the lender buys or repossesses assets they will not yield any dividend. A crisis then results in all of the assets being repossessed by lenders (instead of being seized and then redistributed by the government) and a resulting fall in aggregate output. To prevent output fall falling to zero I need to assume that investors have at least some positive endowment income beyond their dividends. Lenders now have to form expectations not only about the probability of being repaid but also the expected percentage rate of price increase of the asset, because once this expectation exceeds r^*they will be induced by the prospect of capital gain on possessed assets to lend unlimited quantities even to insolvent borrowers, like some US mortgage lenders before the most recent crisis. To complete this line of investigation I need to examine more closely the ex post bargain that an investor and lender face when bankruptcy occurs.

To endogenize the gain parameter α, I am pursuing an adaptive approach in which the investor keeps track of how well she would have done had she been using a slightly higher or lower gain for the past few periods, and periodically adjusts α in the estimated direction of improvement. Preliminary results suggest that this kind of adaptation heightens the probability of crisis, because, individually, faster adaptation (higher α) is generally perceived to be beneficial when asset prices have been rising, but collectively, as we have seen, higher α makes crises more likely.

References

Arthur, W. Brian, John H. Holland, Blake LeBaron, Richard Palmer, and Paul Taylor. "Asset Pricing Under Endogenous Expectations in an Artificial Stock Market." In *The Economy as an Evolving Complex System, II*, edited by W. Brian Arthur, Steven N. Durlauf, and David Lane, 15-44. : Addison-Wesley, 1997.

Bernanke, Ben, and Mark Gertler. "Agency Costs, Net Worth, and Business Fluctuations." *American Economic Review* 79 (March 1989): 14-31.

Bernanke, Ben, Mark Gertler, and Simon Gilchrist. "The Financial Accelerator in a Quantitative Business Cycle Framework." In *Handbook of Macroeconomics*, edited by John Taylor and Michael Woodford. Vol. 1C. Amsterdam: North-Holland, 1999.

Bresnahan, Timothy F., and Manuel Trajtenberg. "General Purpose Technologies: "Engines of Growth?"." *Journal of Econometrics* 65 (January 1995): 83-108.

Brock, William A., and Cars H. Hommes. "Heterogeneous Beliefs and Routes to Chaos in a Simple Asset Pricing Model." *Journal of Economic Dynamics and Control* 22 (August 1998): 1235-74.

Calvo, Guillermo A. "Capital Flows and Capital Market Crises: The Simple Economics of Sudden Stops." *Journal of Applied Economics* 1 (November 1998): 35-54.

Calvo, Guillermo A., Alejandro Izquierdo, and Rudy Loo-Kung. "Relative Price Volatility under Sudden Stops: The Relevance of Balance Sheet Effects." *Journal of International Economics* 69 (2006): 231-54.

Christiano, Lawrence J., Roberto Motto, and Massimo Rostagno. “Financial Factors in Business Cycles.” *American Economic Review* 104 (January 2014): 27-65.

Donaldson, R. Glen, and Mark Kamstra. “A New Dividend Forecasting Procedure that Rejects Bubbles in Asset Prices: The Case of 1929’s Stock Crash.” *Review of Financial Studies* 9 (Summer 1996): 333-83.

Evans, George, and Seppo Honkapohja. *Learning and Expectations in Macroeconomics.* Princeton, NJ: Princeton University Press, 2001.

Fostel, Ana and John Geanakoplos. “Leverage Cycles and the Anxious Economy.” *American Economic Review* 98 (September 2008): 1211-1244.

Freeman, Christopher, and Carlota Perez. “Structural Crises of Adjustment, Business Cycles and Investment Behaviour.” In *Technical Change and Economic Theory*, edited by Giovanni Dosi, Christopher Freeman, Richard Nelson, Gerald Silverberg, and Luc Soete, 38-66. London: Pinter Publishers, 1988.

Guvenen, Fatih. “A Parsimonious Macroeconomic Model for Asset Pricing.” *Econometrica* 77 (November 2009): 1711-50.

Hommes, Cars H. “Heterogeneous Agent Models in Economics and Finance.” In *Handbook of Computational Economics*, edited by Kenneth L. Judd and Leigh Tesfatsion. Vol. 2. Amsterdam: North-Holland, 2006.

Jermann, Urban, and Vicenzo Quadrini. “Macroeconomic Effects of Financial Shocks.” *American Economic Review* 102 (February 2012): 238-71.

Kindleberger, Charles P., and Robert Aliber. *Manias, Panics and Crashes: A History of Financial Crises.* 5th ed. Hoboken, NJ: Wiley, 2005.

Kiyotaki, Nobuhiro, and John Moore. “Credit Cycles.” *Journal of Political Economy* 105 (April 1997): 211-48.

Kushner, H. J., and G. Yin. *Stochastic Approximation and Recursive Algorithms and Applications.* New York: Springer, 2003.

LeBaron, Blake. “Evolution and Time Horizons in an Agent Based Stock Market.” *Macroeconomic Dynamics* 5 (2001): 225-54.

LeBaron, Blake. “Agent-based Computational Finance.” In *Handbook of Computational Economics*, edited by Kenneth L. Judd and Leigh Tesfatsion. Vol. 2. Amsterdam: North-Holland, 2006.

Leijonhufvud, Axel. “Out of the Corridor: Keynes and the Crisis.” *Cambridge Journal of Economics* 33 (July 2009): 741-57.

Ljung, L., and T. Soderstrom. *Theory and Practice of Recursive Identification.* Cambridge, MA: MIT Press, 1983.

Lucas, Robert E. Jr. “Asset Prices in an Exchange Economy.” *Econometrica* 46 (November 1978): 1429-45.

Martin, Philippe, and Helene Rey. “Globalization and Emerging Markets: With or Without Crash?” *American Economic Review* 96 (December 2006): 1631- 51.

Mendoza, Enrique G. “Financial Crises, Sudden Stops and Leverage.” *American Economic Review* 100 (December 2010): 1941-66.

Minsky, Hyman P. “The Financial Instability Hypothesis: Capitalistic Processes and the Behavior of the Economy.” In *Financial Crises: Theory, History and Policy*, edited by Charles P. Kindleberger and J.-P. Laffargue, 13-29. Cambridge, MA: Cambridge University Press, 1982.

Nabar, Malhar. “Information, Investor Expectations, and Productivity Growth.” Unpublished, Wellesley College 2007.

Pastor, Lubos, and Pietro Veronesi. “Learning in Financial Markets.” *Annual Review of Financial Economics* 1 (2009a): 361-81.

Pastor, Lubos, and Pietro Veronesi. “Technological Revolutions and Stock Prices.” *American Economic Review* 99 (September 2009b): 1451-83.

Perez, Carlota. *Technological Revolutions and Financial Capital: The Dynamics of Bubbles and Golden Ages.* Cheltenham, UK: Edward Elgar, 2002.

Reinhart, Carmen M., and Kenneth S. Rogoff. *This Time is Different: Eight Centuries of Financial Folly.* Princeton: Princeton University Press, 2009.

Sargent, Thomas J. *Bounded Rationality in Macroeconomics.* Oxford: Clarendon Press, 1993.

Sargent, Thomas J. *The Conquest of American Inflation.* Princeton: Princeton University Press, 1999.

Scheinkman, José A., and Wei Xiong. “Overconfidence and Speculative Bubbles.” *Journal of Political Economy* 111 (December 2003): 1183- 1219.

Thurner, Stefan, Doyne Farmer, and John Geanakoplos. “Leverage Causes Fat Tails and Clustered Volatility.” arXiv:0908.1555v1 [q-fin.ST] 11 Aug 2009.