EN221: HW #10, Due Wednesday, 12/3.

1. Problem 8, Page 162, Chadwick. (The constititutive equation for a compressible Newtonian fluid is given by Eq.(35), page 144, Chadwick. Also, $\sigma^E = (\kappa - \frac{2}{3}\mu)tr(\mathbf{D})\mathbf{I} + 2\mu\mathbf{D}$ is the stress in excess of the pressure. For part(ii), note that the kinetic energy decreases at the rate $\int_B div(\sigma)\cdot\mathbf{v}dv$. You can use the following vector identity that is satisfied by the incompressible velocity field: $\nabla\times\nabla\times\mathbf{v} = -\nabla^2\mathbf{v}$.)

2. Problem 9, Page 162, Chadwick.

3. Problem 12, Page 164, Chadwick.

4. (a) If I_1, I_2 and I_3 are principal invariants of the left stretch tensor $\mathbf{V}$, prove the following identities:

$$\frac{\partial I_1}{\partial \mathbf{V}} = \mathbf{I}, \qquad \frac{\partial I_2}{\partial \mathbf{V}} = \mathbf{I}\mathrm{tr}(\mathbf{V}) - \mathbf{V}, \qquad \frac{\partial I_3}{\partial \mathbf{V}} = \det(\mathbf{V})\mathbf{V}^{-1}. \tag{1}$$

   (b) In class we showed that the Cauchy stress can be expressed as

$$\mathbf{T} = \phi_0\mathbf{I} + \phi_1\mathbf{V} + \phi_2\mathbf{V^2}, \tag{2}$$

   where ϕ_i are functions of the principal invariants of $\mathbf{V}$. Show that the Cauchy stress can also be expressed as

$$\mathbf{T} = \psi_0\mathbf{I} + \psi_1\mathbf{V^2} + \psi_2\mathbf{V^4}, \tag{3}$$

   and derive expressions for ψ_is in terms of ϕ_is and the principal invariants of $\mathbf{V}$. (Hint: Use Cayley-Hamilton theorem)