EN221: HW #11.

This set is not due. Please go through the solutions and make sure that you understand all the steps for solving problems involving non-linear deformations.

1. Consider the simple shear deformation of a material with the constitutive relation

$$W = \sum_{n=1}^{N} \mu_n(\lambda_1^n + \lambda_2^n + \lambda_1^n)/n. \tag{1}$$

   If the deformation gradient is given by

$$\mathbf{F} = \mathbf{I} + \gamma \mathbf{e}_1 \otimes \mathbf{e}_2, \tag{2}$$

   show that the shear stress is given by

$$(\lambda + \lambda^{-1})T_{12} = \sum_{n=1}^{N} \mu_n(\lambda^n - \lambda^{-n}), \tag{3}$$

   where

$$\gamma = \lambda - \lambda^{-1}. \tag{4}$$

2. In class we considered (some time in October) the deformation gradient for a long rubber cylinder (made out of an incompressible material) when it was turned inside out. Let A and B be, respectively, the inner and outer radii of the cylinder before it was turned inside out and let a and b be, respectively, the outer and inner radii after it has been turned inside out. Assume that the constitutive relation for the material is

$$W = \frac{\mu}{2}(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3). \tag{5}$$

   (a) Derive an equation that relates the old radii A and B and the new radius a. You need not solve this equation.

   (b) Obtain the stress components $\sigma_{rr}(r)$, $\sigma_{\theta\theta}(r)$ and $\sigma_{zz}(r)$ for $b \leq r \leq a$. Your answer should only depend on the material parameter μ, the initial radii A, B and the radius a. Note that if a is known from part (a), the stresses only depend on μ,A and B.

3. Read Example 14, page 156, Chadwick and use the constitutive relation obtained here to solve Problem 13, Page 164.

4. Problem 14, Page 164, Chadwick.

5. Problem 15, Page 165, Chadwick.