EN221: HW #5, Due Wednesday, 10/22.

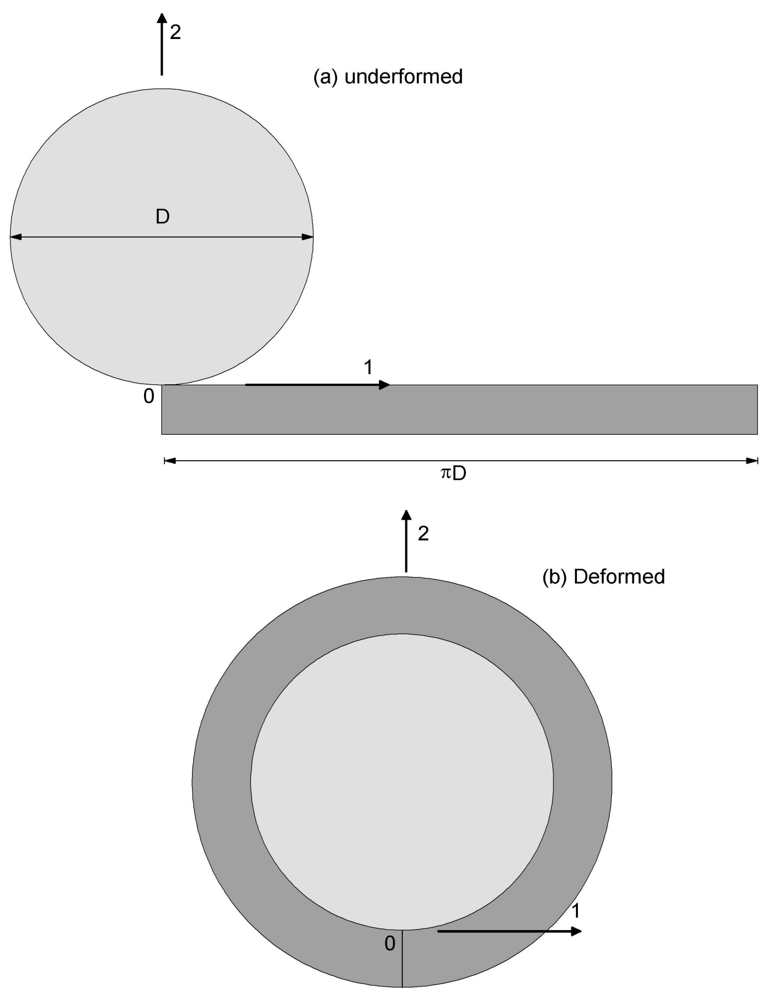
1. For the bending of a rectangular block on page 114 of Ogden,

   (a) Derive the expression for the deformation gradient (second equation on page 115).

   (b) Find the stretch tensors $\mathbf{U}$, $\mathbf{V}$ and the rotation tensor $\mathbf{R}$.

   (c) For isochoric deformations, derive Equation 2.2.80. Disuss why this relation is different from the result for pure bending derived in class.

2. An initially planar sheet of deformable material is to be wrapped uniformly around a non-deformable cylinder as shown in the figure below (like a rubber insulating layer wrapped around a metal wire). Assume that the deformation is plane strain, so that strains involving the 3-component vanish. The initial length of the sheet is equal the circumference of the cylinder, πD, where D is the diameter of the cylinder.



(a) With the choice of coordinate axes and origin shown in the sketch, view the deformation as a compound (2-stage) deformation, $\mathbf{X} \to \mathbf{q} \to \mathbf{x}$. First, impose a *non-uniform* extension of the sheet in the 2-direction given by

$$\mathbf{q} = X_1\mathbf{e}_1 + h(X_2)\mathbf{e}_2 + X_3\mathbf{e}_3, \tag{1}$$

where $h(X_2)$ is a function to be determined, subject to the condition $h(0) = 0$. This deformation is followed by *pure bending*, so that the sheet wraps around the cylinder as shown in part (b) of the figure below. By writing the final deformed coordinates $\mathbf{x}$ in terms of $\mathbf{X}$, calculate the deformation gradient $\mathbf{F}$ and the stretch tensor $\mathbf{U}$. Note that $\mathbf{U}$ should depend on $h(X_2)$ and $h'(X_2)$, where prime denotes differentiation with respect to X_2.

(b) Assume that there is no volume change at any point in the body. From the condition that $J = \det(\mathbf{F}) = 1$, determine the ordinary differential equation that the function $h(X_2)$ must satisfy, and solve that equation.

3. Consider the combined axial and azimuthal shear deformation of a circular tube defined by

$$r = R, \quad \theta = \Theta + \phi(R), \quad z = Z + w(R), \tag{2}$$

where upper(lower) case symbols indicate undeformed(deformed) coordinates.

(a) Calculate the deformation gradient in cylindrical coordinates.

(b) Obtain the principal stretches and verify that the deformation is isochoric (J=1).

4. Problem 2.2.16, Page 116, Ogden.