EN221: HW #8, Due Wednesday, 11/19.

1. (a) Consider a static situation in which a body occupies the region $\mathcal{B}_0$ for all time. Assume that the body force $\mathbf{b} = 0$ and that the body is bounded by surfaces $\mathcal{S}_0$ and $\mathcal{S}_1$ as shown in Fig. (a) below. Assume further that $\mathcal{S}_0$ and $\mathcal{S}_1$ are acted on by uniform pressures π_0 and π_1. Show that the *average* stress in $\mathcal{B}_0$ is a pressure of amount

$$\frac{\pi_1 v_1 - \pi_0 v_0}{v_1 - v_0}, \tag{1}$$

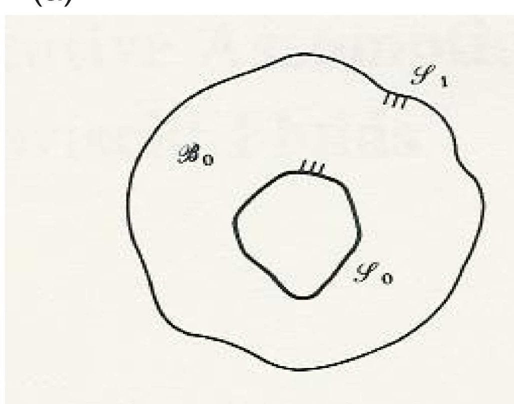
where v_0 and v_1 are, respectively, the volumes enclosed by $\mathcal{S}_0$ and $\mathcal{S}_1$.

(b) Consider a steady, irrotational flow of an ideal fluid of density ρ_0 over an obstacle $\mathcal{R}$, where $\mathcal{R}$ is a bounded regular region whose interior lies outside the flow region $\mathcal{B}_0$ (see Fig. (b)). Assume that the body force is zero. Show that the total force exerted on $\mathcal{R}$ by the fluid is equal to

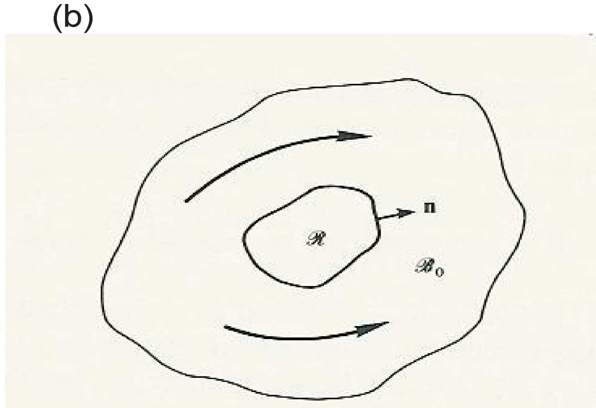
$$\frac{\rho_0}{2}\int_{\partial\mathcal{R}} \mathbf{v}^2 \mathbf{n} dA, \tag{2}$$

where $\partial\mathcal{R}$ is the boundary of $\mathcal{R}$.

(a)



(b)



2. Derive the boundary conditions (26.2c) for the flow problem sketched in Fig. 2.5 of the notes from the OCAIM institute distributed in class.

3. Using the traveling wave solutions (2.63 of the OCAIM notes) derive the dispersion relation (relation between ω and k, 2.64 in the notes) and verify the conditions (2.67) and (2.68) for the *Rayleigh-Taylor* and *Kelvin-Hemholtz* instabilities, respectively.

4. In class we derived the Rayleigh-Plesset equation for bubble dynamics in an incompressible fluid using the cauchy equation of motion (you can find a similar analysis in Chadwick, Problem 7, page 101). Derive this equation using the Bernoulli equation for incompressible fluids given in the class notes. Note that the flow in this case is not steady, but is irrotational.

   References on bubble dynamics and solutions of the Rayleigh-Plesset equation:

   Z. C. Feng and L. G. Leal, "Non-linear bubble dynamics", Annual Reviews of Fluid Mechanics, vol. 29, pp 201-243 (1997).

   Bubble Puzzles : http://www.aip.org/pt/vol-56/iss-2/p36.html