EN221: HW #9, Due Thursday, 11/16.

1. Consider the observer transformation discussed in class defined by the relation
$$\mathbf{x}^* = \mathbf{c}(t) + \mathbf{Q}(t)\mathbf{x}, \tag{1}$$
where $\mathbf{Q}$ is an othrogonal tensor.
   (a) Show that
   $$\mathrm{div}_{\mathbf{x}^*}\mathbf{T}^*(\mathbf{x}^*, t) = \mathbf{Q}(t)\mathrm{div}_{\mathbf{x}}\mathbf{T}(\mathbf{x}, t), \tag{2}$$
   where $\mathbf{T}$ is the Cauchy stress.
   (b) Show that the acceleration transforms according to
   $$\dot{\mathbf{v}}^*(\mathbf{x}^*, t) = \mathbf{Q}(t)\dot{\mathbf{v}}(x, t) + \ddot{\mathbf{c}}(t) + 2\dot{\mathbf{Q}}(t)\mathbf{v}(\mathbf{x}, t) + \ddot{\mathbf{Q}}(t)\mathbf{x} \tag{3}$$
   (c) Show further that if the body force transforms according to
   $$\mathbf{b}^*(\mathbf{x}^*, t) = \mathbf{Q}(t)\mathbf{b}(\mathbf{x}, t), \tag{4}$$
   then the equation of motion is given by
   $$\mathrm{div}_{\mathbf{x}^*}\mathbf{T}^* + \rho^*\mathbf{b}^* + \mathbf{k} = \rho^*\dot{\mathbf{v}}^*, \tag{5}$$
   where $\rho^*(\mathbf{x}^*, t) = \rho(\mathbf{x}, t)$ and determine a relation for $\mathbf{k}$.
   (d) Because of the additional term $\mathbf{k}$, the equation of motion is *not invariant* under all changes in observer. Show that $\mathbf{k}$ vanishes for observers for whom $\mathbf{Q}$ and $\dot{\mathbf{c}}$ are constant. Such observers are called *Galilean* and have the property of being accelerationless with respect to the underlying inertial observer.

2. (a) The transformation rule for the Cauchy stress tensor $\mathbf{T}$ for relative motion of observers is
   $$\mathbf{T}^* = \mathbf{Q}\mathbf{T}\mathbf{Q}^T. \tag{6}$$
   Tensors that tranform in this manner are called objective tensors. Is the material rate of change (or material derivative) of Cauchy stress $\dot{\mathbf{T}}$ objective ?, i.e is $\dot{\mathbf{T}}^* = \mathbf{Q}\dot{\mathbf{T}}\mathbf{Q}^T$?
   (b) Show that if the Cauchy stress tensor is objective, then the *Jaumann stress rate* defined by
   $$\breve{\mathbf{T}} = \dot{\mathbf{T}} - \mathbf{W}\mathbf{T} + \mathbf{T}\mathbf{W}, \tag{7}$$
   is objective. Note that $\mathbf{W}$ in the above equation represents the spin tensor, $\frac{1}{2}(\mathbf{L} - \mathbf{L}^T)$. This stress rate is also called the *co-rotational rate* of Cauchy stress. Why is this a reasonable name for this stress rate ?

3. Problem 3, Page 161, Chadwick. (note that objectivity simply means that the constitutive laws should be consistent with the axiom that constitutive laws must be invariant under changes of observer.)

4. Problem 4, Page 161, Chadwick. (Hint: For the constitutive relation to be objective, $\sigma^* = \mathbf{f}(\mathbf{F}^*, \dot{\mathbf{F}}^*) = \mathbf{Q}\sigma\mathbf{Q}^T$, for any $\mathbf{Q}$ in the notation used in class. Use the fact that $\mathbf{F} = \mathbf{R}\mathbf{U}$ to choose an appropriate $\mathbf{Q}$ that will lead to the required result.)