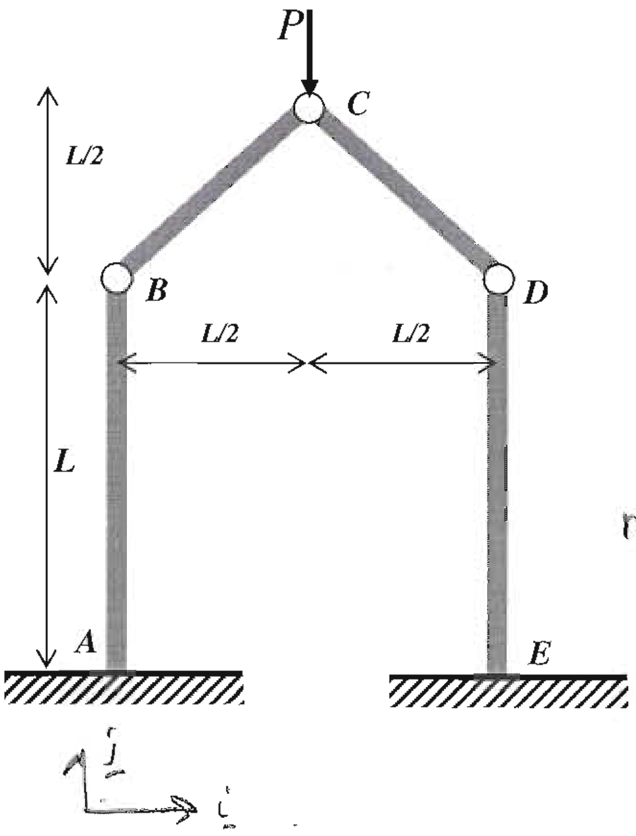
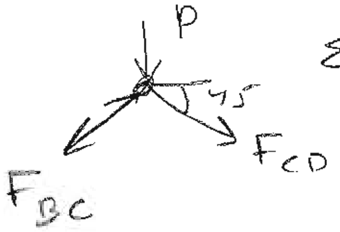
1. The structure shown below is subjected to a load P. Determine the base reactions and the deflection at point C. All members have identical values of E, I, and A. You can neglect deformations due to axial extension.



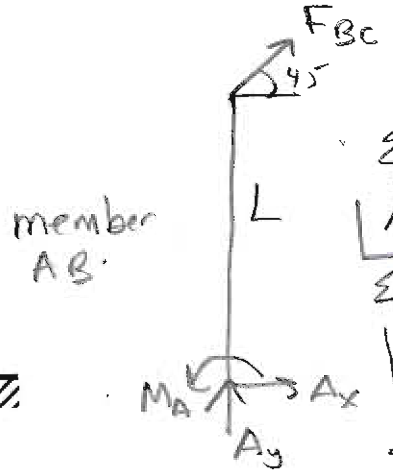


$\Sigma F_y = 0$

$\frac{2F_{CD}}{\sqrt{2}} = -P$

$F_{CD} = -P/\sqrt{2}$

$= F_{BC}$



$\Sigma F_x = 0$

$\boxed{A_x = -F_{BC}/\sqrt{2} = P/2}$

$\Sigma F_y = 0$

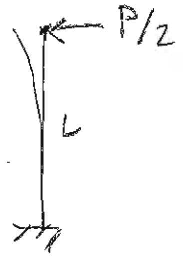
$\boxed{A_y = -F_{BC}/\sqrt{2} = P/2}$

$\Sigma M_A = 0$

$\Rightarrow M_A - F_{BC}/\sqrt{2}\, L = 0$

$\boxed{M_A = -PL/2}$

Deflection @ B (neglect axial)



$u_x^B = -\frac{PL^3}{6EI}$ $\quad u_y^B = 0$

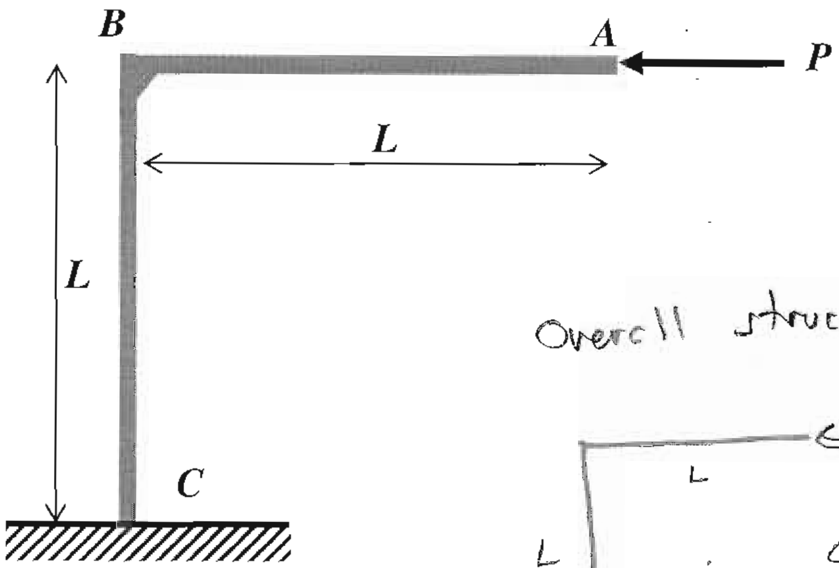
0 due to symmetry

$$\delta_{BC} = \underline{n}^{BC}\cdot(\underline{u}^C - \underline{u}^B) = \frac{1}{\sqrt{2}}(\underline{i}+\underline{j})\cdot[(\cancel{u_x^C} - u_x^B)\underline{i} + (u_y^C - \cancel{u_y^B})\underline{j}] = 0$$

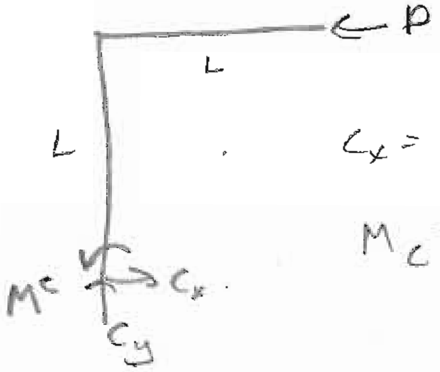
0

$$\frac{1}{\sqrt{2}}(-u_x^B + u_y^C) = 0 \qquad \boxed{u_y^C = u_x^B = \frac{-PL^3}{6EI}}$$

3. The structure is subjected to a lateral load P at point A. Determine the vertical and lateral deflection at point A. Again, both members have identical properties EI and joint B is a full moment connection.

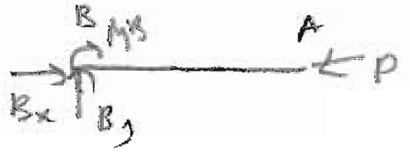


Overall structure

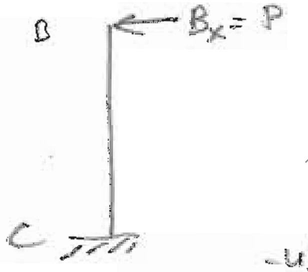


$C_x = P \quad C_y = 0$

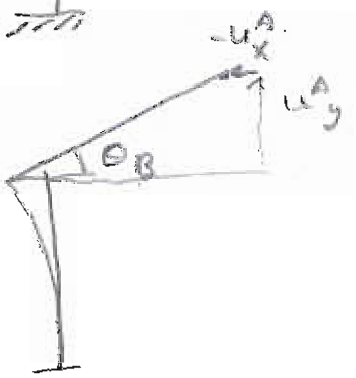
$M_C = -PL$



$B_x = P \quad B_y = M^B = 0$



$u_x^B = -\frac{PL^3}{3EI} \qquad \theta_B = \frac{PL^2}{2EI}$

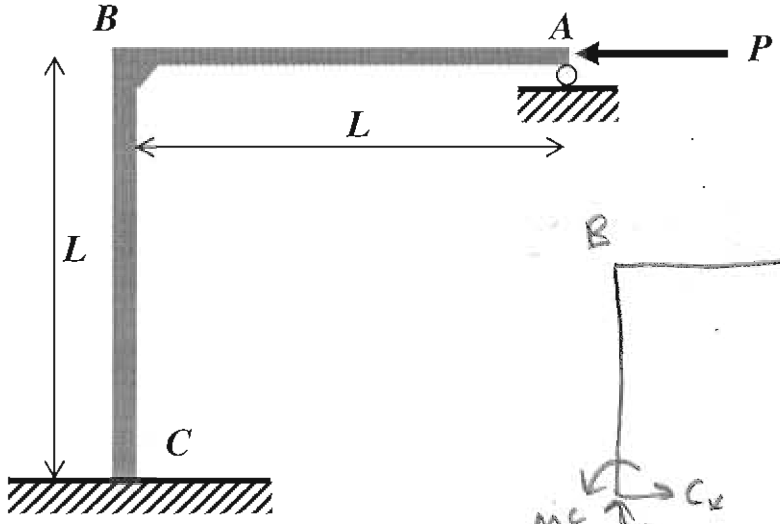


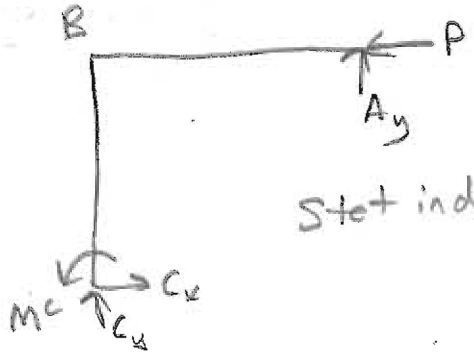
$-u_x^A = -u_x^B$ (small θ_B)

$u_x^A = \frac{PL^3}{3EI}$

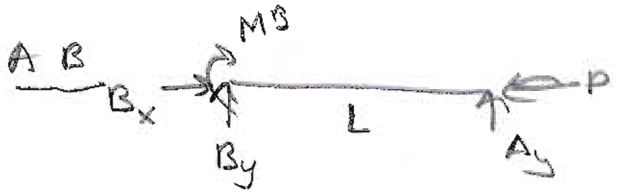
$u_y^A = \theta_B L = \frac{PL^3}{2EI}$

4. A roller support is added at point A. Determine the reactions at point A and C.





Stat indet

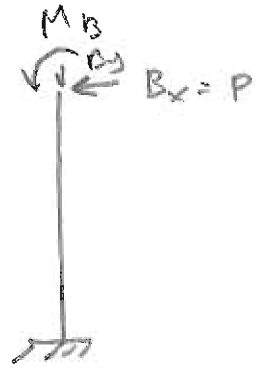


$B_x = P$

$M_B = A_y L \quad B_y = -A_y$

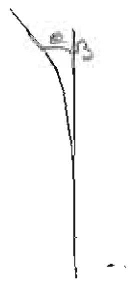
rot. @ B $= \theta_B = -\dfrac{M_B L}{3EI}$

BC



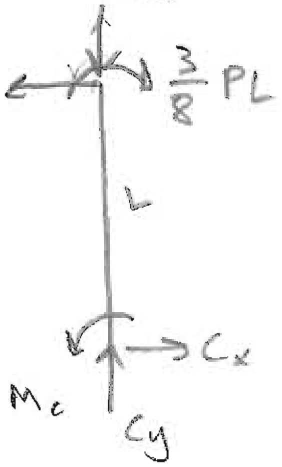
rotation @ B

$$\theta_B = \frac{PL^2}{2EI} + \frac{M_B L}{EI}$$



$$\frac{PL^2}{2EI} + \frac{M_B L}{EI} = -\frac{M_B L}{3EI} \qquad \frac{PL}{2} = -M_B \frac{4}{3}$$

3/8 P

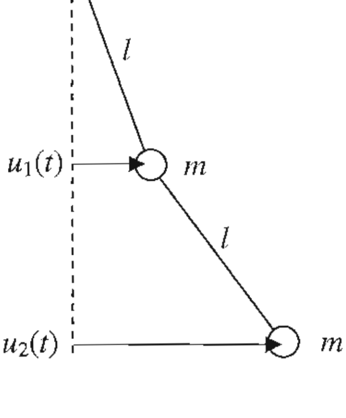


$$\boxed{M_B = \frac{-3}{8}PL}$$

$$\boxed{A_y = -\frac{3}{8}P \quad B_y = \frac{3}{8}P}$$

$C_x = P \quad C_y = \frac{3}{8}P$ $\boxed{M_C = \frac{3}{8}PL - PL = -\frac{5}{8}PL}$

5. A double pendulum is shown below.



The equations of motion are:

$$2m\ddot{u}_1 + m\ddot{u}_2 + 2m(g/l)u_1 = 0$$
$$m\ddot{u}_1 + m\ddot{u}_2 + m(g/l)u_2 = 0$$

a. Put these equations in matrix form..

b. Determine the natural frequencies and mode shapes Sketch the mode shapes. A rough hand-drawn sketch is fine.

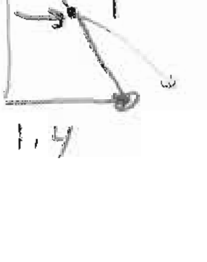
$$\begin{bmatrix} 2m & m \\ m & m \end{bmatrix}\begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{bmatrix} + \begin{bmatrix} 2mg/l & 0 \\ 0 & mg/l \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$M\ddot{\underline{u}} + K\underline{u} = \underline{0}$$

$$M^{-1}K = H = \frac{g}{l}\begin{bmatrix} 2 & -1 \\ -2 & 2 \end{bmatrix}$$

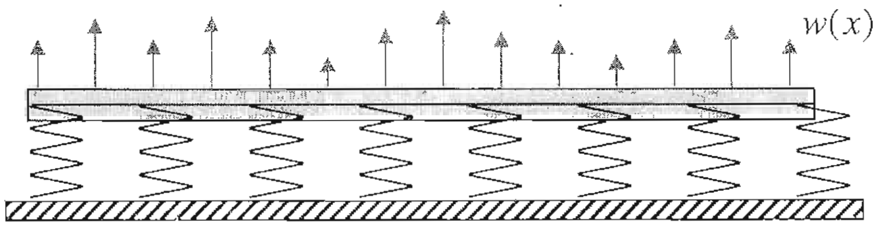
eigenvalues $\omega_1^2 = (2-\sqrt{2})\frac{g}{l}$ $\omega_2^2 = (2+\sqrt{2})\frac{g}{l}$

$\underline{A}_1 = \begin{bmatrix} 1 \\ 1.41 \end{bmatrix}$



$\underline{A}_2 = \begin{bmatrix} 1 \\ -1.41 \end{bmatrix}$

5. A beam on an elastic foundation (eg a footing or roadway on a soil base) can be represented as a beam resting on a distribution of elastic springs with stiffness k per unit length (force/length^2). When the beam deflects with displacement $u(x)$ the springs provide a reaction force distribution $-ku(x)$ (force per unit length).

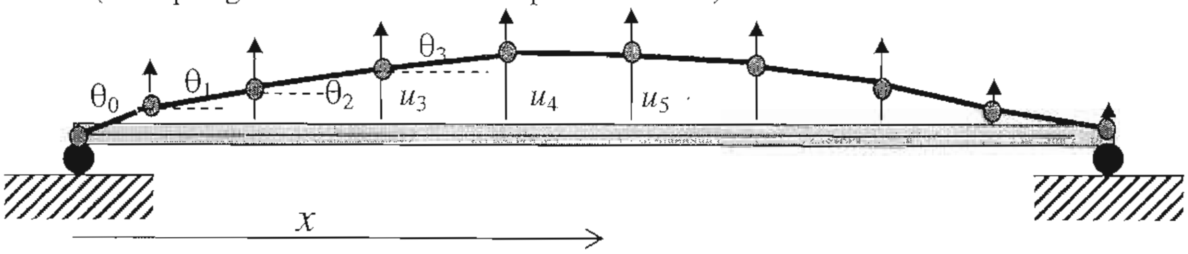


The potential energy of the beam is

$$V = \int_0^{L_0} \left(\frac{1}{2} EI\kappa(x)^2 + \frac{1}{2} ku(x)^2 - w(x)u(x) \right) dx,$$

where u is the upwards deflection and κ is the curvature.

If you were to modify the excel beam bending calculator to handle a simply supported beam on an elastic foundation, you would have to include the contribution due to the elastic base. (The springs are not shown in the picture below.)



As before the beam deflections are determined by the angles θ_i:

$u_0 = 0,\ \ u_1 = s\sin\theta_0,\quad u_2 = u_1 + s\sin\theta_1\ \ ....\ \ u_i = u_{i-1} + s\sin\theta_{i-1}.\quad (*)$

and the curvature at segment i is $\kappa_i=(\theta_i-\theta_{i-1})/s$. ($s=L/N$). The elastic energy due to bending is computed at node i as $V_i = \frac{s}{2} EI\kappa_i^2$. The load at node i is $W_i=w(x_i)s$ (i=1,2,...N-1), $W_0=w(0)s/2$, $W_N=w(L_0)s/2$, and so the associated energy is W_iu_i.

The total energy for the beam is the sum of elastic and load energy:

$$V = \frac{1}{2} EIs \sum_{i=1}^{N} \frac{(\theta_i - \theta_{i-1})^2}{s^2} - \sum_{i=0}^{N} W_i u_i + \text{terms due to the elastic foundation.}$$

a. Specifically, what are the terms that should be included to take into account the elastic foundation? Explain your answer. Be careful to treat the end joints properly.

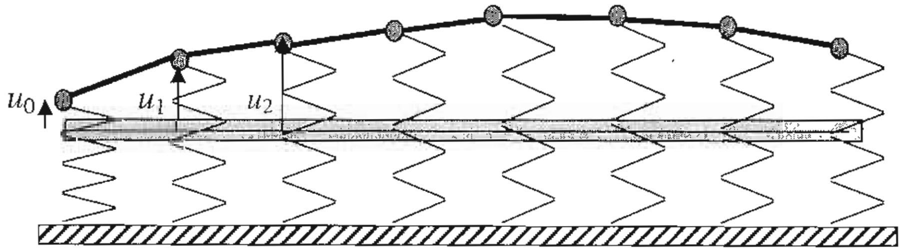
i

$V_i = \frac{1}{2} k u_i^2 s$ inner joint

ends $V_0 = \frac{1}{2} k u_0^2 \, s/2$

$V_N = \frac{1}{2} k u_N^2 \, s/2$] zero for the pinned-pinned beam

b. Suppose you want to calculate the deflection of a beam on an elastic foundation that has **no** additional end supports. The beam rests solely on the elastic foundation. Assuming that you have the proper expression for the potential energy in part a, explain how you would modify equation (*) above and what variables/constraints you would enter into the solver.



$u_0 \neq 0$, & becomes a <u>solver</u> <u>variable</u>

$u_1 = u_0 + s \sin \theta_0$ $u_2 = u_1 + s \sin \theta_1$, etc

No constraints!