Division of Engineering
Brown University

EN130: Structural Analysis

Exam I
Wednesday, March 23, 2005

NAME: ____

General Instructions

- No collaboration of any kind is permitted on this examination.
- You may consult your own written lecture notes and homework solutions during the course of this examination, but no other material. **PRINTED OR XEROXED NOTES ARE NOT PERMITTED**
- Write all your solutions in the space provided. No sheets should be added to the exam.
- Make diagrams and sketches as clear as possible, and show all your derivations clearly. Incomplete solutions will receive only partial credit, even if the answer is correct.
- If you find you are unable to complete part of a question, proceed to the next part.

Please initial the statement below to show that you have read it

`By affixing my name to this paper, I affirm that I have executed the examination in accordance with the Academic Honor Code of Brown University.

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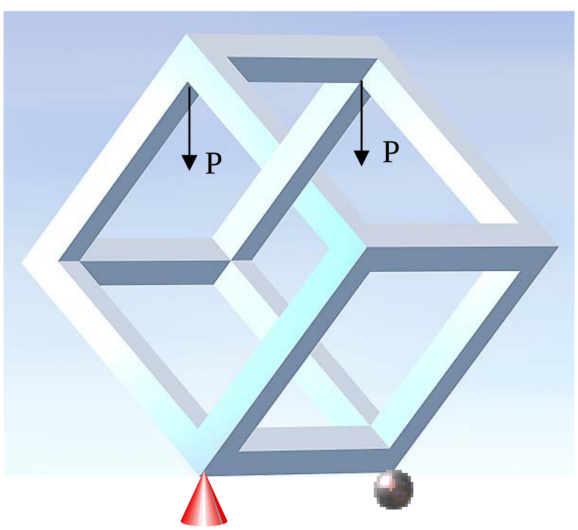


Analyze This!!

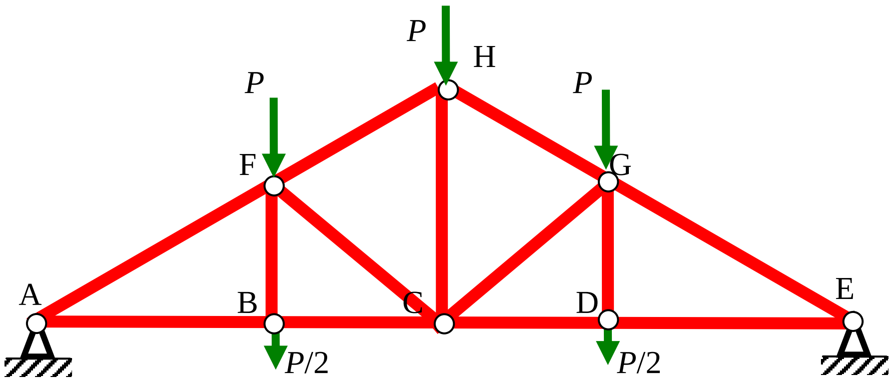
1-8 (16 points) ______________

9 (9 points) ______________

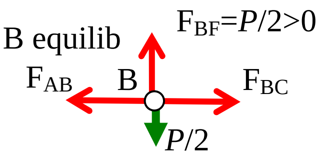
TOTAL (25 points)____________

Multiple Choice/Short Answer Problems. Write your answer to each problem in the space provided. Only the answer will be graded.

Problems 1 and 2 refer to the roof truss shown below. *P*>0.



1. Under the loads shown, member BF:
(a) is in tension
(b) is in compression
(c) is a zero-force member
(d) cannot be determined without further information.



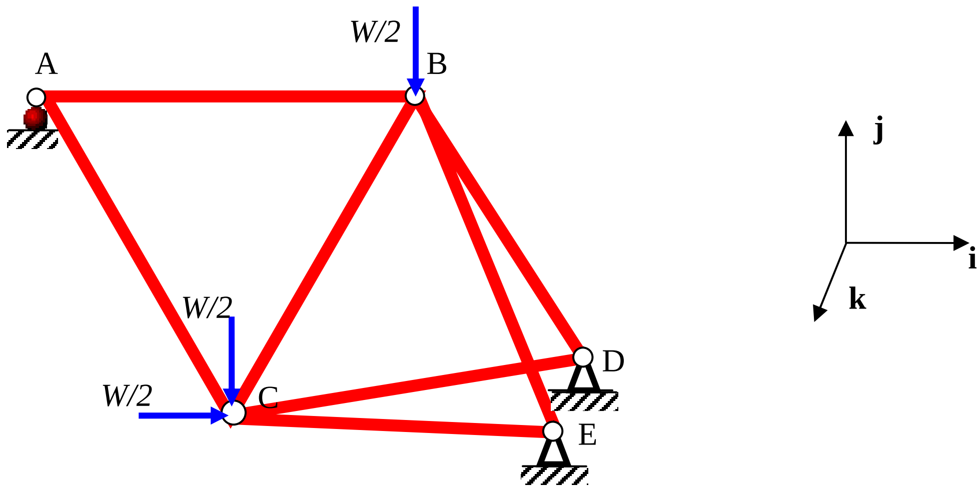
ANSWER ______A_______(2 points)

2. According to Maxwell's law,
(a) The truss shown above is statically determinate
(b) The truss shown above is a mechanism
(c) The truss shown above is statically indeterminate
(d) Bang, bang Maxwell's silver hammer came down upon her head….

J=8, R=4, M=13:
2J=16<M+R=17

ANSWER ______C_________(2 points)

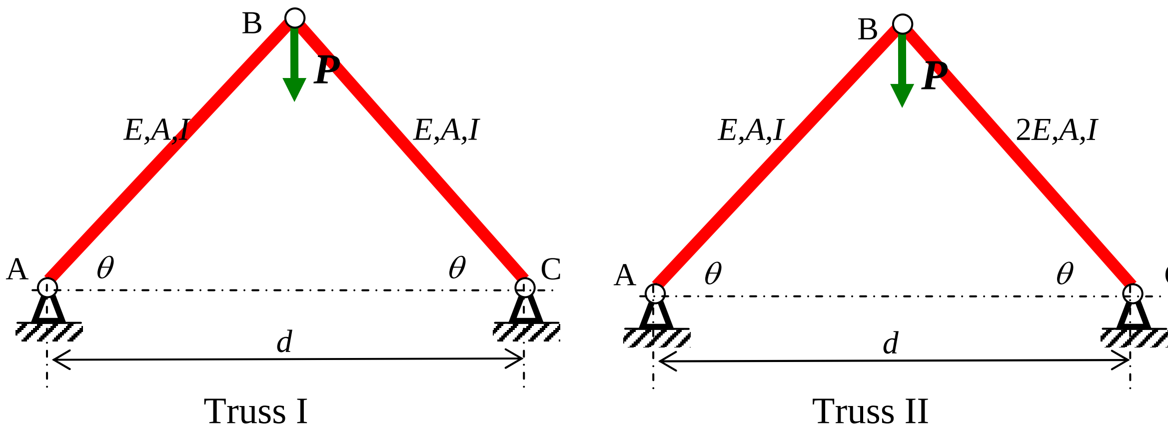
3. You are running a 3-D finite element truss analysis program to calculate the deflections in the 3-D bicycle frame shown. Joints D and E are constrained to have no displacement; joint A is constrained to have no displacement in the **j** direction. The program bombs. Explain **briefly** how you would adjust the problem statement and input file.



Answer: (2 points)

The problem is insufficiently constrained (mechanism). As stated, joint A can move in the **k** direction without inducing member forces. Add the constraint $u_z^A = 0$.

Problems 4 and 5 refer to the two trusses shown below. The cross sectional areas of the members are shown in the figures. The internal forces in the members are denoted by F^{I}_{AB}, F^{I}_{BC} and F^{II}_{AB}, F^{II}_{BC}.



4. Which of the following is true?

(a) $F^{I}_{BC} < F^{II}_{BC}$

(b) $F^{I}_{BC} > F^{II}_{BC}$

(c) $F^{I}_{BC} = F^{II}_{BC}$

(d) Insufficient information

Statically determinate:
$F_{AB}=F_{BC}=P/(2\sin\theta)$ in either case

ANSWER ___C__________(2 points)

5. Let P^{I} and P^{II} be the loads that induce buckling in truss I and truss II, respectively. Which of the following is true?

(a) $P^{I} < P^{II}$

(b) $P^{I} > P^{II}$

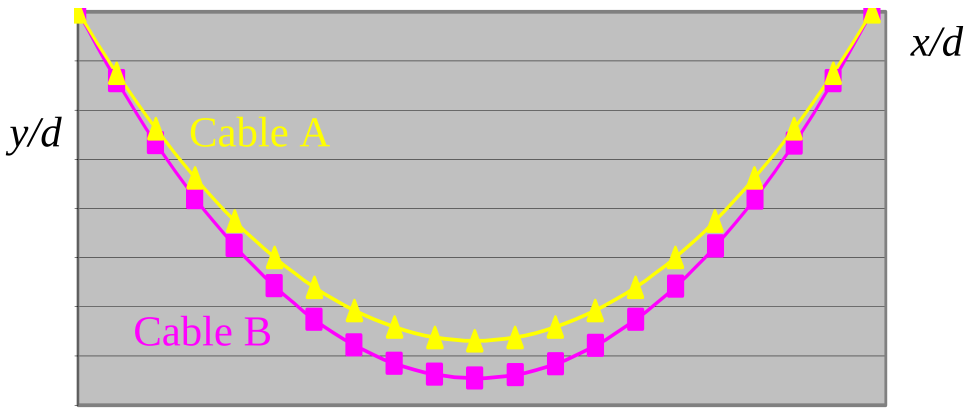
(c) $P^{I} = P^{II}$

(d) Insufficient information.

Buckling load of weakest member dominates.

ANSWER _______C______(2 points)

6. Two inextensible cables are hung between fixed points (x,y)=(0,0) and $(x_e,y_e)=(d,0)$. The cables have equal weight. Cable A has length L_A, cable B has length L_B, with $L_A<L_B$. The maximum tensions in each cable are given by $T^A_{\max}$ and $T^B_{\max}$. Which of the following is true?



(a) $T^A_{\max} < T^B_{\max}$

(b) $T^A_{\max} > T^B_{\max}$

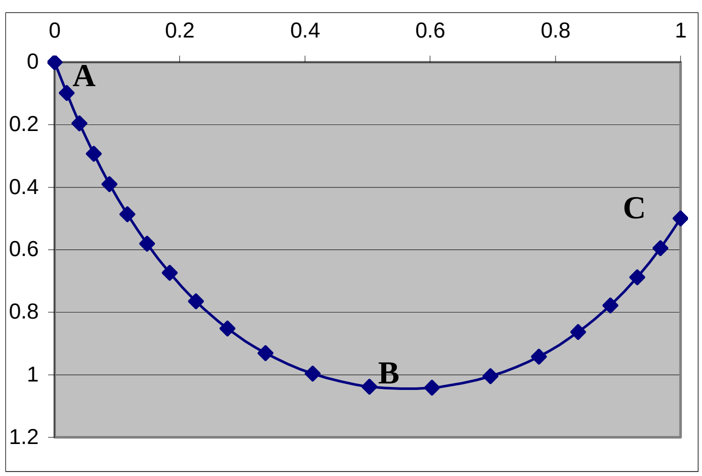
(c) $T^A_{\max} = T^B_{\max}$

(d) Insufficient information

$T_{\max}=T_0= W/(2\sin\theta_0)$ and the angle θ_0 is smaller for cable A.

ANSWER ___B__________(2 points)

7. An inextensible cable is hung between fixed points (x,y)=(0,0) and $(x_e,y_e)=(1,0.5)$ as shown. At which point is the cable tension highest?



(a) Point A

(b) Point B

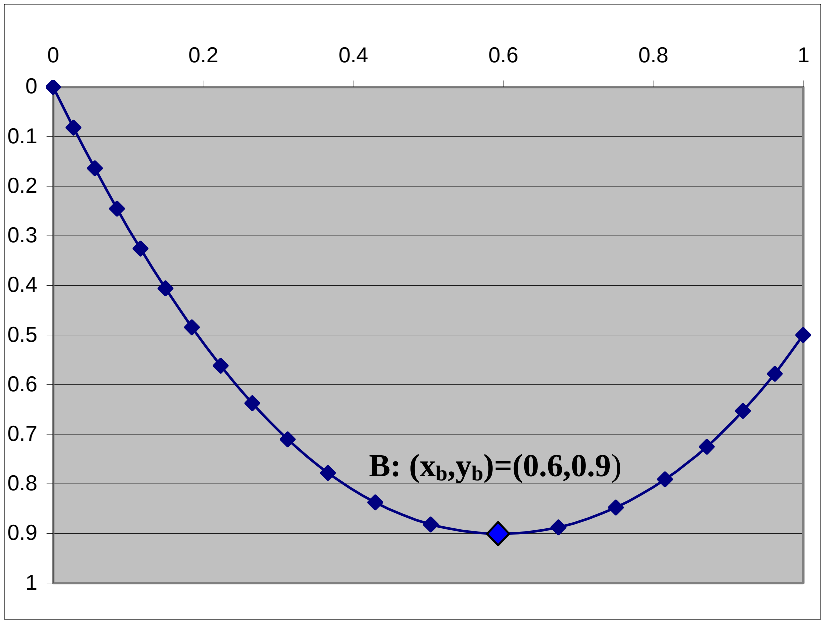
(c) Point C

(d) Insufficient information

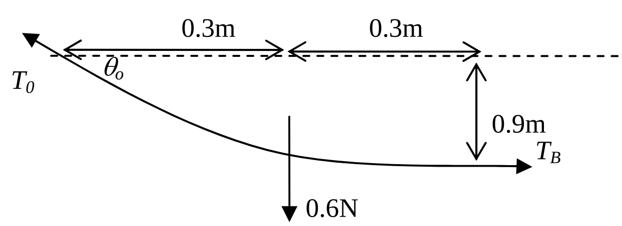
$T(x)=T_0\cos\theta_0/\cos\theta(x)$. $|\theta|$ is at it maximum at $x=0$

ANSWER ____A_______(2 points)

8. An inextensible cable is hung between fixed points (x,y)=(0,0) and (x_e,y_e)=(1m,0.5m) as shown. The cable carries a load $w(x)$=1 Newton/meter **distributed evenly across its span**. The cable shape is shown below. The maximum sag occurs at point B and is found to be 0.9 meters, occurring a distance 0.6 meters from the right endpoint.



The tension in the cable at point B is
(a) T_B=0.185 Newton
(b) T_B=0.2 Newtons
(c) T_B=0.5 Newtons
(d) T_B=1 Newton
(e) None of the above

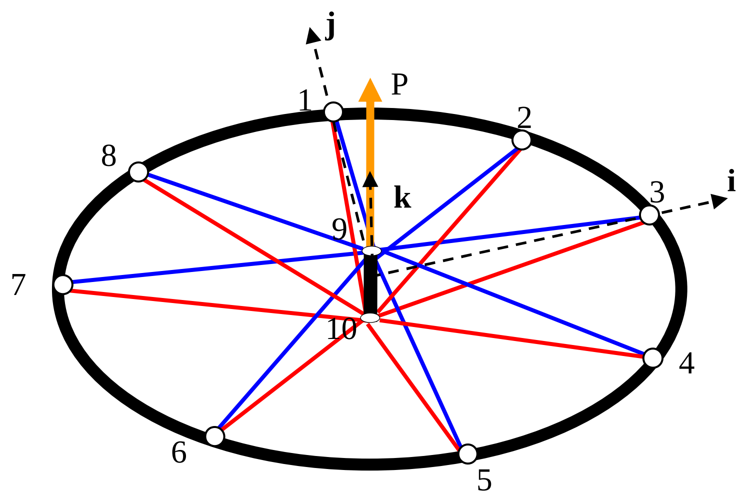


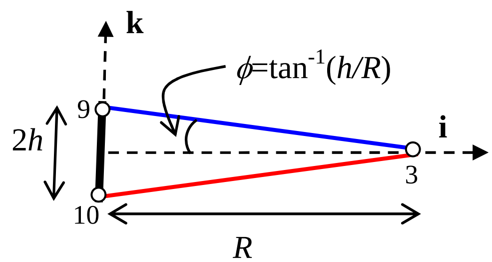
Sum moments about point O to find $0.9T_B=0.3(0.6N)$

ANSWER ___B__________(2 points)

9. (3 points) In this problem, you will analyze the out-of-plane stiffness of the Shimano WH-M540 front wheel, which has 8 pairs of radial spokes. Assume that the wheel rim (radius R) and hub (height $2h$) are effectively rigid compared with the spokes. Each spoke has unstretched length $L_0 = \sqrt{R^2 + h^2}$, area A, and Young's Modulus E.

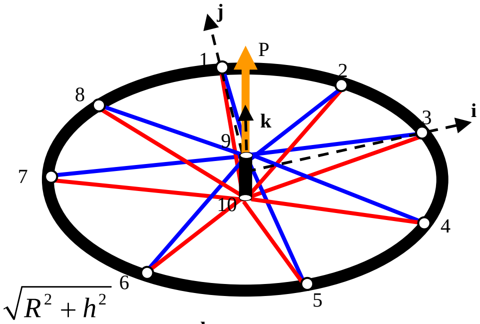
The geometry is shown below. The origin of the coordinate system is in the midplane of the wheel and the hub length is $2h$. The position vectors of nodes 9 and 10 are therefore $\mathbf{r}_9 = h\boldsymbol{k}$ and $\mathbf{r}_{10} = -h\boldsymbol{k}$ Node 1 has position $\mathbf{r}_1 = R\boldsymbol{j}$, node 3 $\mathbf{r}_3 = R\boldsymbol{i}$, etc.





A load P is applied to the hub (node 9) in the **k**-direction. As a result, nodes 9 and 10 undergo corresponding displacements $(u_1^9, u_2^9, u_3^9) = (u_1^{10}, u_2^{10}, u_3^{10}) = (0, 0, u_z), \quad u_z \ll L_0$.

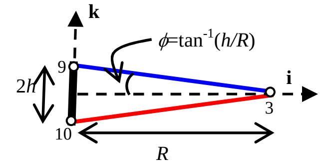
a. Find the potential energy of the loaded wheel (elastic energy of the spokes + the load) as a function of the hub displacement u_z and any of the parameters P, E, A, R , and h. Assume $u_z \ll L_0$. (2 points)



$$\delta_{\text{upper}} = \mathbf{u}^9 \bullet \mathbf{n}_{i9} = u_z \sin\phi = u_z \frac{h}{L_0} \quad (i = 1..8) \text{ for the upper spokes}$$

$$\delta_{\text{lower}} = \mathbf{u}^9 \bullet \mathbf{n}_{i,10} = -u_z \sin\phi = -u_z \frac{h}{L_0} \quad (i = 1..8) \text{ for the lower spokes}$$

$$V = 8\left(\frac{\text{EA}}{2\text{L}_0}\right)(\delta_{\text{lower}}^2 + \delta_{\text{upper}}^2) - Pu_z = 8\frac{EAh^2}{\left(R^2+h^2\right)^{3/2}}u_z^2 - Pu_z \text{ since } L_0 = \sqrt{R^2+h^2}$$



b. Find the hub displacement u_Z as a function of P, E, A, h, and R. (2 points)

$$V = 8\frac{EAh^2}{\left(R^2+h^2\right)^{3/2}}u_z^2 - Pu_z$$

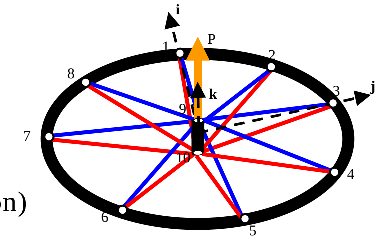
$$\frac{dV}{du_z} = 16\frac{EAh^2}{\left(R^2+h^2\right)^{3/2}}u_z - P = 0 \Rightarrow$$

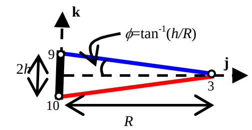
$$u_z = P\frac{\left(R^2+h^2\right)^{3/2}}{16EAh^2}$$

c. Find the force in the spokes as a function of *P*, *E*, *A*, *h*, and *R*. (2 points)

$$F_{i9} = u_z \frac{EAh}{L_0^2} = P\frac{\sqrt{R^2+h^2}}{16h} \quad (i=1..8) \text{ upper spokes (tension)}$$

$$F_{i,10} = -u_z \frac{EAh}{L_0^2} = -P\frac{\sqrt{R^2+h^2}}{16h} \quad (i=1..8) \text{ lower spokes (compression)}$$





d. Suppose you want to take into account the hub elasticity. The hub is made from the same material (Young's modulus *E*) and has cross sectional area of A_h=16*A*. Write the potential energy of the wheel (spokes, hub, and load) as a function of the displacement components of nodes 9 and 10. The answer may also involve the parameters *P*, *E*, *A*, *h*, and *R*. Use symmetry! (3 points)

$$u_x^{(10)} = u_x^{(9)} = u_y^{(10)} = u_y^{(9)} = 0, \quad u_z^{(10)} \neq u_z^{(9)}$$

$$\delta_{\text{upper}} = \mathbf{u}^9 \bullet \mathbf{n}_{i,9} = u_z^{(9)} \sin\phi = u_z^{(9)} \frac{h}{L_0} (i=1..8) \text{ for the upper spokes}$$

$$\delta_{\text{lower}} = \mathbf{u}^{10} \bullet \mathbf{n}_{i,10} = -u_z^{(10)} \sin\phi = -u_z^{(9)} \frac{h}{L_0} (i=1..8) \text{ for the lower spokes}$$

$$\delta_{\text{hub}} = (\mathbf{u}^{10} - \mathbf{u}^9) \bullet \mathbf{n}_{9,10} = u_z^{(9)} - u_z^{(10)}$$

$$V = 8\left(\frac{\text{EA}}{2\text{L}_0}\right)\delta_{\text{upper}}^2 + 8\left(\frac{\text{EA}}{2\text{L}_0}\right)\delta_{\text{lower}}^2 + 16\frac{\text{EA}}{2(2h)}\delta_{\text{hub}}^2 - Pu_z^{(9)} \Rightarrow$$

$$V = 4\frac{EAh^2}{\left(R^2+h^2\right)^{3/2}}\left[\left(u_z^{(9)}\right)^2 + \left(u_z^{(10)}\right)^2\right] + 4\frac{\text{EA}}{h}\left(u_z^{(9)} - u_z^{(10)}\right)^2 - Pu_z^{(9)}$$