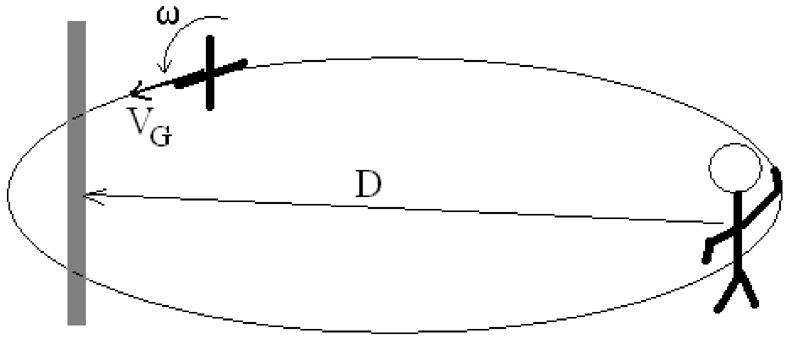
Design of a Boomerang

EN 137 Design Project, Spring 2005

A boomerang is a rigid body that undergoes rotation and translation in a coupled manner, leading to circular motion that can be controlled by appropriate design. The goal of this project is to design a symmetric boomerang that, when thrown appropriately, flies around a pole located at a distance D=10m from the thrower and returns to the thrower.

We will hold a contest to test the boomerangs the morning of Thursday, May 5.



The design of the boomerang involves several pieces of the dynamics of rigid bodies and some aerodynamics. We will proceed through the design in steps, as follows:

1. Given a law for the aerodynamic lift force of an airfoil, determine the forces and moments acting on an symmetric "cross" boomerang that is spinning at rate ω and translating at speed v_G, as shown in the figure.
2. Using Euler's equations for rigid body motion, determine the relationship between the velocity and the processional circular motion of the boomerang.
3. Relate the radius of the orbit to the motion and the design parameters of the boomerang (size, airfoil shape, density of material, etc.) and the throwing conditions (initial velocity, initial rotation rate).
4. Use an airfoil lift simulator to design the airfoil shape needed to obtain the desired radius of orbit (D/2 above). Consider various likely choices of material from which the airfoil will be constructed.
5. Perform preliminary design calculations to understand tradeoffs in various parameters on performance.
6. Build a boomerang according to the selected design.
7. Test boomerang performance and empirically address issues not dealt with in the design (e.g. gravity, air resistance).

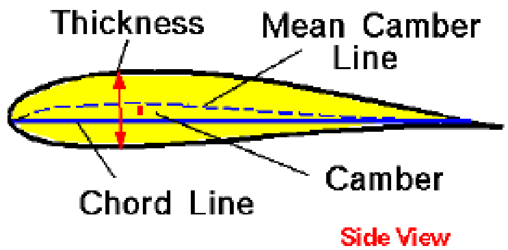
We will go through these steps in sequence, with calculations handed in each week over the next few weeks.

For the design and construction phase,
students will work in teams of 3

Step 1. Forces and Moments on a Symmetric Cross-shaped Boomerang

Lift Force:

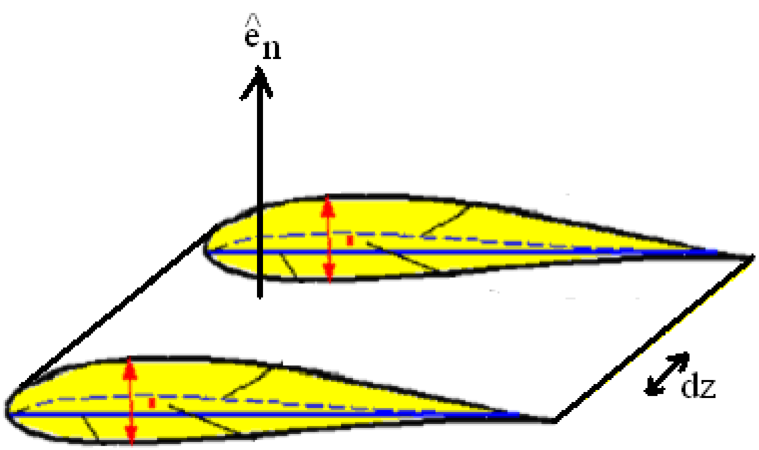
An airfoil provides a lift force that depends on the shape of the airfoil and the velocity of the air passing around it.



For the moment, it suffices to know that the lift force per unit width dz of airfoil is proportional to the square of the velocity. That is,

$$F_{lift} = cv^2 \, dz \, \hat{e}_n$$

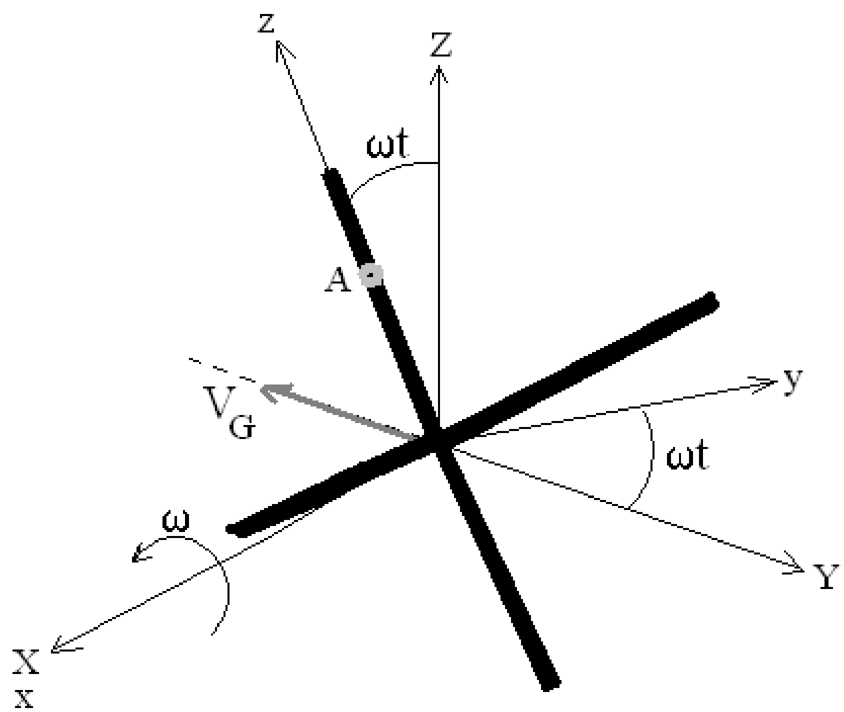
where $\hat{e}_n$ is normal to the flow direction (upward in the figure shown).



The constant c depends on the shape (length, thickness, camber) and orientation (angle of attack). We will return to this later. For now we will use the general force result above to study a boomerang

Forces and Moments on a Boomerang:

Consider a boomerang consisting of two crossed slender rods at right angles, forming four arms each of length L. Ignore the detail shape of the rods and their finite width. The boomerang is spinning at rate ω and also translating with velocity v_G. We establish two coordinate systems. System xyz is fixed in the body of the boomerang so that the boomerang lies in the y-z plane, with one rod along z and one along y. The spinning at rate ω thus occurs around the x axis, as indicated. System XYZ fixed in space, as shown. As time t progresses, the y-z axes rotate relative to the Y-Z axes to form an angle ωt.



To calculate the total force and moment on the boomerang at any instant, we need to calculate the force on each small element dz or dy along the arms of the boomerang, find the moment associated with each increment, and then sum up over all elements (i.e. integrate) to obtain the total force and moment.

We design the airfoil shape of the boomerang such that the lift forces are in the $\hat{i}$ direction.

Since the force is proportional to the velocity, each point along an arm has a different velocity and hence a different contribution to the force. To obtain the total force and moment in terms of the magnitudes of v_G, ω, L, and the airfoil constant c, proceed as follows. *** Neglect gravity ***

a. First, express the fixed axis vectors $\hat{J}$ and $\hat{K}$ in terms of the rotating axis unit vectors, ω, and time. This will be needed below.

b. For a point A at distance z along one arm of the airfoil, calculate the absolute velocity vector $\vec{v}_A$ using relative velocity, and expressed in terms of the xyz coordinate system only.

c. Given this velocity, calculate the force vector acting on an element of width dz at position z. This force will be in the $+\hat{i}$ direction.

d. Calculate the moment around G due to this force.

e. Integrate the force along the length of the arm (0 to L) to obtain the total force acting on the arm.

f. Integrate the moment along the length of the arm to obtain the total moment acting on the arm.

g. Repeat this procedure for the other three arms, noting all the relevant vector changes and keeping careful track of the signs of various terms. Note that $\vec{v}_G = -v_G\hat{J}$ for all four arms.

h. Sum up the contributions from each arm to obtain the total force and total moment on the boomerang.

Please carry out this set of calculations by April 18