EN137 : HW# 11 SOLUTIONS

1. $T = \frac{1}{2}m\left(\frac{l}{2}\dot{\theta}\right)^2 + \frac{1}{2}m\dot{x}^2$

$V = \frac{1}{2}k\left(\frac{l}{2}\theta\right)^2 + \frac{1}{2}k(l\theta - x)^2$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = 0 \Rightarrow \begin{bmatrix} \frac{ml^2}{4} & 0 \\ 0 & m \end{bmatrix}\begin{Bmatrix} \ddot{\theta} \\ \ddot{x} \end{Bmatrix} + \begin{bmatrix} \frac{5kl^2}{4} & -kl \\ -kl & k \end{bmatrix}\begin{Bmatrix} \theta \\ x \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$M\ddot{\underline{u}} + K\underline{u} = \underline{0}$ — ①

Trial solution $\left.\begin{array}{l} \underline{u} = A\underline{U}\sin(\omega t - \phi) \\ \ddot{\underline{u}} = -\omega^2 A\underline{U}\sin(\omega t - \phi) \end{array}\right\} [-\omega^2 M + K][\underline{U}] = \underline{0}$

For non-trivial solution $\Rightarrow \det([-\omega^2 M + K]) = 0$

$\Rightarrow \omega^4 m^2 - 6\omega^2 mk + k^2 = 0 \Rightarrow \omega^2 = (3 \pm 2\sqrt{2})\frac{k}{m}$

Natural Frequency		Mode Shape
$\omega_1 = \sqrt{(3-2\sqrt{2})\frac{k}{m}}$	$\omega_1 \to$ ①	$U_2^{(1)} = \left(\frac{1+\sqrt{2}}{2}\right)l\,U_1^{(1)}$ $\underline{U}^{(1)} = \begin{Bmatrix} 1 \\ \frac{1+\sqrt{2}}{2}l \end{Bmatrix}$
$\omega_2 = \sqrt{(3+2\sqrt{2})\frac{k}{m}}$	$\omega_2 \to$ ①	$U_2^{(2)} = \left(\frac{1-\sqrt{2}}{2}\right)l\,U_1^{(2)}$ $\underline{U}^{(2)} = \begin{Bmatrix} 1 \\ \frac{1-\sqrt{2}}{2} \end{Bmatrix}$

2. $T = \frac{1}{2}m_1\dot{u}_1^2 + \frac{1}{2}m_2\dot{u}_2^2$ $\quad Q_1^{NC} = F_0 \sin\Omega t$ $\quad m_1 = 2m$

$V = ku_1^2 + k(u_2 - u_1)^2$ $\quad Q_2^{NC} = 0.$ $\quad m_2 = m$

$$\begin{bmatrix} 2m & 0 \\ 0 & m \end{bmatrix}\begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{bmatrix} 4k & -2k \\ -2k & 2k \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_0 \sin\Omega t \\ 0 \end{Bmatrix}$$

(i) Trial particular solution $\quad \underline{u}_p = \underline{U}_p \sin\Omega t$

$\Rightarrow [K - \Omega^2 M][\underline{U}_p] = [\underline{F}]$ where $[\underline{F}] = \begin{Bmatrix} F_0 \\ 0 \end{Bmatrix}.$

$$U_{p_1} = \frac{\begin{vmatrix} F_0 & -2k \\ 0 & 2k - \Omega^2 m \end{vmatrix}}{D} = \frac{(2k - \Omega^2 m)F_0}{D}$$

$$U_{p_2} = \frac{\begin{vmatrix} 4k - 2m\Omega^2 & F_0 \\ -2k & 0 \end{vmatrix}}{D} = \frac{2kF_0}{D}$$

where

$$D = \begin{vmatrix} 4k - 2m\Omega^2 & -2k \\ -2k & 2k - \Omega^2 m \end{vmatrix} = \left\{(\Omega^2)^2 - \frac{4k}{m}\Omega^2 + 2\left(\frac{k}{m}\right)^2\right\} 2m^2$$

(ii) Now let $\Omega = \sqrt{\frac{2K}{m}} \Rightarrow U_{P_1} = 0$

Hence, the driving force is applied to the floor however it doesn't move. The top floor acts as a vibration absorber.

(iii) <u>Forces due to unsymmetric load.</u>

$$\boxed{F_0 = \Omega^2 \Delta m a}$$

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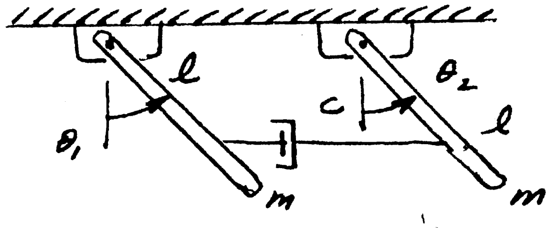

$$T = \tfrac{1}{2} m \left(\tfrac{l}{2}\dot{\theta}_1\right)^2 + \tfrac{1}{2} m \left(\tfrac{l}{2}\dot{\theta}_2\right)^2$$

$$V = -mg\tfrac{l}{2}\cos\theta_1 - mg\tfrac{l}{2}\cos\theta_2$$

and $\cos\theta \approx 1 - \frac{\theta^2}{2}$

$$\Rightarrow V = -mgl + mg\tfrac{l}{4}\left(\theta_1^2 + \theta_2^2\right)$$

$$F = \tfrac{1}{2} c \left(\tfrac{l}{2}\dot{\theta}_2 - \tfrac{l}{2}\dot{\theta}_1\right)^2 = \frac{cl^2}{8}\left(\dot{\theta}_2 - \dot{\theta}_1\right)^2$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_1}\right) - \frac{\partial \mathcal{L}}{\partial \theta_1} + \frac{\partial F}{\partial \dot{\theta}_1} = 0$$

$$\frac{ml^2}{4}\ddot{\theta}_1 + mg\frac{l}{2}\theta_1 - \frac{cl^2}{4}\left(\dot{\theta}_2 - \dot{\theta}_1\right) = 0$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}_2}\right) - \frac{\partial \mathcal{L}}{\partial \theta_2} + \frac{\partial F}{\partial \dot{\theta}_2} = 0$$

$$\frac{ml^2}{4}\ddot{\theta}_2 + mg\frac{l}{2}\theta_2 + \frac{cl^2}{4}\left(\dot{\theta}_2 - \dot{\theta}_1\right) = 0.$$

in matrix form,

$$\begin{bmatrix} \frac{ml^2}{4} & 0 \\ 0 & \frac{ml^2}{4} \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} \frac{cl^2}{4} & -\frac{cl^2}{4} \\ -\frac{cl^2}{4} & \frac{cl^2}{4} \end{bmatrix} \begin{Bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} \frac{mgl}{2} & 0 \\ 0 & \frac{mgl}{2} \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} =$$

$$M\ddot{\underline{u}} + C\dot{\underline{u}} + Ku = \underline{0}$$