EN137

Homework 1

1. General expression for velocity and acceleration

$$\underset{\sim}{v}(t) = \dot{\underset{\sim}{R}}_o + \underset{\sim}{v}_{rel} + \underset{\sim}{\omega} \times \underset{\sim}{r}$$

$$\underset{\sim}{a}(t) = \ddot{\underset{\sim}{R}}_o + \underset{\sim}{a}_{rel} + 2(\underset{\sim}{\omega} \times \underset{\sim}{v}_{rel}) + \dot{\underset{\sim}{\omega}} \times \underset{\sim}{r} + \underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \underset{\sim}{r})$$

Any pt. $\underset{\sim}{r} = (x\underset{\sim}{i} + y\underset{\sim}{j}) = R\cos\theta\, \underset{\sim}{i} + R\sin\theta\, \underset{\sim}{j}$ (wrt. rotating frame)

Passanger 1 $\theta = 90°$,

$\underset{\sim}{v}(t) = -20\underset{\sim}{i}$ (m/s)

$\underset{\sim}{a}(t) = 2\underset{\sim}{i} - 20\underset{\sim}{j}$ (m/s²)

$\left.\right\}$ $\underset{\sim}{a}_{rel} = 0$, $\dot{\underset{\sim}{\omega}} = 0$, $\underset{\sim}{v}_{rel} = 0$, $\dot{\underset{\sim}{R}}_o = 0$, $\ddot{\underset{\sim}{R}}_o = 0$

Passanger 2 $\theta = 0°$

$\underset{\sim}{v}(t) = 20\underset{\sim}{j}$ (m/s), $\underset{\sim}{a}(t) = -20\underset{\sim}{i} - 2\underset{\sim}{j}$ (m/s²)

Passanger 3 $\theta = -90°$

$\underset{\sim}{v}(t) = 20\underset{\sim}{i}$ (m/s), $\underset{\sim}{a}(t) = 20\underset{\sim}{j} - 2\underset{\sim}{i}$ (m/s²)

2. Use the same general expressions for $\underset{\sim}{v}(t)$ & $\underset{\sim}{a}(t)$

$\underset{\sim}{v}_{rel} = v_0\underset{\sim}{j}$, $\underset{\sim}{a}_{rel} = a_0\underset{\sim}{j}$, $\underset{\sim}{r}_p = d\underset{\sim}{i} + h\underset{\sim}{j}$

$$\underset{\sim}{v}(t) = \cancelto{0}{\dot{\underset{\sim}{R}}_o} + \underset{\sim}{v}_{rel} + \underset{\sim}{\omega} \times \underset{\sim}{r}_p = \underline{(v_0 + \omega d)\underset{\sim}{j} + (-\omega h)\underset{\sim}{i}}$$

$$\underset{\sim}{a}(t) = \cancelto{0}{\ddot{\underset{\sim}{R}}_o} + \underset{\sim}{a}_{rel} + 2(\underset{\sim}{\omega} \times \underset{\sim}{v}_{rel}) + \dot{\underset{\sim}{\omega}} \times \underset{\sim}{r}_p + \underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \underset{\sim}{r}_p)$$

$$= (-2\omega v_0 - \dot{\omega} h - \omega^2 d)\underset{\sim}{i} + (a_0 + \dot{\omega} d - \omega^2 h)\underset{\sim}{j}$$

3. Two ways of doing this problem.

Way 1

Consider $Ox_1y_1z_1$ as basis and evaluate velocity and acceleration of point P. Use these values as relative velocity and acceleration when treating $OX_1Y_1Z_1$ as the basis. (Here $OX_1Y_1Z_1$ is fixed system aligned with $Ox_1y_1z_1$).

$$v_{P(Ox_1y_1z_1)} = \cancelto{0}{\dot{R}_Q} + \cancelto{0}{v_{rel}} + \omega \times r_{P(Ox_2y_2z_2)}$$

$$= \omega r_2 \, i_2$$

Similarly $a_{P(Ox_1y_1z_1)} = \omega^2 r_2 \, j_2$

(Radius of smaller circle is r_2 and bigger circle is r_1)

Now

$$a_{P(OX_1Y_1Z_1)} = \cancelto{0}{\ddot{R}_O} + a_{rel} + 2(\omega \times v_{rel}) + \cancelto{0}{\dot{\omega}} \times r_{P(OX_1Y_1Z_1)} + \omega \times (\omega \times r_{P(OX_1Y_1Z_1)})$$

$$r_{P(OX_1Y_1Z_1)} = r_1 i_1 - r_2 j_1 \,, \quad \omega = \Omega k$$

$$\Rightarrow \; a_{P(OX_1Y_1Z_1)} = (\omega + \Omega)^2 r_2 j_1 - \Omega^2 r_1 i_1$$

For $a_{P(OX_1Y_1Z_1)}$ to lie along i_1, we should have

$$\boxed{\Omega = -\omega}$$

Way 2

Directly consider P with respect to $OX_1Y_1Z_1$,

$$a_P = \ddot{R}_2 + \cancelto{0}{a_{rel}} + 2(\Omega + \omega) k \times \cancelto{0}{v_{rel}}$$

$$+ (\dot{\Omega} + \dot{\omega}) k \times r + (\Omega + \omega) k \times [(\Omega + \omega) k \times r]$$

$$= \ddot{R}_2 + (\Omega + \omega)^2 (r_2 j_2)$$

But $\ddot{R}_2 = \cancelto{0}{\ddot{R}_1} + (a_{rel})_2 + (\Omega) k \times \cancelto{0}{(v_{rel})_2} + \cancelto{0}{(\dot{\Omega})} k \times r_2$
$+ (\Omega k) \times (\Omega k \times r_2)$

$$\Rightarrow \; a_P = \underbrace{-\Omega^2 r_1 i_1}_{a_Q} + \underbrace{(\Omega + \omega)^2 r_2 j_1}_{a_{P/Q}}$$

For a_P to be along i_1 or a_Q,

$$\boxed{\Omega = -\omega}$$