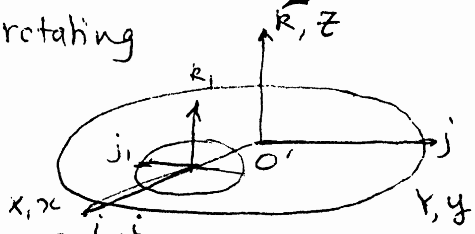
EN 134

HW 2

1\. $O'XYZ$ is fixed frame, $O'xyz$ is rotating with the big disc.



$$\underset{\sim}{v}^A_{O'xyz} = \cancel{\dot{\underset{\sim}{R}}_0}^{\,0} + \cancel{\underset{\sim}{v}_{rel}}^{\,0} + \underset{\sim}{\omega} \times \underset{\sim}{r}$$

$$\underset{\sim}{a}^A_{O'xyz} = \cancel{\ddot{\underset{\sim}{R}}_0}^{\,0} + \cancel{\underset{\sim}{a}_{rel}}^{\,0} + \cancel{\dot{\underset{\sim}{\omega}} \times \underset{\sim}{r}}^{\,0} + 2\underset{\sim}{\omega} \times \cancel{\underset{\sim}{v}}^{\,0}_{rel} + \underset{\sim}{\omega} \times \underset{\sim}{\omega} \times \underset{\sim}{r}$$

$$\underset{\sim}{\omega} = \omega_2 \underset{\sim}{k} = 15\, \underset{\sim}{k}_1 \text{ rad/s}, \quad \underset{\sim}{r} = -\underset{\sim}{j}_1 \text{ ft}$$

v_{rel} for a^A_{OXYZ} $\qquad$ a_{rel} for a^A_{OXYZ}

$$\underset{\sim}{v}^A_{O'xyz} = 15\, \underset{\sim}{i}_1 \text{ (ft/s)}, \quad \underset{\sim}{a}^A_{O'xyz} = 225\, \underset{\sim}{j}_1 \text{ (ft/s}^2)$$

$$\underset{\sim}{a}^A_{OXYZ} = \cancel{\ddot{\underset{\sim}{R}}_0}^{\,0} + \underset{\sim}{a}_{rel} + 2\underset{\sim}{\omega} \times \underset{\sim}{v}_{rel} + \dot{\underset{\sim}{\omega}} \times \underset{\sim}{r} + \underset{\sim}{\omega} \times \underset{\sim}{\omega} \times \underset{\sim}{r}$$

$$\underset{\sim}{\omega} = 10\underset{\sim}{k} \text{ (rad/s)}, \quad \dot{\underset{\sim}{\omega}} = -4\, \underset{\sim}{k} \text{ (rad/s}^2), \quad \underset{\sim}{r} = 2\underset{\sim}{i} - \underset{\sim}{j} \text{ (ft)}$$

$$\underset{\sim}{a}^A_{OXYZ} = 617\, \underset{\sim}{j} - 204\, \underset{\sim}{i} \quad \text{ft/sec}^2$$

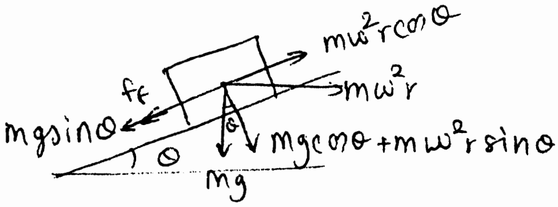
$$\underset{\sim}{f}^A = \frac{1}{4}\, \frac{617\underset{\sim}{j} - 204\underset{\sim}{i}}{32.2} + \frac{1}{4}(-\underset{\sim}{k}) = 4.79\,\underset{\sim}{j} - 1.58\,\underset{\sim}{i} - 0.25\,\underset{\sim}{k}$$

Axial force = force along $\underset{\sim}{j}$ = $4.79\,\underset{\sim}{j}$

Shear force = force along $\underset{\sim}{i}\,\underset{\sim}{k}$ = $-1.58\,\underset{\sim}{i} - 0.25\,\underset{\sim}{k}$

(Answer i.e. signs and vectors depend on your axes.)

3\. (a) $\qquad$ 1 mi = 5280 ft



FBD

$$f_f = 0 \Rightarrow mg\sin\theta = m\omega^2 r\cos\theta$$

$$\Rightarrow \tan\theta = \frac{\omega^2 r}{g} = \frac{v^2}{rg}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{(55 \times 5280/3600)^2}{2000 \times 32.2}\right) = 5.769^\circ$$

(b) Now $f_f = \mu(mg\cos\theta + m\omega^2 r\sin\theta)$

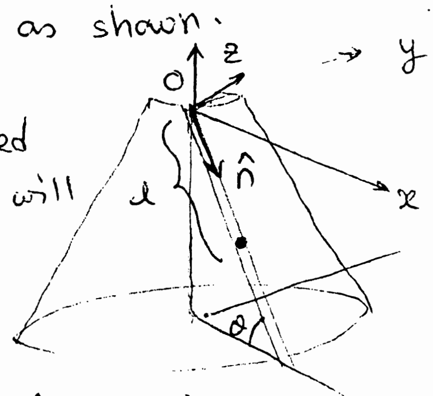
$$\Rightarrow \mu(mg\cos\theta + m\omega^2 r\sin\theta) + mg\sin\theta = m\omega^2 r\cos\theta$$

Hence, $\quad \omega^2 r = \frac{V^2}{r} = \frac{\mu m g\cos\theta + mg\sin\theta}{m\cos\theta - \mu m\sin\theta}$

$$\Rightarrow \quad V = \left[r \cdot \frac{\mu g\cos\theta + g\sin\theta}{\cos\theta - \mu\sin\theta} \right]^{1/2}$$

For $\mu = 0.5$, $V = 137.7$ mi/hr
$\mu = 0.25$, $V = 103.84$ mi/hr
$\mu = 0.05$, $V = 67.42$ mi/hr

4. We can choose axes X and Y as shown. $\hat{n}$ is the unit vector along the groove. The body is constrained to move in y direction. so we will not consider forces or acc^n in $\underset{\sim}{j}$ direction. forces and acc^n only along the groove will be considered.



$\hat{n} = \cos\theta \, \underset{\sim}{i} - \sin\theta \, \underset{\sim}{k}$
$\hat{m} (\perp \hat{n}) = \sin\theta \, \underset{\sim}{i} + \cos\theta \, \underset{\sim}{k}$

$\underset{\sim}{r} = l (\cos\theta \, \underset{\sim}{i} - \sin\theta \, \underset{\sim}{k})$
$\underset{\sim}{v}_{rel} = \dot{l} (\cos\theta \, \underset{\sim}{i} - \sin\theta \, \underset{\sim}{k})$
$\underset{\sim}{a}_{rel} = \ddot{l} (\cos\theta \, \underset{\sim}{i} - \sin\theta \, \underset{\sim}{k})$

Here we are ignoring $\underset{\sim}{F}$ or $\underset{\sim}{a}$ along $\underset{\sim}{j}$. considering only $\underset{\sim}{i}, \underset{\sim}{k}$

$$\underset{\sim}{a} = \cancel{\ddot{\underset{\sim}{R}}_0}^{\,0} + \underbrace{\underset{\sim}{a}_{rel}}_{\text{this acc}^n \text{ is along the } \hat{n} \text{ vector}} + \underbrace{2\underset{\sim}{\omega} \times \underset{\sim}{v}_{rel}}_{\text{along } \underset{\sim}{j}} + \cancel{\dot{\underset{\sim}{\omega}}}^{\,0} \times \underset{\sim}{r} + \underbrace{\underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \underset{\sim}{r})}_{\text{along} -\underset{\sim}{i}} \quad \text{—— (1)}$$

component of the last term along $\hat{\underset{\sim}{n}} = -\omega^2 l \cos^2\theta$

$\underset{\sim}{F} = m\underset{\sim}{a}$

$$\underset{\sim}{F} = \underbrace{-mg\,\underset{\sim}{k}}_{\text{component along } \hat{n} \,=\, mg\sin\theta} + (-k(l - l_0))\,\hat{\underset{\sim}{n}} + \underbrace{f_m \hat{m}}_{\text{force } f_m \text{ constraining motion in } \hat{m} \perp \text{ to the groove}} \quad \text{—— (2)}$$

component along $\hat{n}$ = $mg\sin\theta$

force f_m constraining motion in $\hat{m}$ $\perp$ to the groove

Eq^n of motion $\quad \ddot{l} - \omega^2 l\cos^2\theta = -k(l - l_0) + mg\sin\theta$

we can also write two equations of motion along $\underset{\sim}{i}$ and $\underset{\sim}{k}$ by taking components of $\underset{\sim}{F}$ and $\underset{\sim}{a}$ along $\underset{\sim}{i}$ and $\underset{\sim}{k}$ and equating them respectively.