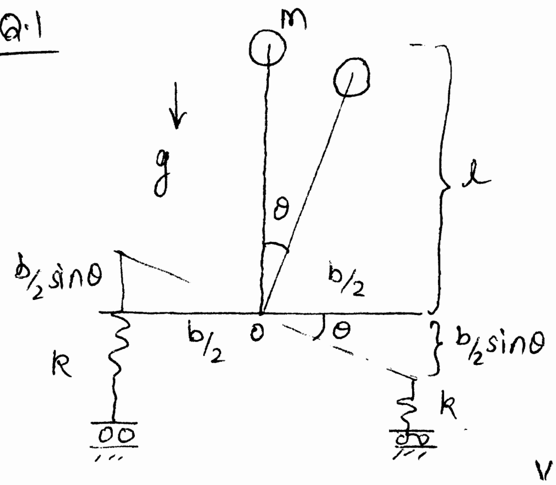
EN 137

HW4

Q.1



let θ be any rotation about point O. Since we can express the motion using θ, degrees of freedom=1. (Also that is the only admissible variation → δθ)

velocity of mass m = $l\dot{\theta}$

$$T = \frac{1}{2}m(l\dot{\theta})^2 = \frac{1}{2}ml^2\dot{\theta}^2$$

Potential energy of a spring = $\frac{1}{2}kx^2$ where x is the change in length from equilibrium point.

Let the springs have compression Δl at the equilibrium position of the pendulum. (It is not equal to zero in general.)

$$V_{spring} = \frac{1}{2}k\left(\Delta l + \frac{b}{2}\sin\theta\right)^2 + \frac{1}{2}k\left(\Delta l - \frac{b}{2}\sin\theta\right)^2$$

$$V_{mass} = -\int \underset{\sim}{F}\cdot d\underset{\sim}{r} = -mgl(1-\cos\theta)$$

$$V = \frac{1}{2}k\left(\Delta l + \frac{b}{2}\sin\theta\right)^2 + \frac{1}{2}k\left(\Delta l - \frac{b}{2}\sin\theta\right)^2 - mgl(1-\cos\theta)$$

$$\mathcal{L} = \frac{1}{2}ml^2\dot{\theta}^2 - \frac{1}{2}k\left(\Delta l + \frac{b}{2}\sin\theta\right)^2 - \frac{1}{2}k\left(\Delta l - \frac{b}{2}\sin\theta\right)^2 + mgl(1-\cos\theta)$$

(Note: Δl being constant will not appear in the answer.)

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}}\right) - \frac{\partial \mathcal{L}}{\partial \theta} = 0$$

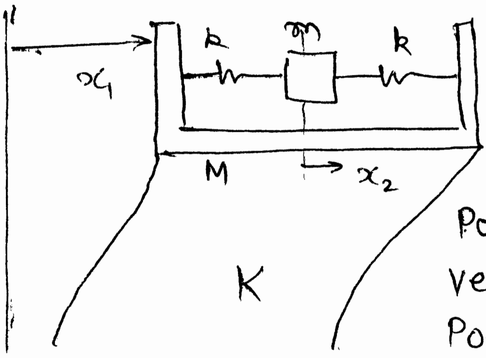
$$\Rightarrow \quad ml^2\ddot{\theta} + \frac{kb^2}{2}\sin\theta\cos\theta - mgl\sin\theta = 0$$

Q.2

Since there are two independent generalized coordinates and two independent admissible variations, degrees of freedom = 2.

x_1 - defined stationary left end

x_2 - defined wrt center of moving mass M.



Again let Δl be the initial compression/extension.

Position of m = $x_1 + l + x_2$

velocity of m = $\dot{x}_1 + \dot{x}_2$

Position of M = x_2

velocity of M = $\dot{x}_2$

$$T = \frac{1}{2} m (\dot{x}_1 + \dot{x}_2)^2 + \frac{1}{2} M \dot{x}_2^2$$

Potential energy $V = \frac{1}{2} K x_1^2 + \frac{1}{2} k (\Delta l + x_2)^2 + \frac{1}{2} k (\Delta l - x_2)^2$

$$\mathcal{L} = \frac{1}{2} m (\dot{x}_1 + \dot{x}_2)^2 + \frac{1}{2} M \dot{x}_2^2 - \frac{1}{2} K x_1^2 - \frac{1}{2} k (\Delta l + x_2)^2 - \frac{1}{2} k (\Delta l - x_2)^2$$

Using x_1 : $m(\ddot{x}_1 + \ddot{x}_2) + M \ddot{x}_1 + K x_1 = 0$

Using x_2 : $m(\ddot{x}_1 + \ddot{x}_2) + 2k x_2 = 0$

Note: Δl will not appear because it is in both the springs. Assumption $\Delta l = 0$ is valid but not always true. (for prob. 1 and 2)