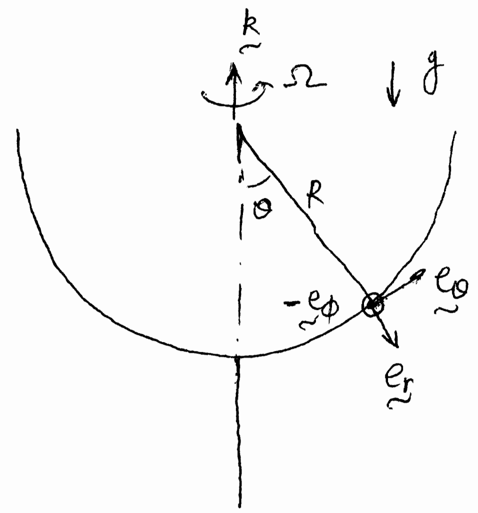
1.



generalized coordinate θ

$$\underset{\sim}{r} = R\,\underset{\sim}{e_r}$$

$$\underset{\sim}{v} = \underset{\sim}{v_{rel}} + \Omega\,\underset{\sim}{k}\times\underset{\sim}{r}$$

$$= R\dot\theta\,\underset{\sim}{e_\theta} + \Omega R\sin\theta\,\underset{\sim}{e_\phi}$$

$$\underset{\sim}{F}^{NC} = -c\,\underset{\sim}{v_{rel}} = -cR\dot\theta\,\underset{\sim}{e_\theta}$$

$$T = \frac{1}{2}m\,\underline{v}\cdot\underline{v} = \frac{1}{2}m\left[R^2\dot\theta^2 + \Omega^2R^2\sin^2\theta\right]$$

$$V = mgR\,(1-\cos\theta)$$

$$Q_\theta^{NC} = \underset{\sim}{F}^{NC}\cdot\frac{\partial \underset{\sim}{r}}{\partial\theta} = -cR\dot\theta\,\underset{\sim}{e_\theta}\cdot R\,\underset{\sim}{e_\theta} = -cR^2\dot\theta$$

$$\frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot\theta}\right) - \frac{\partial\mathcal{L}}{\partial\theta} = Q_\theta^{NC}$$

$$\Rightarrow\quad mR^2\ddot\theta - m\Omega^2R^2\sin\theta\cos\theta + mgR\sin\theta + cR^2\dot\theta = 0$$

2. generalized coordinates r, θ

$$\underset{\sim}{r} = r\,\underset{\sim}{e_r}\,,\qquad \underset{\sim}{v} = \dot r\,\underset{\sim}{e_r} + r\dot\theta\,\underset{\sim}{e_\theta}$$

$$T = \frac{1}{2}m\left[\dot r^2 + r^2\dot\theta^2\right]$$

$$V = V_{mass} + V_{spring} = mg\,(l_0 - r\cos\theta) + \frac{1}{2}k(r-l_0)^2$$

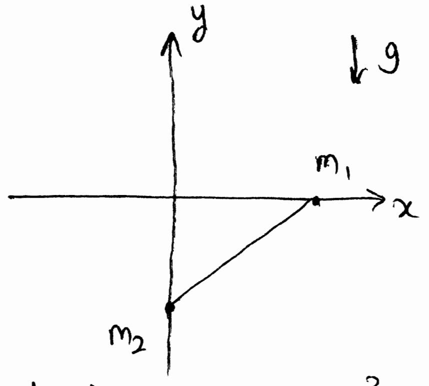
$$Q_i^{NC} = \underset{\sim}{f}^{NC}\cdot\frac{\partial \underset{\sim}{r}}{\partial q_i}\quad\text{where } \underset{\sim}{f}^{NC} = -c\dot r\,\underset{\sim}{e_r}$$

$$Q_r^{NC} = -c\dot r\,,\qquad Q_\theta^{NC} = 0$$

$\underline{\theta}$: $$mr^2\ddot\theta + mgr\sin\theta = 0$$

$\underline{r}$: $$m\ddot r - mr\dot\theta^2 - mg\cos\theta + k(r-l_0) + c\dot r = 0$$

Q.1



$$T = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} m_2 \dot{y}^2$$

$$V = m_2 g y$$

$$\mathcal{L} = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} m_2 \dot{y}^2 - m_2 g y$$

constraint $\quad f \equiv x^2 + y^2 - l^2 = 0$

x: $\quad m_1 \ddot{x} = \lambda(2x)$ — ① $\qquad$ y: $\quad m_2 \ddot{y} + m_2 g = \lambda(2y)$ — ②

Differentiating f twice wrt time, $x\ddot{x} + y\ddot{y} + \dot{x}^2 + \dot{y}^2 = 0$ — ③
Solving ①, ②, ③ to eliminate λ and then separating expressions for $\ddot{x}, \ddot{y}$, we get

$$\dot{x} = u \quad ; \quad \dot{y} = v$$

$$\dot{u} = \frac{m_2 x\,(gy - (u^2+v^2))}{m_1 y^2 + m_2 x^2} \quad \text{— ④}$$

$$\dot{v} = \frac{-m_1 y\,(u^2+v^2) - m_2 g x^2}{m_1 y^2 + m_2 x^2} \quad \text{— ⑤}$$

Solve using Maple
$x(0) = l$, $y(0) = 0$
$u(0) = 0$, $v(0) = 0$
$m_1 = m_2 = 1$, $l = 1$

Motion is periodic with period ~ 2.35 sec.

Tension is equal to the constraint force S.

$$S_x = m_1 \ddot{x} = \lambda(2x) = m_1 \dot{u}$$

$$S_y = \lambda(2y) = m_2 \ddot{y} + m_2 g = m_2 \dot{v} + m_2 g$$

Use ④ & ⑤ to express in terms of $x, y, \dot{x}, \dot{y}$

$$S = \sqrt{S_x^2 + S_y^2} = \left[(m_1 \dot{u})^2 + (m_2 \dot{v} + m_2 g)^2 \right]^{1/2}$$

Q.2 $\quad \underset{\sim}{r}_m = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}, \quad \underset{\sim}{v}_m = \dot{x}\underset{\sim}{i} + \dot{y}\underset{\sim}{j} + \dot{z}\underset{\sim}{k}$

$$\mathcal{L} = T - V = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - mgz$$

$f = z - z(x,y) = 0$ (In this case $z(x,y) = h - \frac{xy}{h}$)

x: $\quad m\ddot{x} = \lambda\left(\frac{y}{h}\right)$ — ① $\qquad y$: $\quad m\ddot{y} = \lambda\left(\frac{x}{h}\right)$ — ②

z: $\quad m\ddot{z} + mg = \lambda$ — ③

Differentiating f twice wrt t,

$$\ddot{z} = -\left(\frac{\ddot{x}y}{h} + \frac{x\ddot{y}}{h} + \frac{2\dot{x}\dot{y}}{h}\right) \text{ — ④}$$

For Maple, we have to reduce this to $\ddot{x}$ and $\ddot{y}$ only. So we need to eliminate λ and $\ddot{z}$ from ①, ②, ③, ④

$$\dot{x} = u, \quad \dot{y} = v$$

$$\dot{u} = \frac{\frac{y}{h}\left(g - \frac{2\dot{x}\dot{y}}{h}\right)}{1 + \frac{x^2}{h^2} + \frac{y^2}{h^2}}$$

$$\dot{v} = \frac{\frac{x}{h}\left(g - \frac{2\dot{x}\dot{y}}{h}\right)}{1 + \frac{x^2}{h^2} + \frac{y^2}{h^2}}$$

solve using Maple

$x(0) = \frac{h}{4}$, $y(0) = 0$

$u(0) = 0$, $v(0) = 0$

$h = 1000$

Constraint force $S = S_x\underset{\sim}{i} + S_y\underset{\sim}{j} + S_z\underset{\sim}{k} = \lambda\left(\frac{y}{h}\right)\underset{\sim}{i} + \lambda\left(\frac{x}{h}\right)\underset{\sim}{j} + \lambda\underset{\sim}{k}$

But this is along the normal to f surface.

So $\underset{\sim}{S} = \underset{\sim}{N}$ (normal force)

If λ changes sign (+ve to −ve), $\underset{\sim}{N}$ becomes opposite in sign resulting in skier losing the contact