EN 137 HW7 Solutions

Problem 1

I_{zz} about center of mass:

For sphere: $= \frac{4}{5} m r_0^2$

For cube: $= \frac{1}{6} m l_0^2$

Parallel axis theorem $\rightarrow$ $I_{zz}^{o} = I_{zz}^{cm} + md^2$

For a sphere: $I_{zz}^{o} = \frac{4}{5} m r_0^2 + 2m(l_2^2 + l_1^2)$

For a cube $I_{zz}^{o} = \frac{1}{6} m l_0^2 + m l_1^2$

$\Rightarrow$ Total moment of intertia @ O:

$$I_{zz}^{o} = m\left[\frac{8}{5} r_0^2 + 4 l_2^2 + 6 l_1^2 + \frac{1}{3} l_0^2\right]$$

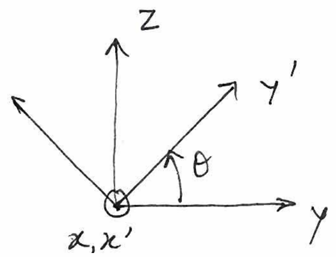
— Ans

Problem 2

(a) $h = 2a$

$$\Rightarrow [\bar{I}] = \frac{3}{10} ma^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ — Ans.

(b)



$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}}_{[\alpha]} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$\theta = \pi/3$

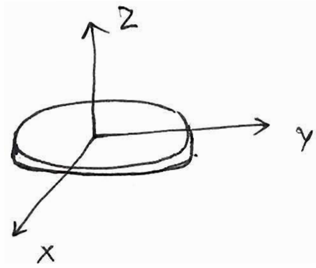
$$[I'] = [\alpha][I][\alpha]^T$$

$$[I] = 3ma^2 \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & 1/4 & 0 \\ 0 & 0 & 1/10 \end{bmatrix}$$ — Ans

gives

$$[I'] = ma^2 \begin{bmatrix} 0.75 & 0 & 0 \\ 0 & 0.4125 & -0.1949 \\ 0 & -0.1949 & 0.6375 \end{bmatrix}$$

Problem 3



MI about y = MI about x

$= I_t$

MI about $Z = I_a = 2I_t$

$$\Rightarrow [I] = I_t \begin{bmatrix} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$[H] = [I]\,\omega$$

$$\Rightarrow \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} = I_t \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \cancel{I_t} \cancel{\left[\omega_x + \omega_y + 2\omega\right]}$$

$$= I_t \begin{bmatrix} \omega_x \\ \omega_y \\ 2\omega_z \end{bmatrix}$$

$\underline{\omega} \cdot \underline{H} = |\omega|\,|H| \cos\phi$ where ϕ = Angle betn $\underline{\omega}$ & $\underline{H}$

$$\Rightarrow \cos\phi = \frac{\omega_x^2 + \omega_y^2 + 2\omega_z^2}{(\omega_x^2 + \omega_y^2 + 4\omega_z^2)^{1/2}\,(\omega_x^2 + \omega_y^2 + \omega_z^2)^{1/2}}$$

Let ω^2 = magnitude of $\underset{\sim}{\omega}$ = constant

$$\Rightarrow \omega_x^2 + \omega_y^2 = \omega^2 - \omega_z^2$$

$$\Rightarrow \cos\phi = \frac{\omega^2 + \omega_z^2}{(\omega^2 + 3\omega_z^2)^{1/2}(\omega^2)^{1/2}}$$

For maximum $\phi \rightarrow$ $\cos\phi$ should be minimum

$$\Rightarrow \frac{\omega^2 + \omega_z^2}{(\omega^2 + 3\omega_z^2)^{1/2}}$$ should be minimum

Let $\frac{w_2}{w_1} = \delta$ where $0 \leq \delta \leq 1$

$$\Rightarrow \cos\phi = \frac{1+\delta^2}{\sqrt{1+3\delta^2}}$$

$\frac{\partial}{\partial\delta}(\cos\phi) = 0 \Rightarrow \delta = 0$ or $\delta = \frac{1}{\sqrt{3}}$

$\delta = 0$ gives $\cos\phi = 1$; $\delta = \frac{1}{\sqrt{3}}$ gives $\cos\phi = 0.9428$

Hence max $\phi = \cos^{-1}(0.9428) = 19.47^\circ$ — Ans.