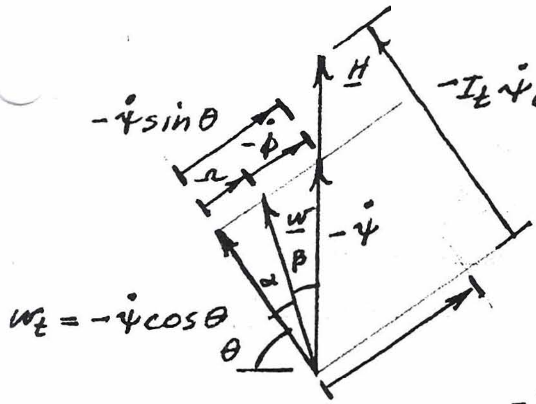
EN 137: ADVANCED ENGINEERING MECHANICS
HW # 9 SOLUTIONS



Observations:

(a.) $\beta = 30^\circ$

(b.) $\dot{\psi} > w$

(c.) $\theta = 45^\circ \Rightarrow \alpha = 15^\circ$

From schematic:

$$\frac{I_a}{-I_t \dot{\psi}\cos\theta} = \tan\theta \Rightarrow \frac{I_a}{I_t} = \frac{-\dot{\psi}\sin\theta}{\Omega}$$

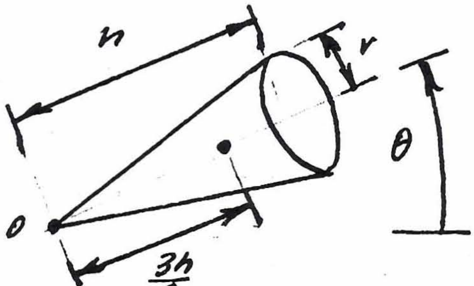
$$w\cos\alpha = -\dot{\psi}\cos\theta$$

$$\Rightarrow \dot{\psi} = \frac{-w\cos\alpha}{\cos\theta} = \boxed{-1.3660\,w}$$

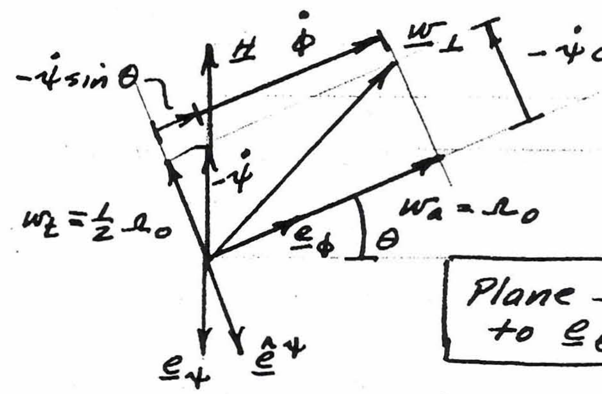
$$\Omega = -\dot{\psi}\sin\theta - (-\dot{\phi}) = w\sin\alpha$$

$$\Rightarrow \dot{\phi} = w(\sin\alpha - \cos\alpha) = \boxed{-\frac{\sqrt{2}}{2}w}$$

so,
$$\frac{I_a}{I_t} = \frac{\left(\frac{w\cos\alpha}{\cos\theta}\right)\sin\theta}{w\sin\alpha} = \frac{1}{\tan\alpha} = \boxed{3.7321}$$



Physical



Plane ⊥ to e_θ

$h = 4r$, $I_a = \frac{3}{10}mr^2$, $I_t = \frac{3}{4}mr^2$, $w_a = \Omega_o$, $w_t = \frac{1}{2}\Omega_o$

(a.) For axisymmetric body subject to free motion,

$$\dot{\psi} = \frac{-I_a}{I_t}\frac{\Omega_o}{\sin\theta} \quad \text{and} \quad w_t = -\dot{\psi}\cos\theta = \frac{1}{2}\Omega_o \;\left(\text{From figure}\right)$$

$$\Rightarrow \frac{-I_a}{I_t}\frac{\Omega_o}{\sin\theta} = \frac{-\Omega_o}{2\cos\theta} \Rightarrow \tan\theta = 0.8;\ \theta = 38.66^\circ$$

so,
$$\dot{\psi} = \frac{-\Omega_o}{2\cos(38.66^\circ)} = \boxed{-0.6403\,\Omega_o}$$

b.) Consider change in angular momentum when vertex is stopped. The angular momentum about the vertex is,

$$\underline{H}_o = \underline{R}_o \times m\underline{v} + \underline{H} \quad \text{where } \underline{R}_o = \frac{3}{4} h\, \underline{e}_\phi$$

Since constraining force acts through the vertex, the angular momentum about the vertex is unchanged,

$$\Delta \underline{H}_o = \underline{H}_o^+ - \underline{H}_o^- = \underline{0}$$

$$\Rightarrow \underline{R}_o \times m\underline{v}^- + \underline{H}^- = \underline{R}_o \times m\underline{v}^+ + \underline{H}^+$$

or,

$$\underline{H}^+ + mR_o^2 \underline{e}_\phi \times (\underline{\omega}^+ \times \underline{e}_\phi) = \underline{H}^- \qquad ①$$

Take ϕ component $(① \cdot \underline{e}_\phi) \Rightarrow \Omega_o^+ = \Omega_o^- = \Omega_o$

now, $\underline{\omega}^+ = \Omega_o \underline{e}_\phi + \dot{\theta}^+ \underline{e}_\theta + \dot{\psi}^+ \underline{e}_\psi$

introduce dual vector, $\hat{\underline{e}}^\psi = |\cos\theta|\, \underline{e}^\psi$

so, $\underline{\omega}^+ = \Omega_o \underline{e}_\phi + \dot{\theta}^+ \underline{e}_\theta + \dot{\psi}^+ \cos\theta\, \hat{\underline{e}}^\psi$

① becomes $\underline{H}^+ + mR_o^2 \{ \dot{\theta}^+ \underline{e}_\theta + \dot{\psi}^+ \cos\theta\, \hat{\underline{e}}^\psi \} = \underline{H}^- \qquad ②$

Take ψ component of ② $(② \cdot \underline{e}_\psi)$

$$p_\psi^+ + mR_o^2 \dot{\psi}^+ \cos^2\theta = p_\psi^-$$

and $p_\psi^+ = -I_a \Omega_o \sin\theta + I_t \dot{\psi}^+ \cos^2\theta$

$\quad\; p_\psi^- = -I_a \Omega_o \sin\theta + I_t \dot{\psi}^- \cos^2\theta$

$$\Rightarrow -I_a \Omega_o \sin\theta + I_t \dot{\psi}^+ \cos^2\theta + mR_o^2 \dot{\psi}^+ \cos^2\theta = -I_a \Omega_o \sin\theta + I_t \dot{\psi}^- \cos^2\theta$$

$$\Rightarrow \dot{\psi}^+ = \frac{I_t \dot{\psi}^-}{I_t + mR_o^2} = \frac{\dot{\psi}^-}{1 + \frac{mR_o^2}{I_t}}$$

$$\boxed{\dot{\psi}^+ = -0.04925\, \Omega_o}$$