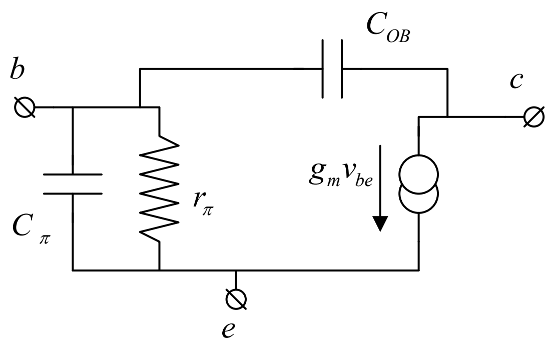
Engineering 1620: High Frequency Effects in BJT Circuits – an Introduction
Especially for the Friday before Spring Break

I have prepared these notes because on the day before a major vacation break some people find it necessary to leave early for travel connections. As this material is only partially covered in Sedra and Smith, I felt it worthwhile to summarize it for you. I hope it will also help you understand the calculations for Lab 5, a cascode amplifier for video signals.

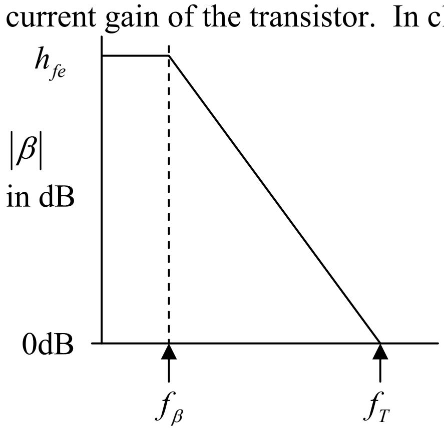
Over earlier classes in the week, I developed a number of formulae first for the capacitances in the junctions of a transistor and then for their effect on the gain, input impedance and output impedance of a generalized common emitter amplifier. Let me begin by recounting the capacitances of a transistor. (I gave you a handout with the full derivation of these results and have added that as an Appendix to this set of notes.)

The small-signal model in its simplest form has two capacitances: the base-emitter capacitance is commonly called C_π while the collector-base capacitance is variously C_{OB} or C_μ. (The first nomenclature is common on data sheets while the latter is more common in papers. The CAD/SPICE usage is "cmu." There is a slight but difficult to determine difference between the two when there is parasitic series resistance in the base. We do not observe this distinction.)

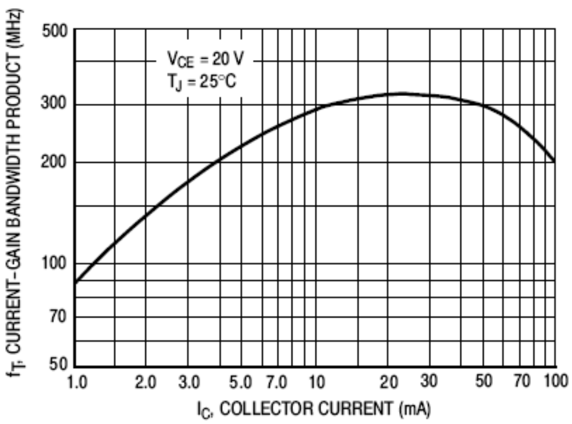


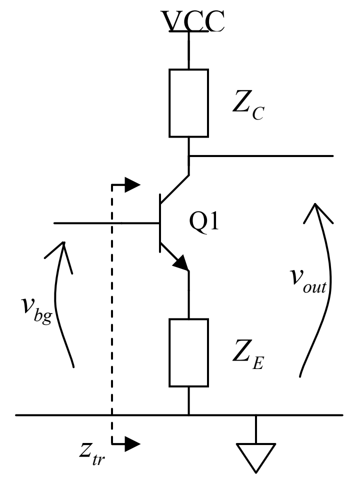
C_{OB} is the capacitance of a reverse biased junction and often has the dependence on voltage of an abrupt-junction. For at least one value of collector-base voltage, it is straightforwardly shown on any datasheet. The transconductance and the dynamic resistance of the base-emitter junction are needed first before one can tease the value of C_π out of the datasheet.

The Q-point determines the remaining model parameters in the usual way: $g_m \simeq \frac{qI_C}{kT}$ and $r_\pi = \left(1+h_{fe}\right) r_e \simeq \frac{\left(1+h_{fe}\right)kT}{qI_E}$ where I am using h_{fe} interchangeably with β as the DC current gain of the transistor. In class we showed that because of the base-emitter capacitance, the current gain becomes frequency dependent. For a grounded-collector circuit, the gain becomes $\beta(s) = \frac{i_c}{i_b} = \frac{h_{fe}}{1+sr_\pi\left(C_\pi + C_\mu\right)}$. This is a single pole function that looks like this sketch when $|\beta|$ is graphed on a log-log Bode plot. Here we have defined two frequencies: f_β at which the gain is down –

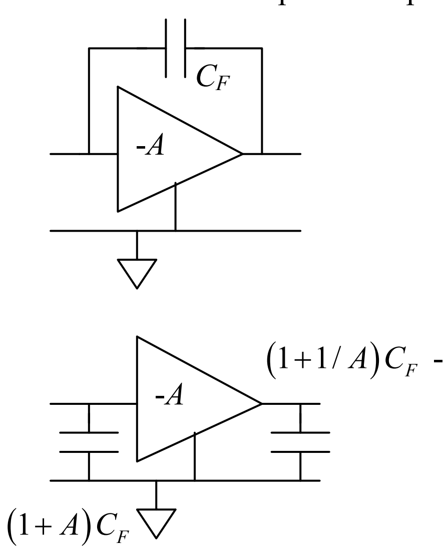


3dB from its DC value and f_T at which the magnitude of the gain is unity. If h_{fe} is large, then $f_\beta \simeq \frac{f_T}{1+h_{fe}}$. The datasheet gives f_T as a function of the quiescent current because that frequency is not sensitive to the DC current gain. (It also helps in advertising the properties of a transistor to advertise the larger number!) The dependence of f_T on current comes from r_π (or equivalently r_e) being inversely proportional to current. Part of the C_π capacitance is due to the depletion layer and is roughly independent of current. The other part of C_π is due to charge in transit from emitter to collector and when that is dominant, $r_e C_\pi = \tau$, the transit time. The typical curve for a 2N2222A device is shown below next to a generalized common emitter circuit.





The capacitance from collector to base complicates the simple calculation of gain and input impedance. To simplify the problem, we take advantage of the Miller effect (named after John W. Miller who published it in 1920). Miller's theorem points out the equivalence for input impedance, output impedance and gain of the two block diagrams below as long as the gain of the amplifier is known with the feedback capacitor in place. (The proof of the theorem simply equates the current in the feedback capacitor to the two currents through the capacitors of the second configuration.) Because the feedback capacitor connects between output and input and the output voltage is often bigger than the input, the current in the capacitor is generally greater than it would be if the capacitor were across the input to ground. The theorem points out that this is equivalent to a larger, possibly frequency-dependent capacitor across the input and a marginally bigger one across the output. What often makes this theorem useful is that the low frequency gain is known and that gain is constant enough to use over most of the useful frequency range of the amplifier.



We also developed a set of formulas for the input impedance and gain of the generalized common emitter amplifier shown at the top right. The idea was to use Miller's theorem to move the collector base capacitance to two places: the (1+1/A)C component simply became part of Z_C and contributes to calculating the gain. The (1+A)C_{OB} element moved to the left and ended in parallel with the input. This transform left only C_π to complicate life. The input impedance with only that parasitic capacitance was:

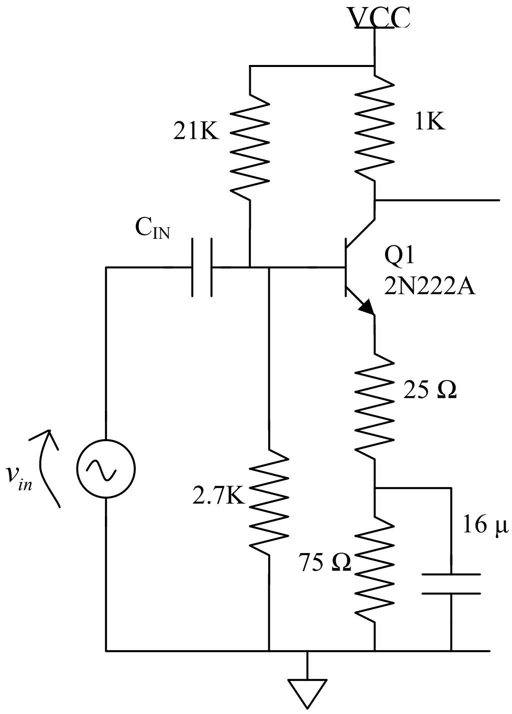
$$z_{tr} = \frac{\left(1+h_{fe}\right)\left(r_e + Z_E\right)}{1+sr_\pi C_\pi} + \frac{sr_\pi C_\pi Z_E}{1+sr_\pi C_\pi}$$

The first term is our old result for input impedance at low frequency but now there is a decrease in impedance with increasing frequency from an extra single pole at $f_\beta = \frac{1}{2\pi r_\pi C_\pi} = \frac{f_T}{1+h_{fe}}$, a nice but not surprising result. The second term assures that even when the base capacitance shorts the base-emitter junction, there will still be Z_E left as part of z_{tr}. This term is not important until roughly f_T and one would not usually try to use a device close to its maximum frequency limit. Notice that the first term has the form of a resistor $R = \left(1+h_{fe}\right)\left(r_e + Z_E\right)$] in parallel with a capacitor of value $C = \frac{r_e C_\pi}{\left(r_e + Z_E\right)}$. The resistance value is the low-frequency result we derived a couple of weeks ago. The capacitance is proportional to but generally smaller than C_π. We will use this result in an analysis example.

The gain formula has a similar form: $G = -\frac{\alpha Z_C'}{\left(r_e + Z_E\right)} \cdot \frac{1}{1+sr_e Z_E C_\pi / \left(r_e + Z_E\right)}$ where Z_C' is Z_C in parallel with Miller's second capacitor $C_\mu\left(1+1/A\right)$. [I am playing fast and loose with exact results here. Actually as A becomes frequency dependent one has to be careful to include the effect of that change on the input capacitance. We will see this more clearly in MOS circuits later.] The first factor is the low frequency gain and the second is a new pole generally somewhat above f_T. In other words, C_π does not have a big effect on the Gain except through a decrease in input impedance that causes loading of the input signal source.

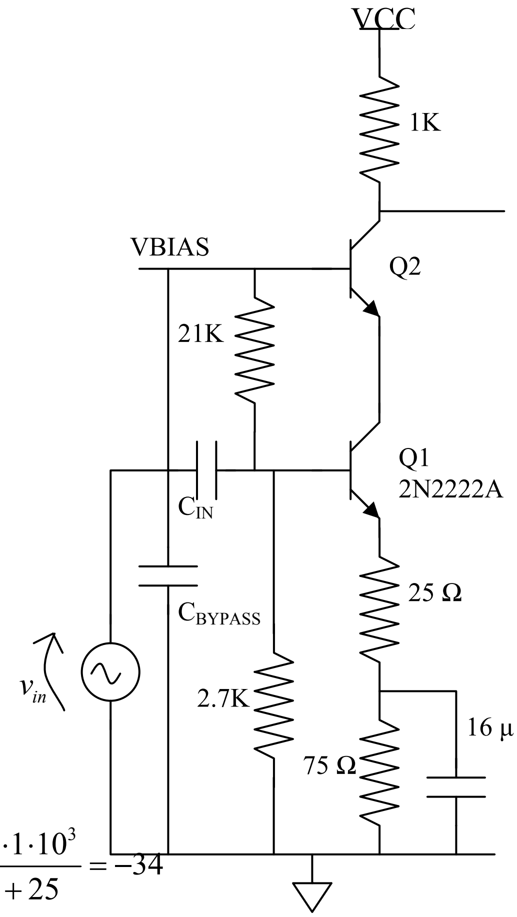
Now let us look at an example of these effects. The circuit on the left below is one we used as a low frequency example some time ago. It has a quiescent point around 6 mA, a current gain about h_{fe} = 120 typical, and therefore r_e = 4.2 ohms and $r_\pi = 500$ ohms. From the graph above, $f_T \approx 250$ MHz. The datasheet value of C_{OB} is 7 pf. From this,

$$C_\pi = \frac{1}{2\pi f_T r_e} - C_{OB} = \frac{1}{6.28 * 4.2 * 2.5 \cdot 10^{+8}} - 7 \cdot 10^{-12} = 146\,\text{pf}.$$

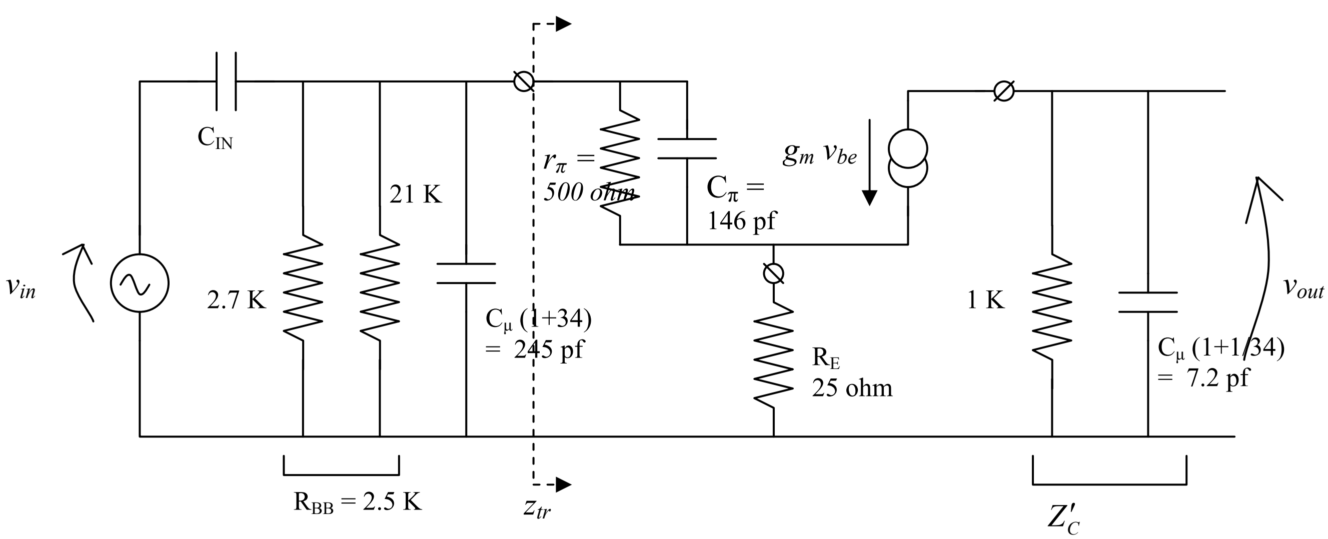


The mid band gain is

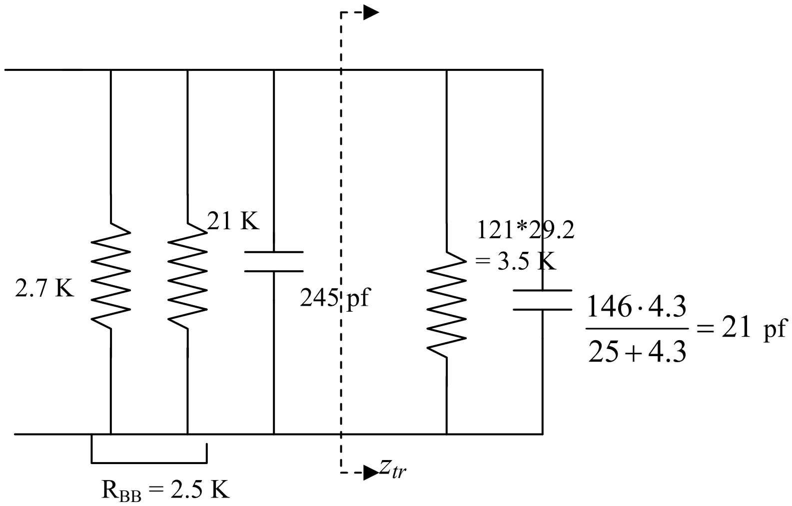
$$\frac{-.99\cdot 1\cdot 10^{3}}{4.3+25}=-34$$



By employing Miller's theorem, we can draw the small signal model as:



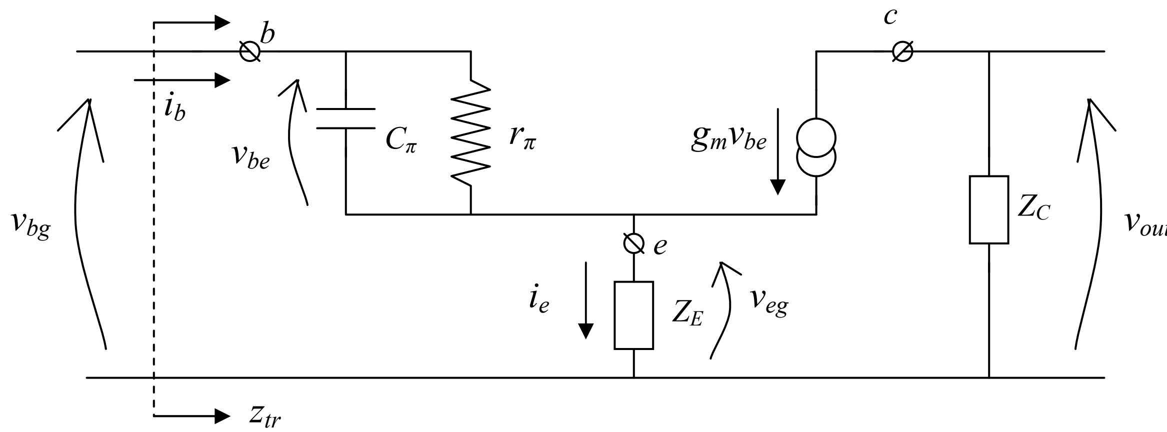
For Z_{in} we have (neglecting C_{IN} because it provides only a low-pass cutoff):



The resulting system is 1.42 K in parallel with 266 pf. The input impedance is dominated by the Miller capacitance (245 pf of 266 pf total) and its magnitude will start to decrease with single pole behavior at 421 KHz. By contrast the effect of C_μ on the gain is to introduce a high-frequency low-pass pole at about 23 MHz from the 1K resistor and 7 pf capacitance in parallel. [If there is appreciable source loading at this frequency, it is even possible that this second pole will be even higher in frequency but that is a later topic.]

An amplifier that has constant gain to some high frequency but has so low an input impedance as to load the signal source well below its gain cutoff frequency is a poor design because one cannot use the gain for the full range of the amplifier's potential usefulness. The circuit on the right above is called a cascode amplifier and it attempts to solve this problem with a second transistor. The tandem arrangement of a common emitter stage, Q1, with a common base stage, Q2, is called a cascode connection. (And no, this is not a spelling error.) The voltage gain of the common emitter Q1 is very low, fractional in this case, because Z_C for that stage is the input impedance of the common base stage, Q2. (That input impedance is $z_{in2} = r_{e2} \| C_{\pi 2}$. The capacitance of Q2 is not important at a few megahertz so the voltage gain of Q1 is $G = -\frac{\alpha r_e}{(r_e + Z_E)} \cong -\frac{0.99 \cdot 4.2}{4.2+25} = -.15$) There is no longer a direct capacitance between input and output. The output load no longer affects the input impedance. For that reason, cascode circuits are sometimes said to be 'unilateral.' This time the input impedance is the same 1.42 K resistive part but the capacitor is only 21+7 = 28 pf and the capacitive part becomes a factor in loading the input only above 4.5 MHz. That is a full order of magnitude improvement in pole placement.

Appendix: The Effect of C_π on Input Impedance and Gain of a BJT CE Circuit



Basic Equations:

KCL at the emitter terminal, *e*: $0 = i_b + g_m v_{be} - \dfrac{v_{eg}}{Z_E}$

Ohm's law across base-emitter: $v_{be} = \dfrac{r_\pi i_b}{1 + sC_\pi r_\pi}$

KVL from input across base-emitter and emitter to ground: $v_{bg} = v_{eb} + v_{bg}$

Definition of z_{tr}: $z_{tr} \equiv \dfrac{v_{bg}}{i_b}$

Connection between hybrid-pi transconductance model and current controlled h-model: $g_m r_\pi = \beta_0$ where β_0 is the low-frequency current gain.

Steps to solve:

$$\frac{v_{eg}}{Z_E} = i_b\left(1 + \frac{g_m r_\pi}{1 + sC_\pi r_\pi}\right) = i_b\left(\frac{1 + \beta_0 + sC_\pi r_\pi}{1 + sC_\pi r_\pi}\right)$$

$$v_{eg} = Z_E\left(\frac{1 + \beta_0 + sC_\pi r_\pi}{1 + sC_\pi r_\pi}\right)i_b \quad \text{and} \quad v_{be} = \frac{r_\pi i_b}{1 + sC_\pi r_\pi}$$

$$v_{bg} = \left(\frac{r_\pi + Z_E\left(1+\beta_0 + sC_\pi r_\pi\right)}{1+sC_\pi r_\pi} \right) i_b \quad \text{(Used: } r_\pi = \left(1+\beta_0\right) r_e \text{)}$$

Input impedance is:

$$z_{tr} = \frac{\left(1+\beta_0\right)\left(r_e + Z_E\right) + sC_\pi r_\pi Z_E}{1+sC_\pi r_\pi}$$

$$z_{tr} = \frac{\left(1+\beta_0\right)\left(r_e + Z_E\right)}{1+sC_\pi r_\pi} + \frac{sC_\pi r_\pi Z_E}{1+sC_\pi r_\pi}$$

Gain is the ratio:

$$H\left(s\right) = \frac{v_{out}}{v_{bg}} = \frac{-Z_C i_C}{v_{bg}} = \frac{-Z_C g_m v_{be}}{v_{bg}} = \frac{-Z_C g_m r_\pi i_b}{v_{bg}\left(1+sC_\pi r_\pi\right)} = \frac{-Z_C g_m r_\pi i_b}{v_{bg}\left(1+sC_\pi r_\pi\right)} = \frac{-Z_C \beta_0}{z_{tr}\left(1+sC_\pi r_\pi\right)}$$

$$H\left(s\right) = \frac{-Z_C \beta_0}{\left(1+\beta_0\right)\left(r_e + Z_E\right) + sC_\pi r_\pi Z_E} = \frac{-Z_C \beta_0}{\left(1+\beta_0\right)\left(r_e + Z_E\right)} \cdot \frac{1}{1 + +sC_\pi r_e Z_E / \left(r_e + Z_E\right)}$$

Summary:

The input impedance is lowered by C_π beginning at the beta cutoff frequency. It is also asymptotic to Z_E for frequencies above f_T. In this equation the first term is the low frequency impedance with a new pole at the beta cutoff frequency. The second term takes care of the behavior that makes Z_E the impedance at very high frequency.

$$z_{tr} = \frac{\left(1+\beta_0\right)\left(r_e + Z_E\right)}{1+sC_\pi r_\pi} + \frac{sC_\pi r_\pi Z_E}{1+sC_\pi r_\pi}$$

Notice that the first term has the form of a resistance in parallel with a capacitance. As you did in the first lab, we can manipulate that into the form of a resistance with an equivalent capacitance in parallel by multiplying and dividing the frequency in the denominator by the resistance from the numerator.

$$z_{tr} = \frac{\left(1+\beta_0\right)\left(r_e + Z_E\right)}{1+sC'R'} \quad \text{if } C' = \frac{C_\pi r_e}{\left(r_e + Z_E\right)} \text{ and } R' = \left(1+\beta_0\right)\left(r_e + Z_E\right)$$

The gain is little affected except there is a new pole at some frequency above f_T. The first factor is the low frequency gain and the second is a new high frequency pole.

$$H(s) = \frac{-Z_C \beta_0}{(1+\beta_0)(r_e + Z_E)} \cdot \frac{1}{1 + +sC_\pi r_e Z_E / (r_e + Z_E)}$$