ENGN 2210 : Continuum Mechanics Fall 2016

2 Mathematical Preliminaries

2.1 Vectors & Index notation

Definitions

* Euclidean space $\mathbb{R}^3$: set of all triads of real #s (x_1, x_2, x_3)

* Point : $\underline{x} \in \mathbb{R}^3$: a particular x_1, x_2, x_3

  Index notation x_i $i = 1, 2, 3$

* Metric : $g(\underline{x}, \underline{y})$ $\forall$ $x \in \mathbb{R}^3$

$$g(x,y) = \sqrt{(x_i - y_i)(x_i - y_i)}$$

Index Notation $(x_i - y_i)(x_i - y_i) = (x_1 - y_1)^2 + (x_2 - y_2)^2 + (x_3 - y_3)^2$

Einstein summation convention

* Vector : $\underline{V} = \underline{x} - \underline{y} \quad \forall\ \underline{x}, \underline{y} \in \mathbb{R}^3$

$V_i = x_i - y_i \Leftarrow$ 3 eqs

Indices must agree

* Vector Operations : Let $\underline{v}, \underline{w}$ be vectors

Dot product : $\underline{V} \cdot \underline{W} = V_i W_i$

Cross product $[\underline{V} \times \underline{W}]_i = \epsilon_{ijk} V_j W_k$

Permutation symbol. $\epsilon_{ijk} = \begin{cases} \epsilon_{123} = \epsilon_{231}, \epsilon_{312} = +1 \\ \epsilon_{321} = \epsilon_{213} = \epsilon_{132} = -1 \\ \text{all others zero} \end{cases}$

Special vectors

Unit vector $\underline{n} \cdot \underline{n} = 1$

Null vector $\underline{n} \cdot \underline{n} = 0$

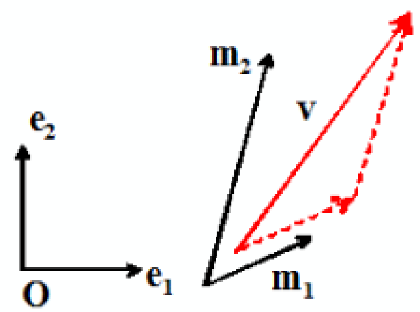
* Linearly Independent vectors $\underline{m}_i$

$\underline{m}_i$ are linearly independent iff $c_i \underline{m}_i \neq 0$ $\forall\, c_i \neq 0$

* Vector Basis: Any 3 linearly independent vectors

Any vector can be expressed as the weighted sum of three basis vectors

$$\underline{V} = V_i^{(m)} \underline{m}_i$$



Note $\underline{V}$, $V_i^{(\underline{m})}$ – 3 real #s
$\underline{m}_i$ – 3 sets of 3 real #s

Given $\underline{V}$, given $\underline{m}_i$, to find $V_i^{(\underline{m})}$

$$\underbrace{\underline{m}_k \cdot \underline{V}}_{\text{3 numbers}} = \underbrace{\underline{m}_k \cdot \underline{m}_i}_{g_{ki}} \; V_i^{(\underline{m})} \leftarrow \text{3 unknowns}$$

$g_{ki} \leftarrow$ 3x3 matrix

$$V_i^{(\underline{m})} = g_{ij}^{-1} \left(\underline{m}_j \cdot \underline{V}\right) \qquad [g]^{-1}[g] = [I]$$

Here g_{ij} is a 3x3 matrix $\begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{bmatrix}$

Row → ← Column

g_{ij}^{-1} are components of $[g]^{-1}$

Row → ← column