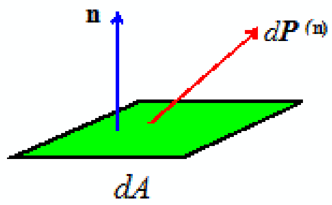
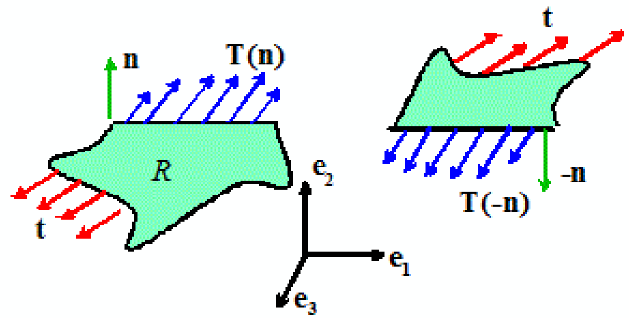
Review

Internal Traction

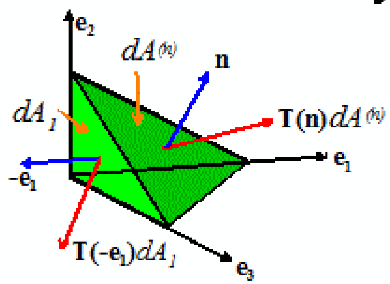
$$\mathbf{T}(\mathbf{n}) = \lim_{dA\to 0} \frac{d\mathbf{P}(\mathbf{n})}{dA}$$





Cauchy Stress $\mathbf{T}(\mathbf{n}) = \mathbf{n}\boldsymbol{\sigma}$

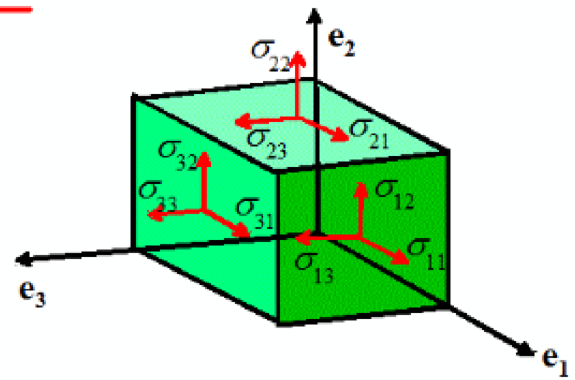
(Follows from Newton's laws for Cauchy Tetrahedron)



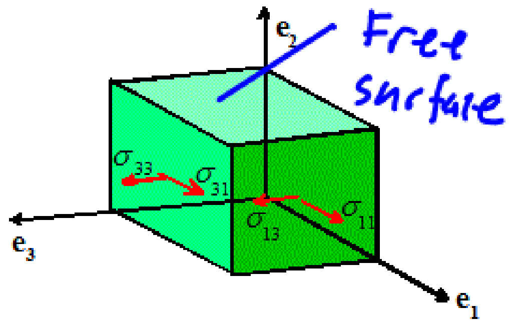
Physical Interpretation of stress components

σ_{ij}

normal to plane — Direction of traction



Stresses at an external surface

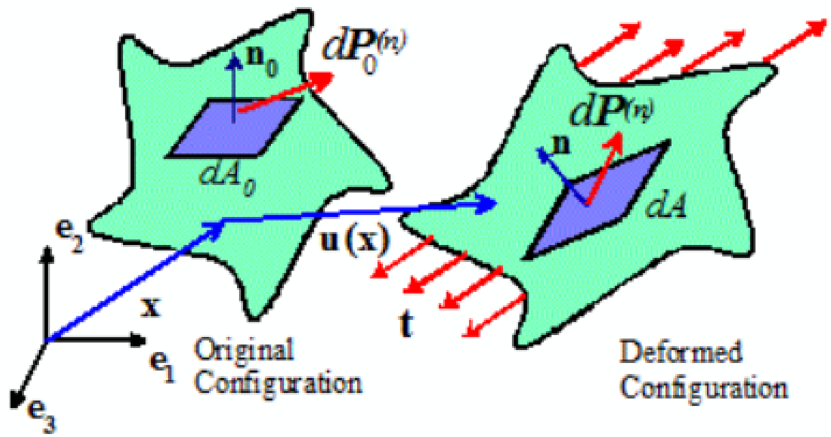


$$\underline{t} = \underline{n}\,\sigma$$

For a stress free surface $n\underline{\sigma} = 0$

(Show later that $\sigma = \sigma^T$ to satisfy angular momentum conservation)

Other stress measures



Let $F = \nabla y = I + \nabla \underline{u}$

$J = \det(F)$

Define :

① Kirchhoff stress $\boxed{\tau = J\sigma}$

Property (show later) $\tau : D$ gives rate of work done on solid per unit ref volume

② Nominal stress (1st Piola-Kirchhoff stress)

$$\boxed{S = J F^{-1} \sigma}$$

[Also some authors use $S = J \sigma F^{-T}$ - transpose of our definition]

Property $\quad d\underline{P} = dA_0 \, \underline{n}_0 \, S$

To see this note $\quad d\underline{P} = dA \, \underline{n} \, \sigma$

Recall: $\quad dA \, \underline{n} = J F^{-T} dA_0 \, \underline{n}_0$

$$\Rightarrow d\underline{P} = dA_0 \left(J F^{-T} \underline{n}_0 \right) \sigma = dA_0 n_0 \underbrace{J F^{-1} \sigma}_{S}$$

③ Material stress (2nd Piola-Kirchhoff stress)

$$\boxed{\Sigma = J F^{-1} \sigma F^{-T}}$$

Physical interpretation: define "reference" force vector satisfying $d\underline{P} = F\, d\underline{P}^0 = d\underline{P}^0 F^T$

$$d\underline{P}^0 = dA_0\, \underline{n}_0\, \Sigma$$

To see this note $d\underline{P} = d\underline{P}^0 F^T$

$$d\underline{P} = dA\, \underline{n}\, \sigma = \underbrace{dA_0\, \underline{n}_0 \underbrace{J F^{-1} \sigma F^{-T}}_{\Sigma}}_{d\underline{P}^0} F^T$$