Review

Solving acoustics problems:

Wave speed $c_s = \sqrt{\left.\frac{\partial p}{\partial \rho}\right|_{s=const}} = \sqrt{\gamma R \theta_0}$

Flow potential $$\frac{\partial^2 \Omega}{\partial t^2} = c_s^2 \frac{\partial^2 \Omega}{\partial y_i \partial y_i}$$

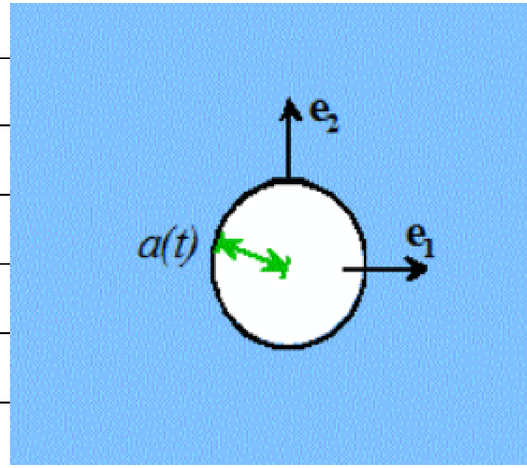
Boundary conditions: $\frac{\partial \Omega}{\partial y_i} n_i = V_n^*$ (surfaces with known velocity)

$-\rho_0 \frac{\partial \Omega}{\partial t} = p^*$ (boundaries with known pressure)

Velocity: $v_i = \frac{\partial \Omega}{\partial y_i}$ Pressure: $p = -\rho_0 \frac{\partial \Omega}{\partial t}$

Example: Vibrating sphere

$a(t) = a_0 + A e^{i\omega t} \qquad A \ll a_0$



Try a potential of form $\Omega = \frac{1}{r} f(t - r/c_s)$

Substitute into wave eq : straightforward to show $\frac{\partial^2 \Omega}{\partial t^2} = c_s^2 \nabla_y^2 \Omega$

$n_i = y_i / a_0$

Boundary conditions

$$i\omega A e^{i\omega t} = v_i n_i = -\left[\frac{f(t - a_0/c_s)}{a_0^2} + \frac{1}{a_0 c_s} f'(t - a_0/c_s)\right] \overset{=1}{\cancel{\frac{y_i}{A}}\frac{y_i}{a_0}}$$

Assume $f(t) = C e^{i\omega t}$

$$-i\omega a_0^2 A e^{i\omega t} = C e^{i\omega t} e^{-i\omega a_0/c_s}\left[1 + \frac{i\omega a_0}{c_s}\right]$$

$$\Rightarrow \boxed{C = \frac{-i\omega a_0^2 A e^{i\omega a_0/c_s}}{\left[1 + \frac{i\omega a_0}{c_s}\right]}}$$

Now $v_i = \frac{\partial \Omega}{\partial y_i} = -C e^{i\omega(t - r/c_s)} \left[\frac{1}{r^2} + \frac{i\omega}{r c_s}\right] \frac{y_i}{r}$

$$p = -\rho_0 \frac{\partial \Omega}{\partial t} =$$

Far field behavior $v_i = \frac{1}{r} f(t - r/c_s) \frac{y_i}{r}$

– "monopole"

Can generate more solutions by taking spatial derivatives of this function

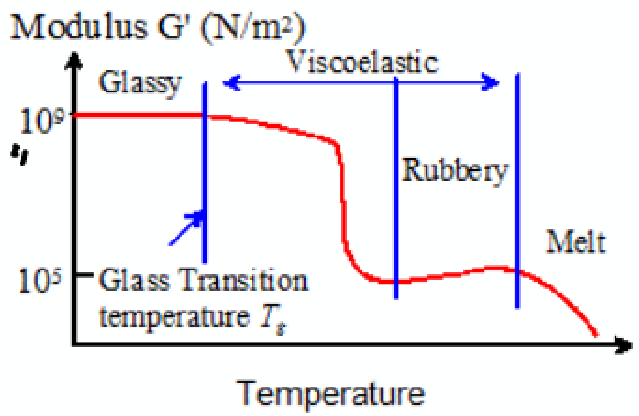
eg $\frac{\partial v_i}{\partial y_j}$ – "dipole"

7) Elasticity

* Goal: understand large strain elasticity
  - Linearize ⇒ linear elasticity

* Material models for elastic materials
  (1) Linear elasticity - models reversible deformation in most solid materials @ low stresses
  (2) Hyperelastic - model polymers experiencing large strains in "rubbery" regime

* General behavior of cross-linked polymers:
  @ low temp stiff / brittle "glassy"
  @ intermediate - softens; rate & history dependent
  @ high temp: "rubbery"



* Features of rubbery regime

(1) Low shear modulus ($\sim 2 GPa$)
(2) High bulk modulus ($\sim 100 GPa$)
(3) Modulus increases with temp
(4) Gives off heat when stretched
(5) History independent
Rate independent
Reversible
} approximately

Hyperelastic material models approximate this regime of behavior

Structure of hyperelastic material models

Assume : Local action

: No history dependence : state is completely specified by F ; θ

We will show that constitutive equations have the following structure

Specific internal energy $\varepsilon = \hat{\varepsilon}(C, \theta)$ $\quad C = F^T F$

" entropy $s = \hat{s}(C, \theta)$

Helmholtz $\psi = \hat{\psi}(C, \theta)$

Stress response function $\Sigma = \hat{\Sigma}(C, \theta)$

Heat transfer response funct $\underline{Q} = \hat{\underline{Q}}(C, \theta, \nabla\theta)$