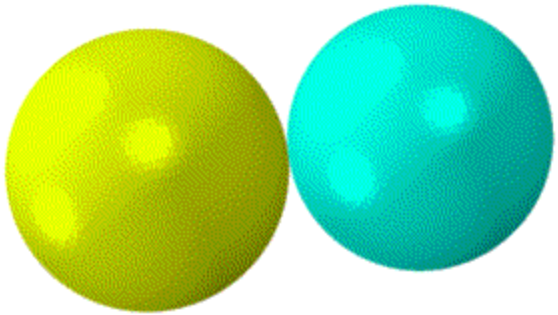
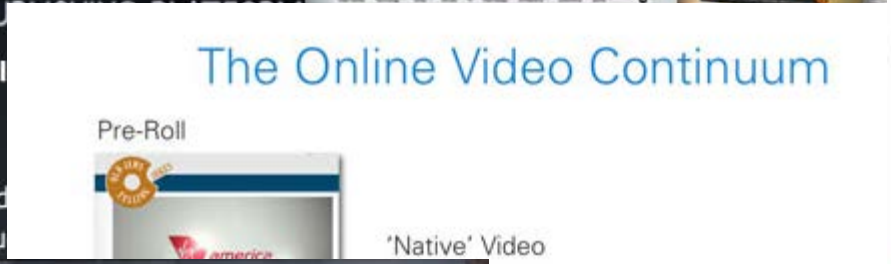
School of Engineering
Brown University

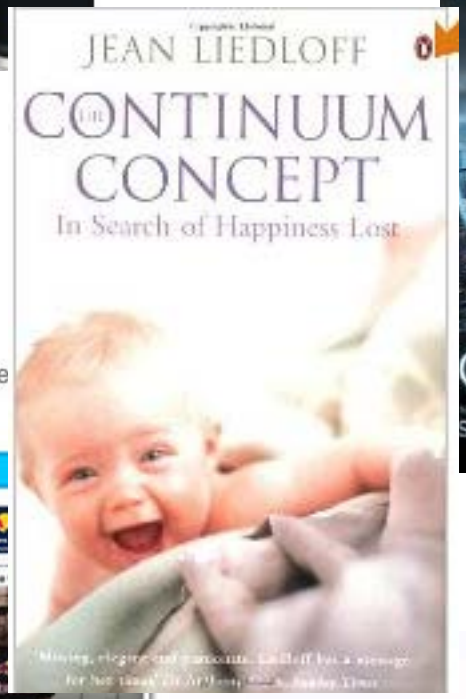
ENGN2210 CONTINUUM MECHANICS

Allan Bower

Fall  2016

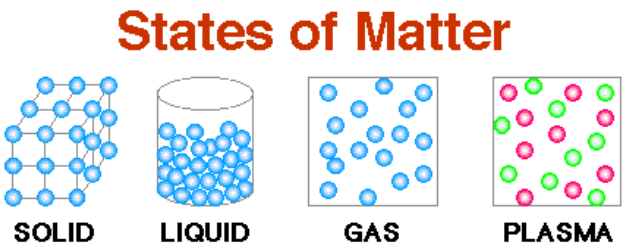
What is continuum mechanics?



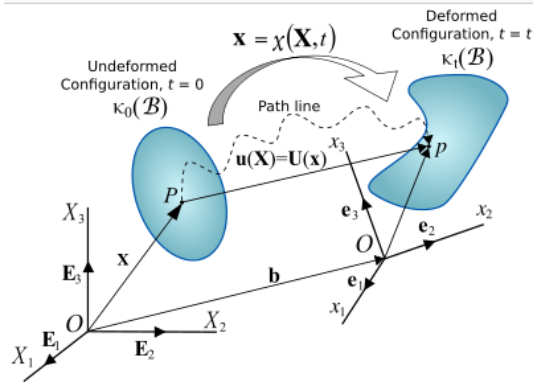


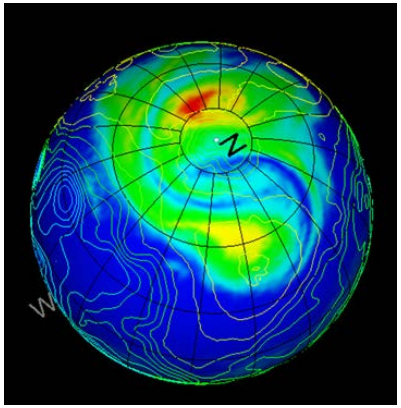
© Continuum Fashion

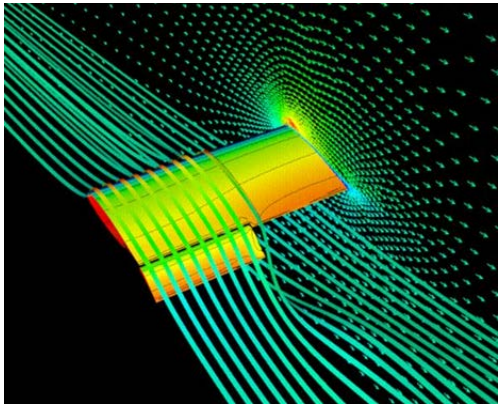
What is continuum mechanics?

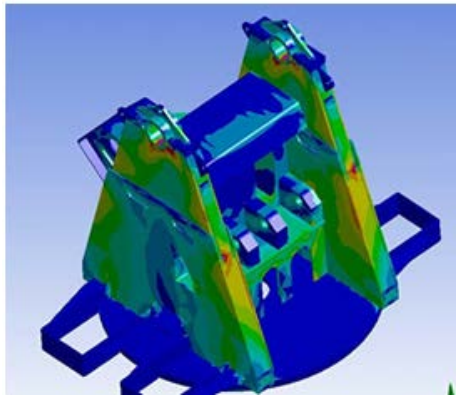


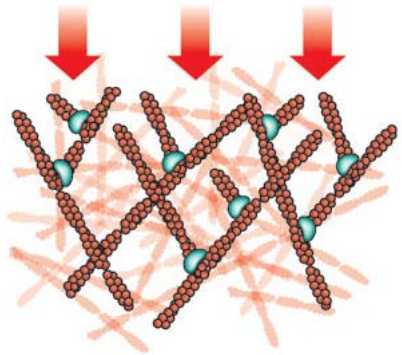
- A continuum – idealized form of matter
- Descriptions of motion and forces for continua
- Physical laws – conservation of mass; momentum; energy
- Thermodynamics – entropy
- Constitutive laws (empirical)
- Mathematical Techniques – tensors, differential geometry;
- Numerical methods
- Applications









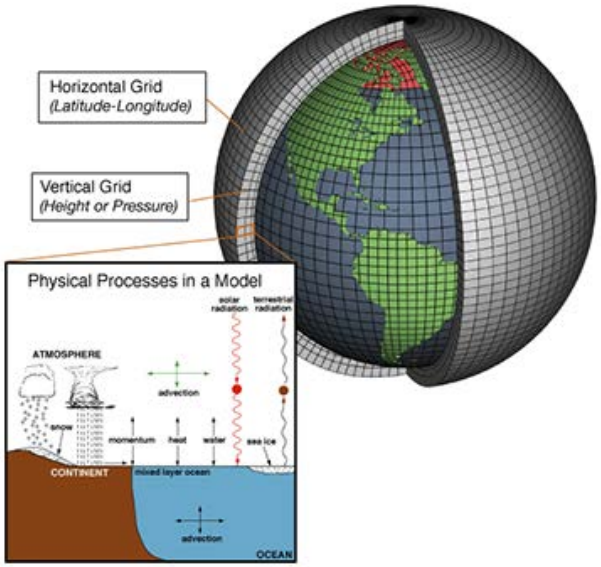


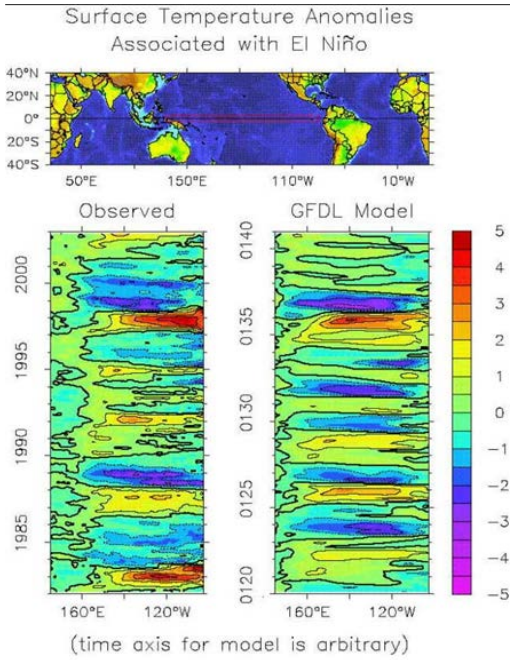
Applications of Continuum Mechanics – climate modeling

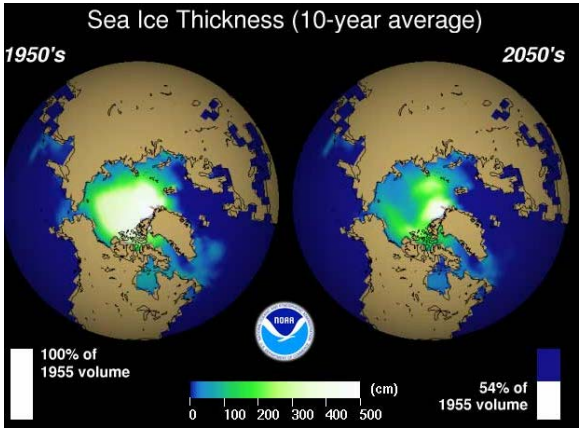
Governing equations

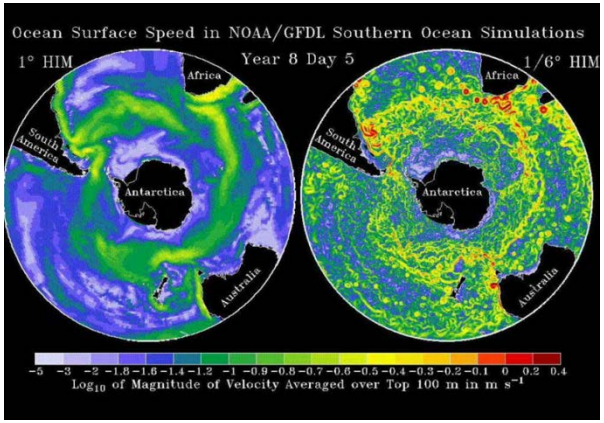
- Momentum balance
- Thermal energy balance
- Mass conservation
- Constitutive eqs for air/water

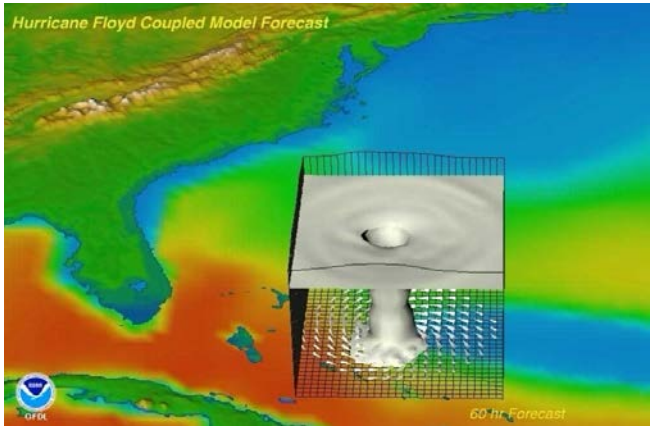
- $\frac{dv}{dt} = -(1/\rho)\nabla p - g(r/r) + (1/\rho)\left[\nabla \cdot (\mu \nabla v) + \nabla(\lambda \nabla \cdot v)\right]$
- $c_v \frac{dT}{dt} + p\frac{d\alpha}{dt} = q + f$
- $\frac{d\rho}{dt} + \rho \nabla \cdot v = 0$
- $p = \rho R T.$







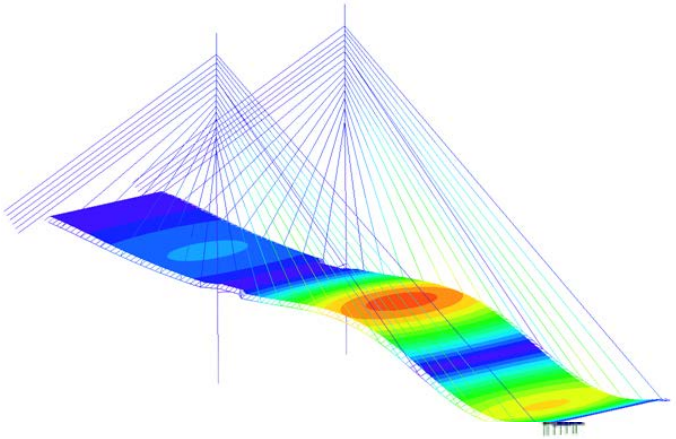


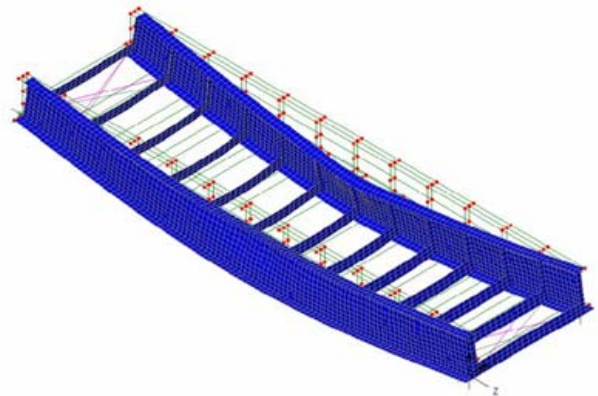


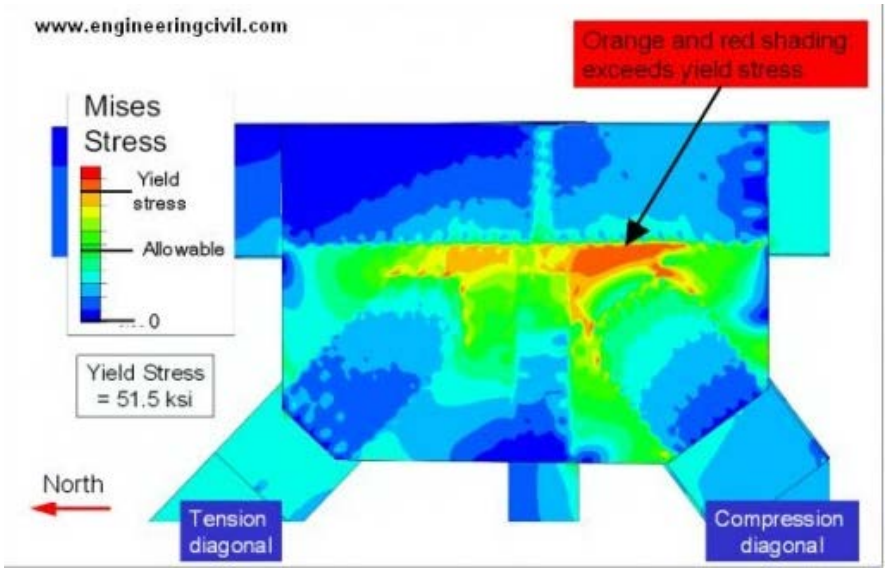
Applications of Continuum Mechanics: Structural Mechanics

Governing equations

- Equilibrium (momentum for vibrations)
- Kinematics (beams, shells, plates)
- Material Models (elasticity, plasticity)



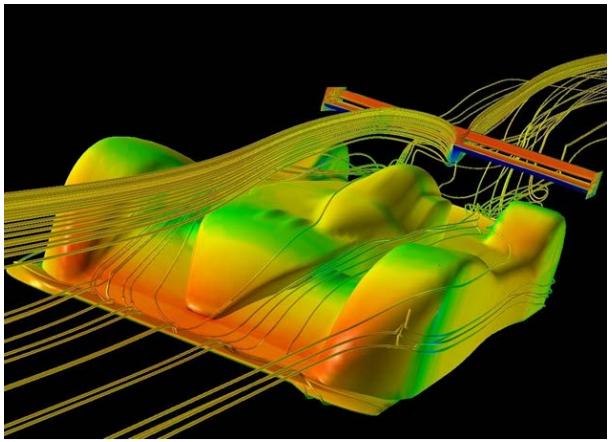


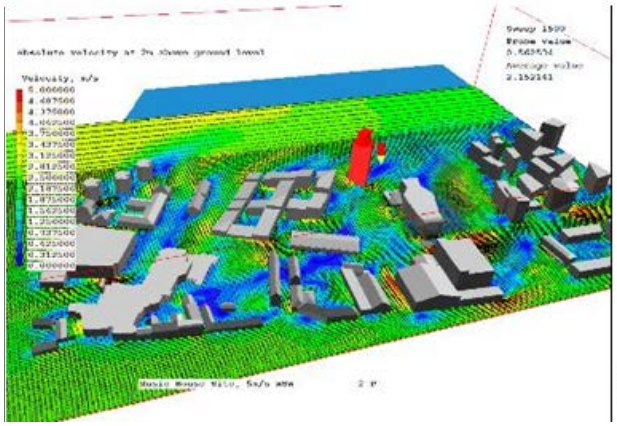


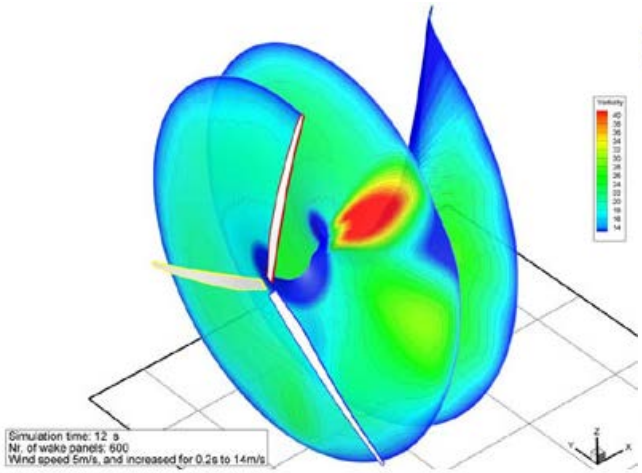
Applications of Continuum Mechanics: Aerodynamics

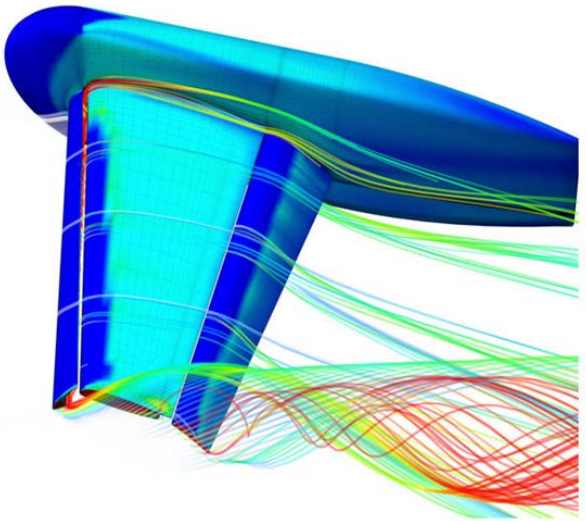
Governing equations

- Momentum balance (Navier Stokes)
- Turbulence closure conditions
- Mass conservation
- Constitutive relations (gas law?)





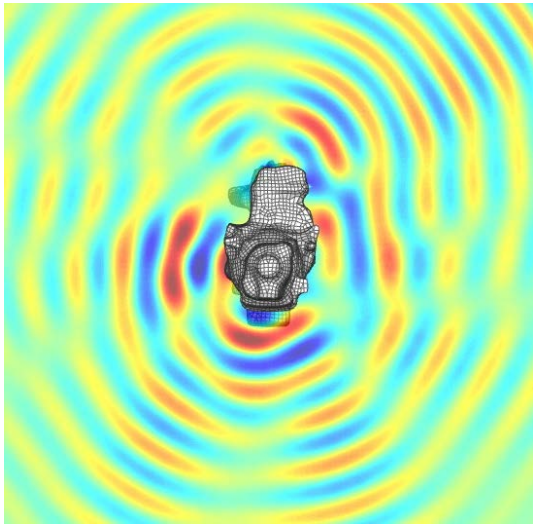


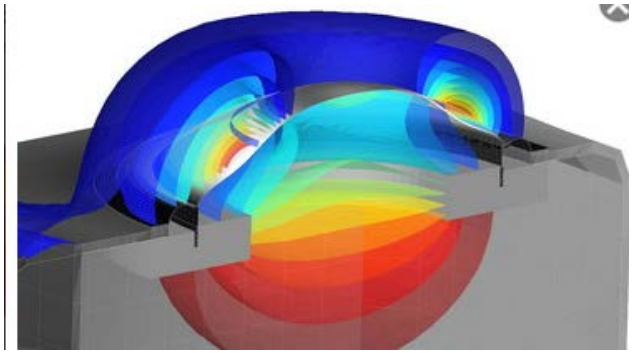


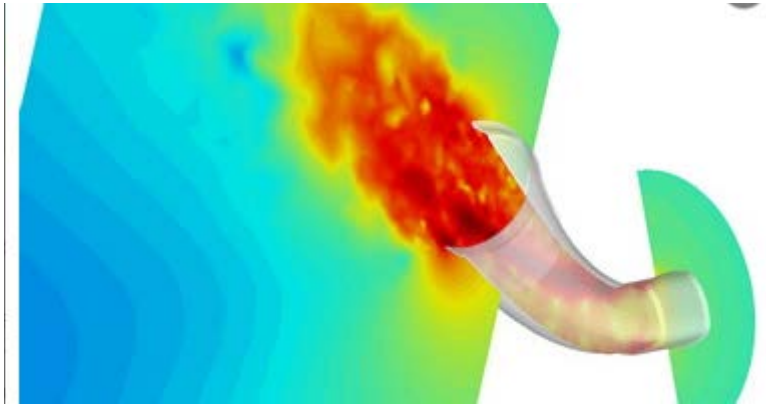
Applications of Continuum Mechanics: Acoustics

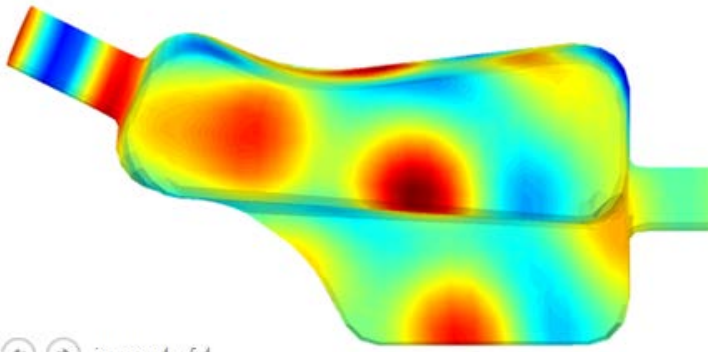
Governing equations

- Linearized Navier-Stokes equations
- Ideal gas law
- Mass conservation





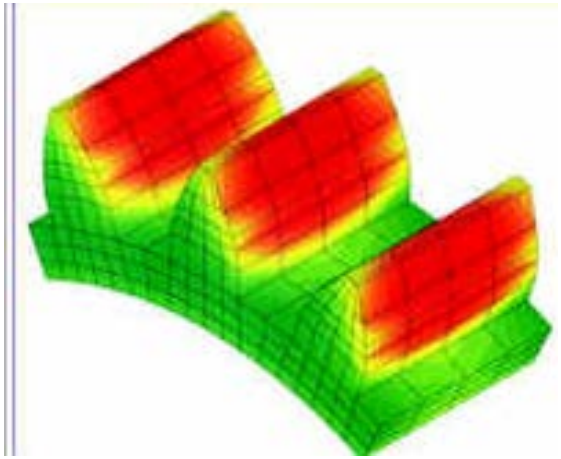


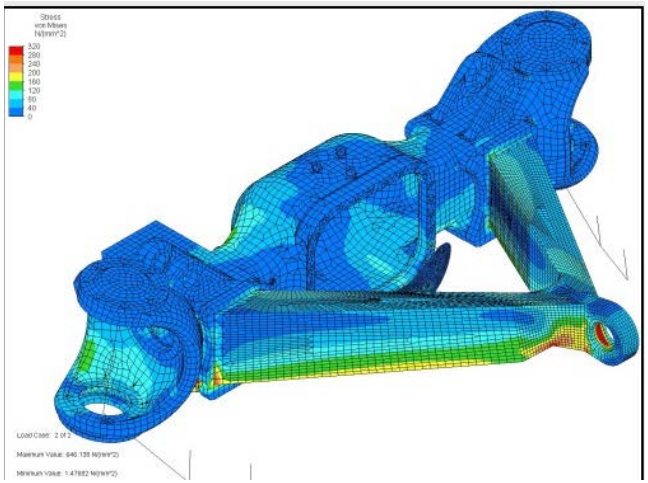


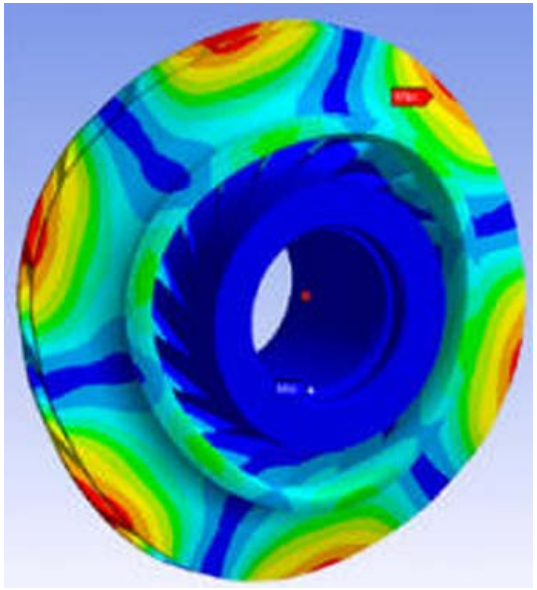
Applications of Continuum Mechanics: Machine Design

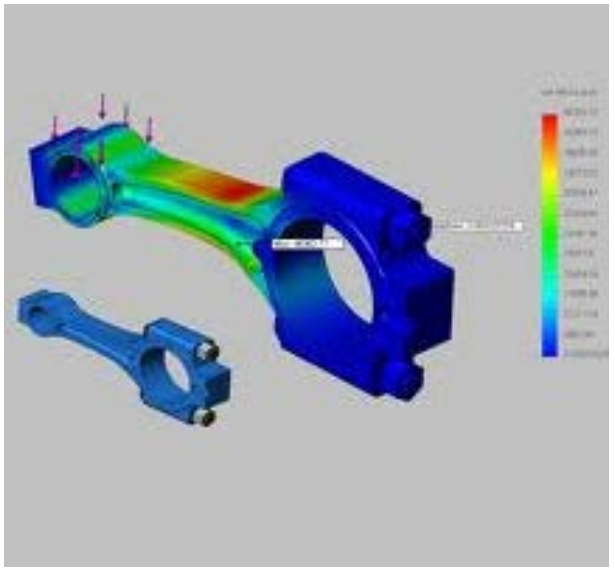
Governing equations

- Equilibrium (momentum for modal analysis)
- Constitutive equations – elasticity/plasticity
- Failure criteria/Fracture Mechanics





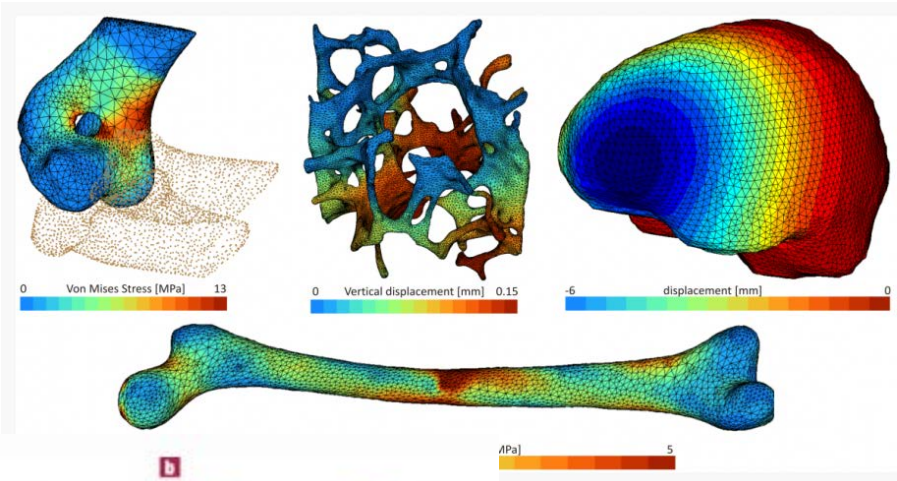


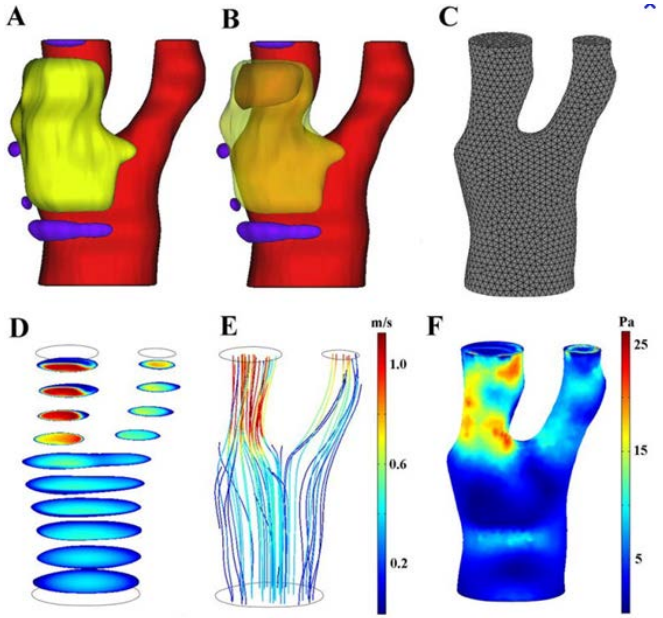


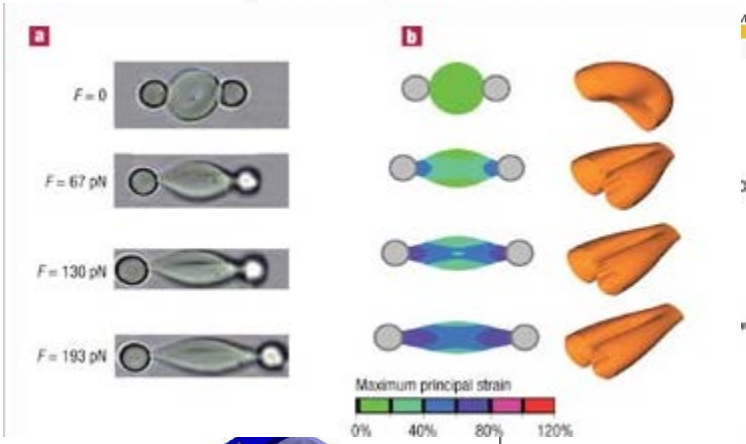
Applications of Continuum Mechanics: Biomechanics

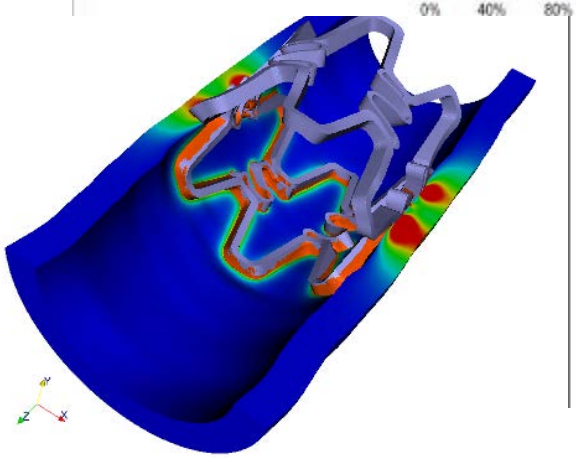
Governing equations

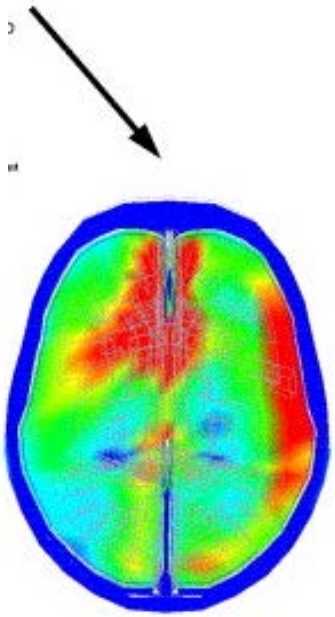
- Equilibrium/Navier Stokes
- Mass conservation
- Constitutive equations
- Growth laws?
- Failure criteria







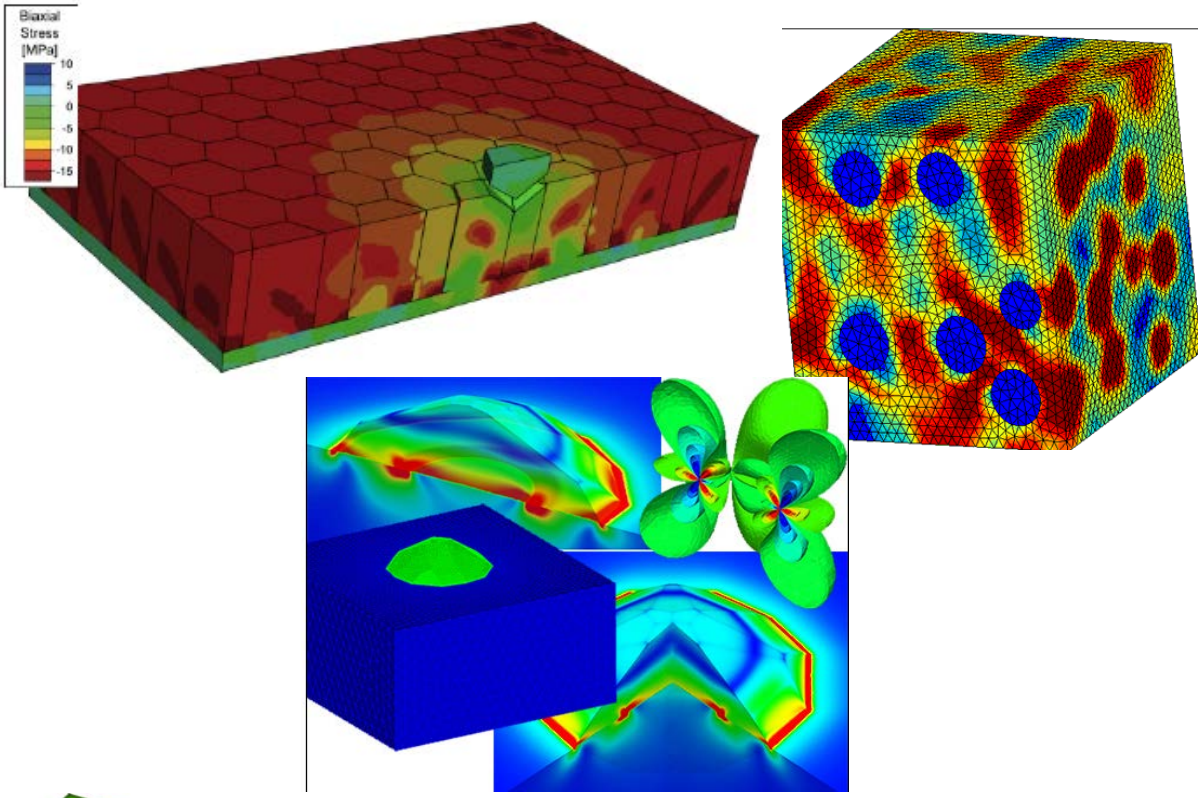


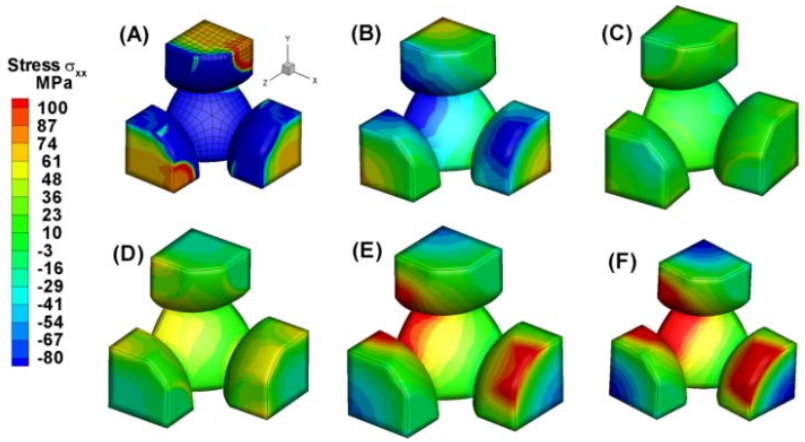


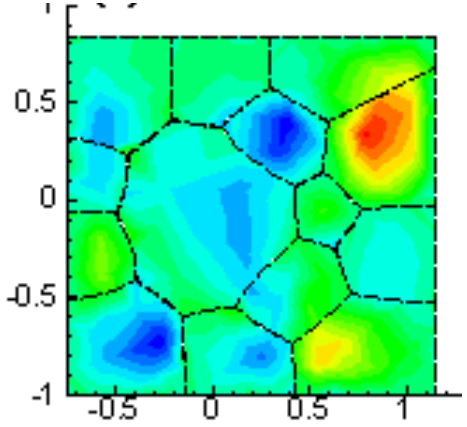
Applications of Continuum Mechanics: Materials Science

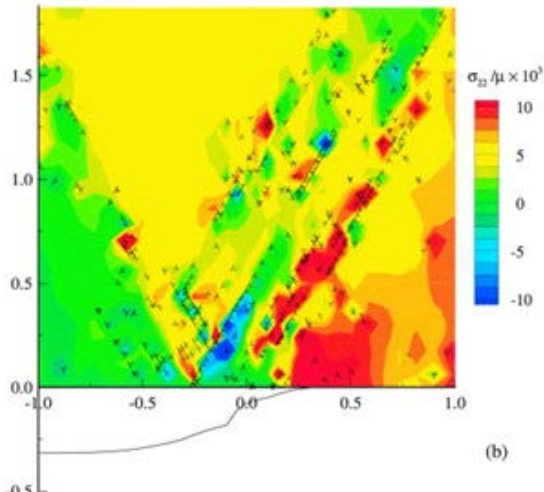
Governing equations

- Equilibrium
- Constitutive relations
- Multiphysics
- Failure criteria





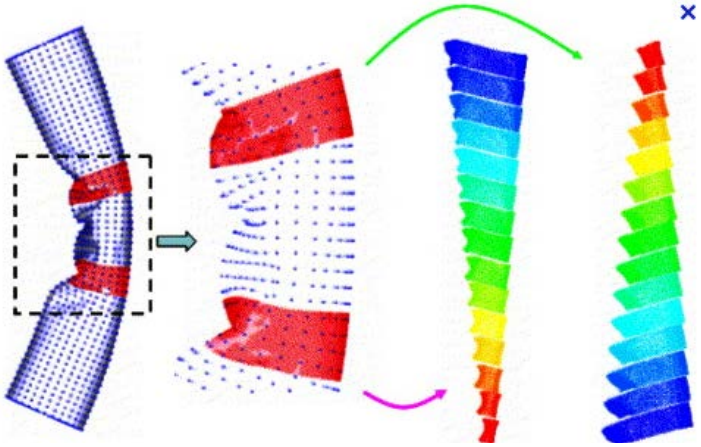


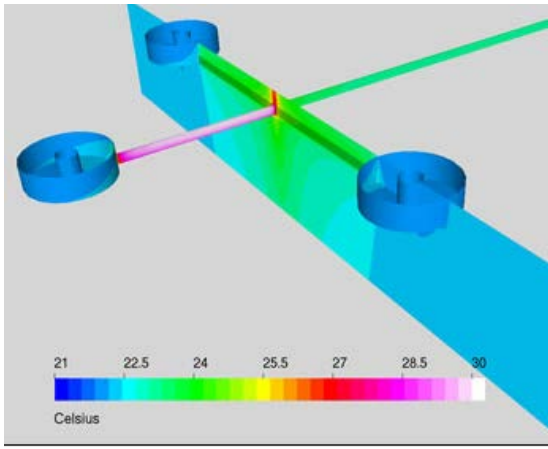


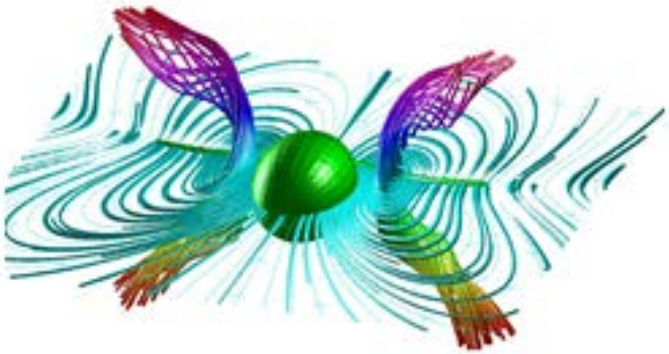
Applications of Continuum Mechanics: Nano/microfluidics

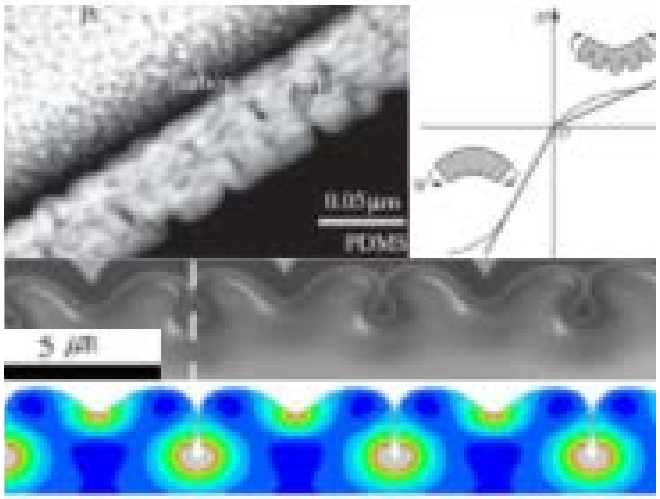
Governing equations

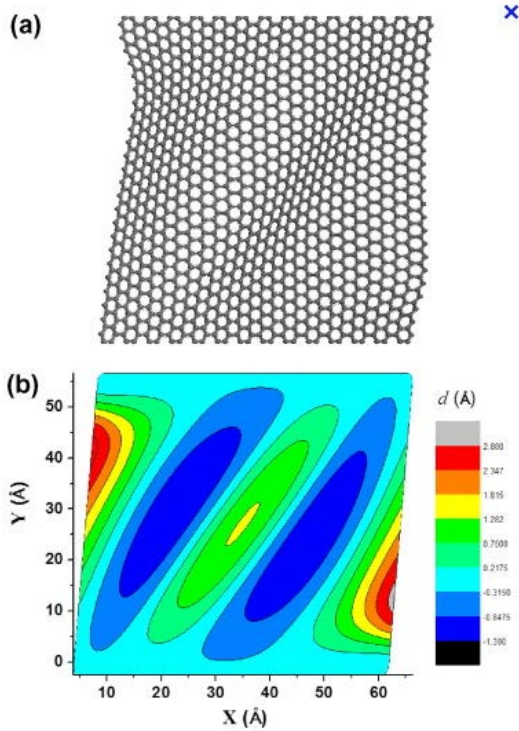
- Equilibrium
- Constitutive relations
- Multi-scale methods



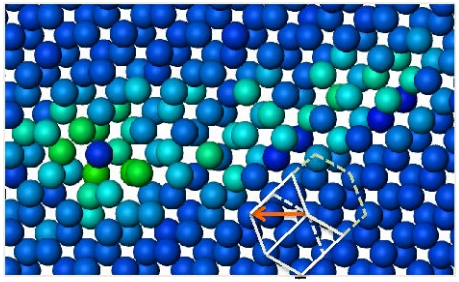






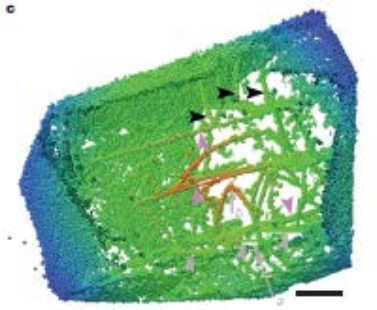


Current research topics in continuum mechanics

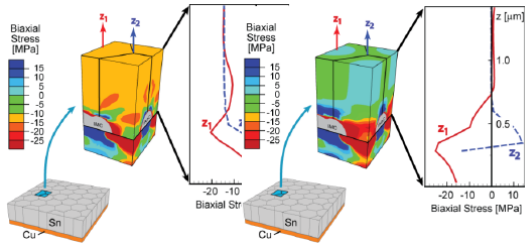


- Coupled problems – multiphysics: eg
  - Li ion batteries (chemistry/diffusion/mechanical stress)
  - Biophysics/mechanics/chemistry - eg mechanotransduction
  - New materials – eg hydrogels
  - Coupled fluids/solids – blood flow; tissue mechanics

- Deriving/understanding constitutive relations from fundamental physics
  - Ab initio atomistics
  - Molecular dynamics
  - Multiscale methods – concurrent or hierarchical



- Major unresolved fundamental issues
  - How to treat kinetics of slow processes at atomic scales or mesoscale?
  - Complexity – eg plasticity (dislocations) – or turbulent flow
  - Uncertainty – existing models nearly always deterministic; applications need eg statistics of failures



- Numerical Methods
- Applications….

Goals of Continuum Mechanics

- Predict the deformation and motion of solid or fluid
- Use predictions to design engineering systems
- Use predictions to understand natural phenomena

Foundations

- Mathematical preliminaries – vectors, tensors, calculus
- Mathematical description of motion and deformation
- Mathematical description of internal forces
- Conservation laws (mass, momentum, energy)
- Thermodynamics – first and second laws
- Constitutive relations and failure criteria – experimental mechanics
- Mathematical methods for solving PDEs
- Numerical methods for solving PDEs

Course Goals

An introduction to the mathematical foundations of continuum mechanics. Vectors and tensors, properties and basic operations. Kinematics of deformation. Conservation laws, thermodynamics. Stress. Constitutive equations. Elastic, viscous, and viscoelastic response. Linearization. Simple problems in finite and linear elasticity, and in Navier-Stokes flows. Creep and relaxation in linear viscoelasticity.

After completing EN2210, you should

- Be able to manipulate tensors effortlessly
- Be able to describe motion, deformation and forces in a continuum
- Be able to derive equations of motion and conservation laws for a continuum
- Understand constitutive models for fluids and viscoelastic solids
- Be able to solve simple boundary value problems for fluids and solids

Course Outline

1. Vectors and index notation
2. Tensors
3. Kinematics
4. Kinetics
5. Conservation Laws
6. Thermodynamics
7. Constitutive models – general restrictions
8. Mechanics of Fluids
9. Mechanics of elastic and (maybe) viscoelastic solids
10. Special topic: Mass transport in solids (time permitting).

Course format

- Lectures – meant to confuse you
- Homework assignments – where you actually figure out what's going on
- Design projects – more open ended applications
- Exams – not sure what these are for

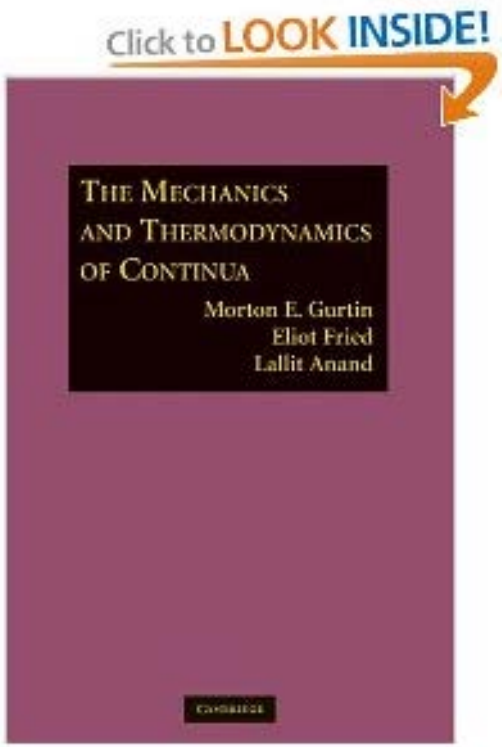
Resources

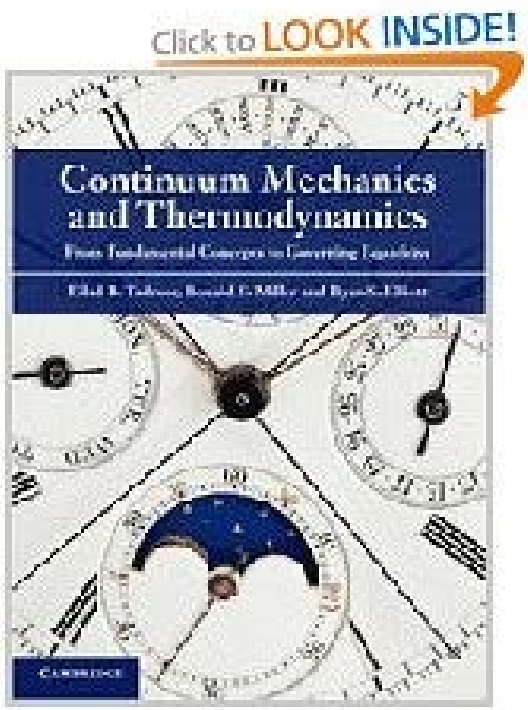
- Course web site –
- http://www.brown.edu/Departments/Engineering/Courses/En221/
- Reference notes and tutorials (see web site)
- Faculty Office Hours – see web

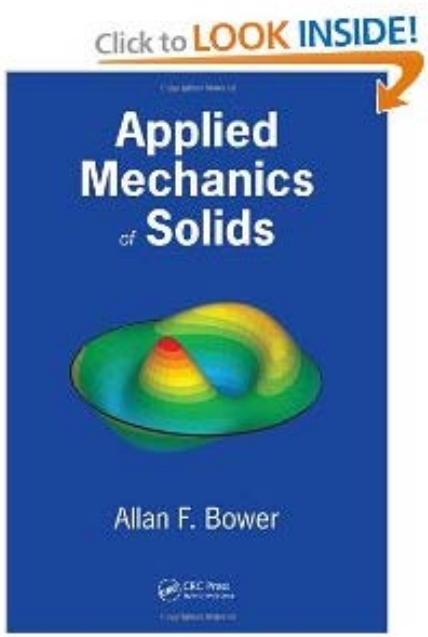
- Ms Gesualdi (stephanie_gesualdi@brown.edu) coordinates administrative matters.

Textbooks

- No required text, but you might find a few supplementary texts helpful. Many good recent books on continuum mechanics!
- See web for suggestions (first two below recommended)

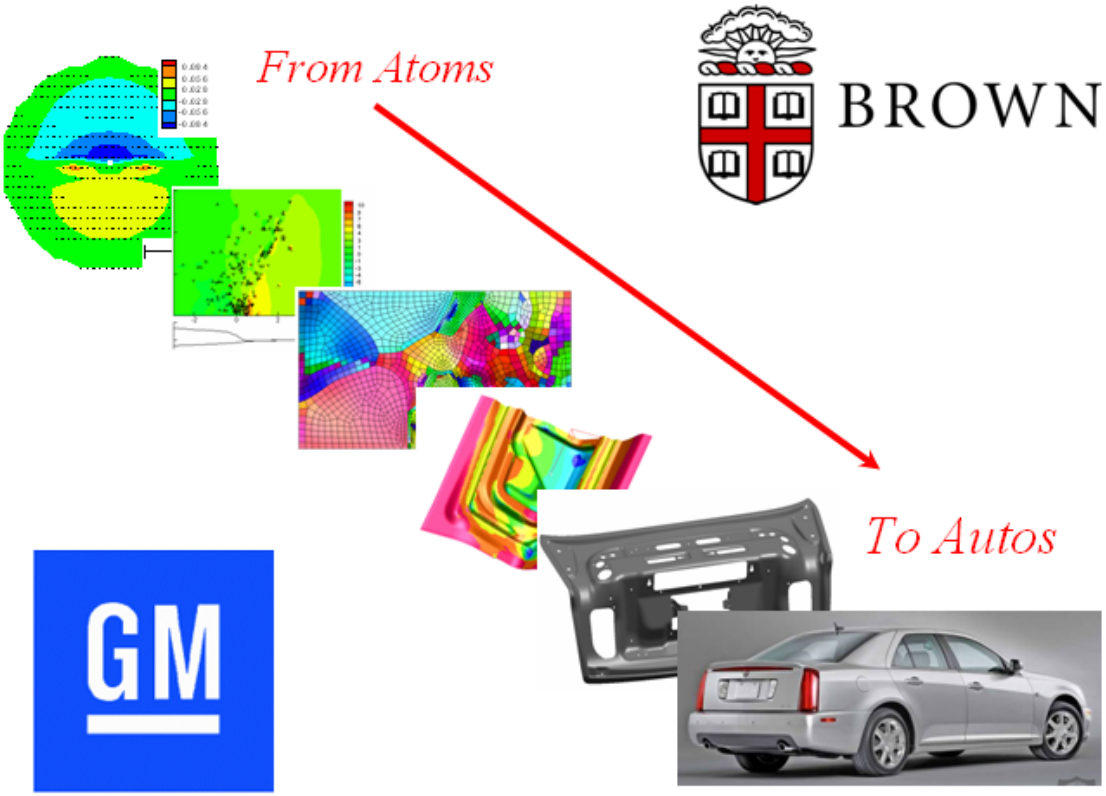






Faculty – Professor Bower

- Professor of Engineering
- PhD Cambridge
- Director, GM/Brown Collaborative Research Lab
- Mechanics of materials – fracture, fatigue, deformation



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Grading

- Homework: 35%
- Midterm Exam: 15%
- Final Examination: 30%
- Design Projects: 10% each