

Diffraction and interference of lights

Introduction

In the class the Fresnel diffraction integral was introduced as

$$u(\mathbf{x}) = \frac{ke^{ikz}}{2\pi iz} e^{\frac{ik}{2z}(x_1^2 + x_2^2)} \iint_{\Sigma} u(\boldsymbol{\xi}) \ e^{\frac{ik}{2z}(\xi_1^2 + \xi_2^2)} e^{\frac{-ik}{z}(x_1\xi_1 + x_2\xi_2)} d\xi_1 d\xi_2$$
(1)

If $\frac{\pi D^2}{\lambda z} \ll 1$ with *D* the diameter of the aperture and λ the wave length of the light, the integral can be reduced to the Fraunhoffer diffraction integral as

$$u(\mathbf{x}) = \frac{ke^{ik_z}}{2\pi i z} e^{\frac{ik}{2z}(x_1^2 + x_2^2)} \iint_{\Sigma} u(\boldsymbol{\xi}) \ e^{\frac{-ik}{z}(x_1\xi_1 + x_2\xi_2)} d\xi_1 d\xi_2$$
(2)

which has a factor of Fourier transformation in its expression. Now, we consider that $u(\xi)$ is constant at the aperture plane and $|u(\xi)|^2 = I_0$ where I_0 is the intensity of the light at the aperture plane.

Experiment 1: Fresnel and Fraounhoffer diffraction of a slit aperture.

The light intensity at the image plane is expressed for the Fraunhoffer diffraction as

$$I(\mathbf{x}) = u(\mathbf{x})\overline{u}(\mathbf{x}) = \frac{I_0}{\lambda^2 z^2} \int_{-a}^{a} e^{\frac{-ik}{z} x_1 \xi_1} d\xi_1 \int_{-b}^{b} e^{\frac{-ik}{z} x_2 \xi_2} d\xi_2 = \left(\frac{\sin ka\overline{x}_1}{ka\overline{x}_1}\right)^2 \left(\frac{\sin kb\overline{x}_2}{kb\overline{x}_2}\right)^2 I_0 \qquad (3)$$

where $\overline{x}_1 = x_1/z$ and $\overline{x}_2 = x_2/z$. In the experiment fix the image plane distance z about 1m and change the slit opening 2a about 1mm, 0.5mm, 0.25mm and 0.1mm to measure the

intensity at the image plane with the TV camera system. Then, normalize equation (3) to plot the four intensity profiles on a master curve and find out how it is deviating from the theoretical prediction. Discuss error sources of the experiment.

Experiment 2: Fresnel and Fraounhoffer diffraction of a circular aperture

Now, use a pin hole of $50 \mu m$ diameter as an aperture and measure the image intensity at the distance z of 1cm, 2.5cm 5cm 10cm and 25cm. Then, compare the five radial intensity distribution with the theoretical predictions of the Fraunhoffer diffraction,

$$I(\mathbf{x}) = \left[\frac{2J_1(ka\overline{r})}{ka\overline{r}}\right]^2 I_0 \tag{4}$$

where $\overline{r} = r/z$ is the normalized radial position on the image plane and $J_1(\dots)$ is the Bessel function of order 1.

Experiment 3: Frounhoffer diffraction field of a grating

Put a slit of about $50 \,\mu m$ width and a grating of 200 lines/mm on the aperture plane and observe the Fraunhoffer diffraction intensities on an image plane. Decide the distance of the image plane from the aperture plane for the best observation condition with the given experimental apparatus.

Experiment 4: Alignment of a pin hole filter for a laser beam expander

This is an application experiment. First observe the diffraction patterns caused by the converging lens in the beam expander without a pin hole filter. Discuss why you have such diffraction patterns. Then, adjust the pin hole filter for the beam expander to have the maximum intensity output.

Notes: Record the specs of laser that you have used, i.e. the wave length and the nominal intensity.

Hand in the report by February 20.