EN234: Computational methods in Structural and Solid Mechanics

Homework 4: Deriving FEA equations, intro to coding elements
Due Wed Oct 2, 2013

School of Engineering
Brown University

1. Derive the weak form of the diffusion equation that you solved in HW2:

$$\frac{\partial^2 c}{\partial x_i \partial x_i} = 0 \qquad c = c^* \quad S_1 \qquad j_i n_i = -D\frac{\partial c}{\partial x_i} n_i = j^* \quad S_2$$

   Show that the weak form is equivalent to the statement that

$$\Pi(\hat{c}) = \frac{1}{2}\int_V \frac{\partial \hat{c}}{\partial x_i}\frac{\partial \hat{c}}{\partial x_i} dV + \int_{S_2} \frac{j^*}{D}\hat{c}\, dA$$

   is a minimized when $\hat{c}$ satisfies the steady-state diffusion equation.

2. Consider a mixed formulation of the diffusion equation in which both c and the flux vector j_i are unknown variables. The strong form of the equations governing these variables is

$$\frac{\partial j_i}{\partial x_i} = 0 \qquad j_i = -D\frac{\partial c}{\partial x_i} \qquad c = c^* \quad S_1 \qquad j_i n_i = j^* \quad S_2$$

   Show that the solution to these equations minimizes the functional

$$\Pi(\hat{c}, \hat{j}_i) = \int_V \left(\hat{j}_i \frac{\partial \hat{c}}{\partial x_i} + \frac{1}{2D}\hat{j}_i \hat{j}_i \right) dV - \int_{S_2} j^* \hat{c}\, dA$$

3. If you have not already done so, install a FORTRAN compiler, TECPLOT and download FEACHEAP from the course website (there are detailed instructions on the programming page of the website). Work through the FEACHEAP tutorial (first link on the programming page). Then implement a general 2D fully integrated (i.e. use enough integration points to evaluate the stiffness exactly) plane elasticity element in the code. As a guide, you can use either the MATLAB example FEA code for general 2D or 3D linear elasticity element, or modify the 3D linear elasticity element provided with FEACHEAP. Be very careful with array dimensions when you modify the code.