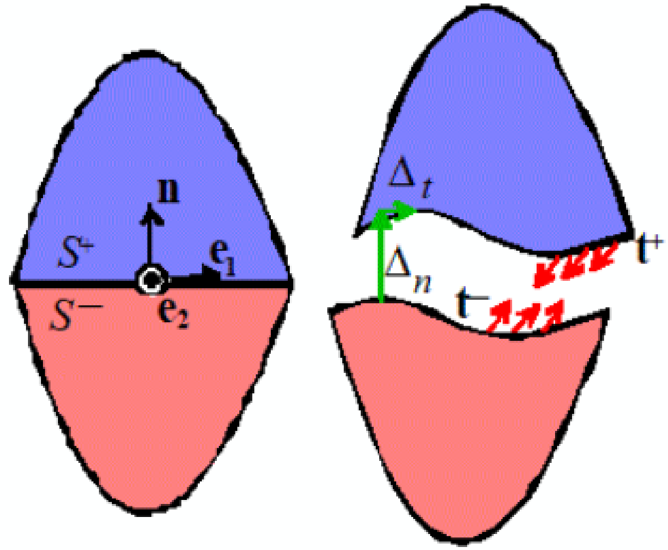
11) Interfaces and Contact

11.1 "Cohesive Zone" models of fracture



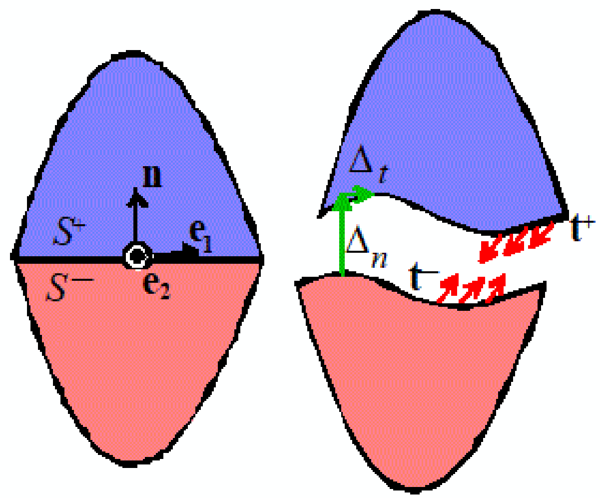
Goal: model crack nucleation and growth along a weak interface

Approach: Introduce planes Γ where solid can separate

Introduce additional constitutive equations that specify tractions acting on material planes as functions of separation

Cohesive Zone Laws

Kinematics & Kinetics



Let $\Delta_n = (\underline{u}^+ - \underline{u}^-) \cdot \underline{n}$

$\Delta_\alpha = (\underline{u}^+ - \underline{u}^-) \cdot \underline{e}^\alpha$

also $T_n = \underline{t}^- \cdot \underline{n}$

$T_\alpha = \underline{t}^- \cdot \underline{e}^\alpha$

We usually assume Δ_n, Δ_α are small

For large relative displacements use contact elements

Constitutive laws must give $\{T_n, T_\alpha\} = f(\Delta_n, \Delta_\alpha)$

Examples: Reversible cohesive zone law

Let $\Phi(\Delta_n, \Delta_\alpha)$ be work done per unit area to separate planes

$$T_n = \frac{\partial \Phi}{\partial \Delta_n} \qquad T_\alpha = \frac{\partial \Phi}{\partial \Delta_\alpha}$$

Example : Xu-Needleman

$$\Phi(\Delta_n, \Delta_t) = \phi_n - \phi_n \left(1 + \frac{\Delta_n}{\delta_n}\right) \exp\left(-\frac{\Delta_n}{\delta_n}\right) \exp\left(-\frac{\beta^2 \Delta_t^2}{\delta_n^2}\right)$$

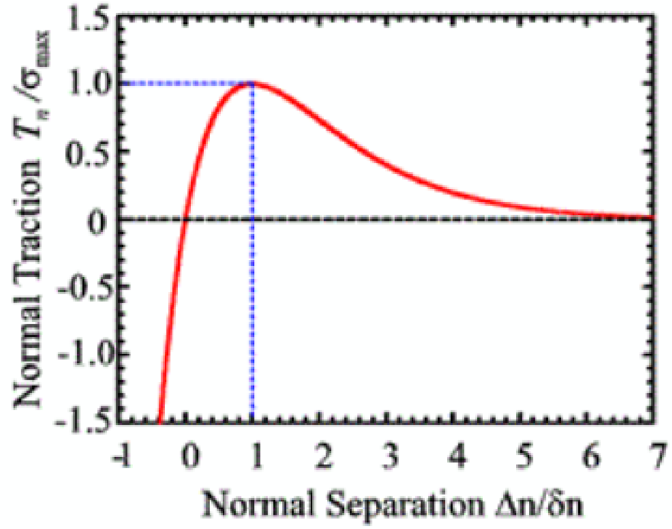
ϕ_n = interface energy

δ_n, β : material props controlling compliance

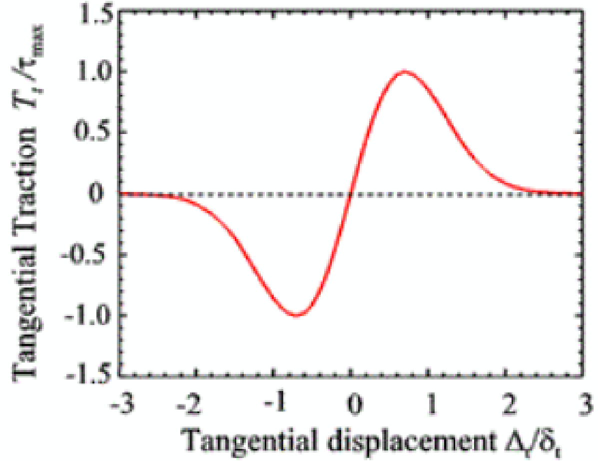
Hence

$$T_n = \sigma_{\max} \frac{\Delta_n}{\delta_n} \exp\left(1 - \frac{\Delta_n}{\delta_n}\right)$$

$$T_\alpha = 2\sigma_{\max} \left(\frac{\beta^2 \Delta_\alpha}{\delta_n}\right)\left(1 + \frac{\Delta_n}{\delta_n}\right) \exp\left(1 - \frac{\Delta_n}{\delta_n}\right) \exp\left(-\frac{\beta^2 \Delta_t^2}{\delta_n^2}\right)$$



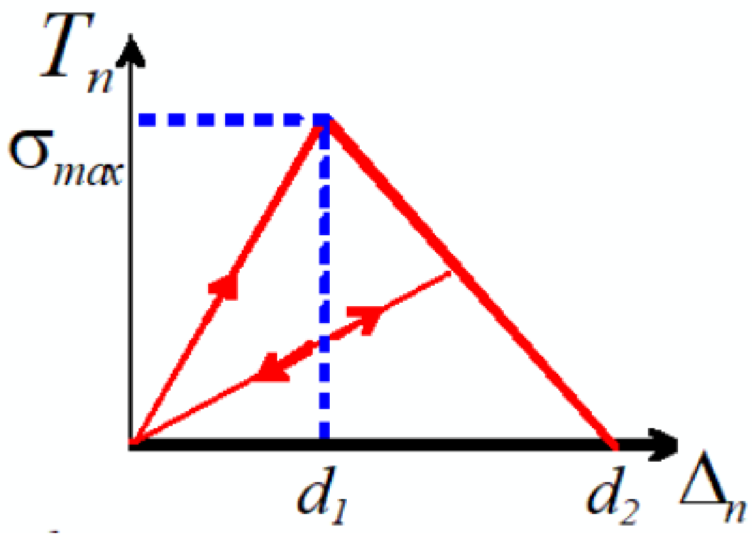
$$\sigma_{max} = \frac{\phi_n}{\delta_n \exp(1)} \qquad \tau_{max} = \beta^2 \sigma_{max}$$



Notes: (1) Interface is reversible:
Heals if interfaces contact after separation

(2) Interface can behave strangely under large pressure

Irreversible Cohesive zone laws



Assumptions :

(1) No inter-penetration $\Delta_n \geq 0$

(2) Irreversible behavior in tension

(1) Undamaged interface has stiffness $k_0 = \frac{\sigma_{max}}{d_1}$

(2) If separation exceeds d_1 interface stiffness drops

Damage is quantified by an internal variable $0 < D < 1$

Undamaged solid has $D = 0$
Total failure $D = 1$

To quantify magnitude of separation and traction

$$\lambda = \sqrt{\Delta_n^2 + \beta^2 (\Delta_1^2 + \Delta_2^2)}$$

$$\tau = \sqrt{T_n^2 + (T_1^2 + T_2^2)/\beta^2}$$

Assume that interface potential $\Phi(\lambda, D)$ has form

$$\Phi = \frac{1}{2} k_0 (1-D) \lambda^2$$

Hence $T_n = \dfrac{\partial \Phi}{\partial \Delta_n} = k_0 (1-D) \Delta_n$

$$T_\alpha = \frac{\partial \Phi}{\partial \Delta_\alpha} = \beta^2 k_0 (1-D) \Delta_\alpha$$

We also need an evolution equation for D

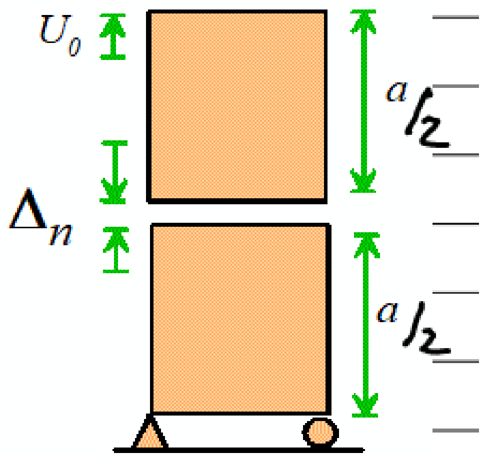
We have that $\underbrace{k_0}_{=\sigma_{max}/d_1}(1-D)\lambda = \sigma_{max}\left(1 - \frac{\lambda - d_1}{d_2 - d_1}\right)$

Take time derivative

$$\frac{dD}{dt} = \begin{cases} 0 & \lambda < \dfrac{d_1 d_2}{(1-D)d_2 + D d_1} \quad \text{or} \quad d\lambda/dt < 0 \quad \text{or} \quad D = 1 \\ \left((1-D) + \dfrac{d_1}{d_2 - d_1}\right)\dfrac{1}{\lambda}\dfrac{d\lambda}{dt} & \text{Otherwise} \end{cases}$$

History dependent - D is a state variable

Viscous Regularization



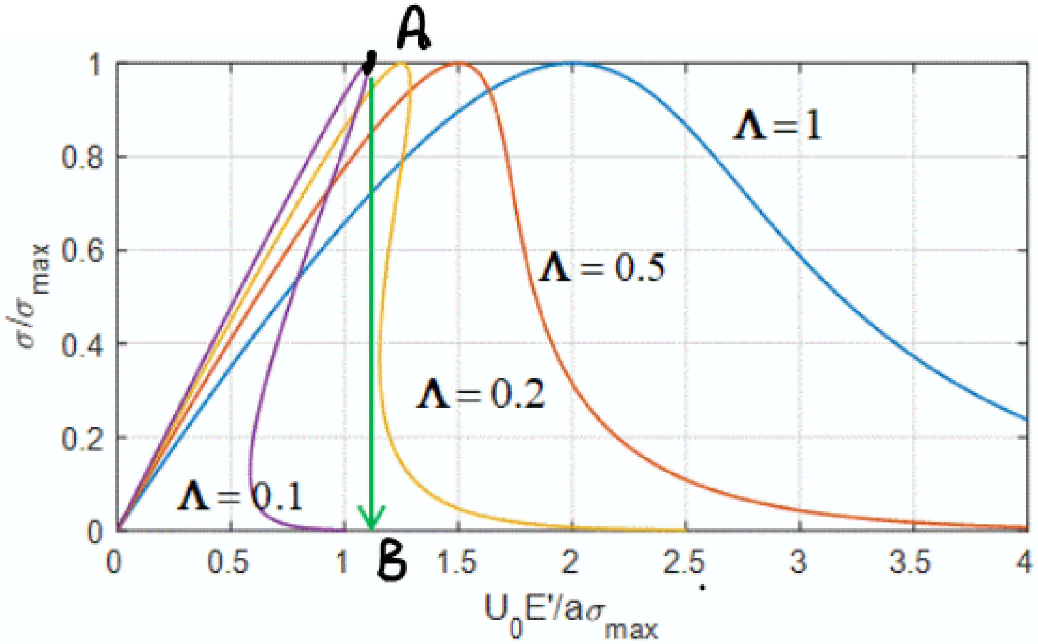
Problem: Crack nucleation can be an unstable process

Example: Xu-Needleman interface between elastic blocks in uniaxial tension

$$\frac{\sigma}{\sigma_{max}} = \frac{\Delta_n}{\delta_n} \exp\left(1 - \frac{\Delta_n}{\delta_n}\right)$$

$$U_0 = \frac{\sigma}{E} a + \Delta_n \Rightarrow \frac{U_0 E}{\sigma_{max} a} = \frac{\sigma}{\sigma_{max}} + \frac{\Delta_n}{\delta_n} \underbrace{\frac{E \delta_n}{\sigma_{max} a}}_{\Lambda}$$

Stress - displacement relation



For large Λ, we see stable separation as U_0 increases

For small Λ we see a snap from A-B

For a displacement controlled static analysis there is no equilibrium path from A-B

Newton iterations will fail at A

We can fix this by adding some viscosity to interface

For example we could use

$$\left.\begin{aligned} T_n &= \frac{\partial \bar{\Phi}}{\partial \Delta_n} + \eta \frac{d\Delta_n}{dt} \\ T_\alpha &= \frac{\partial \bar{\Phi}}{\partial \Delta_\alpha} + \eta \frac{d\Delta_\alpha}{dt} \end{aligned}\right\}$$

η is a small viscosity

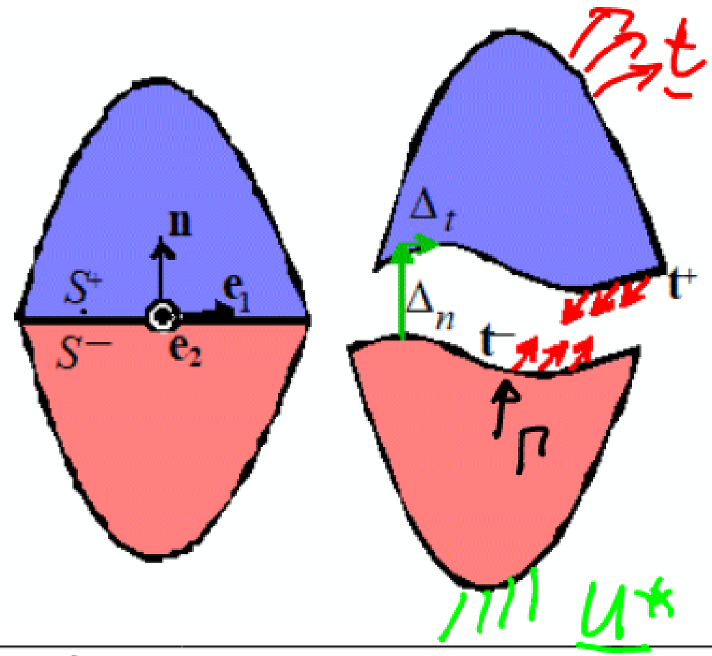
Choice of η dictated by computational cost & accuracy

FE implementation

Assume small strain linear elasticity

Governing Equations

$$\frac{\partial \sigma_{ij}}{\partial x_j} = 0 \qquad \sigma_{ij} = C_{ijkl} \frac{\partial u_k}{\partial x_l}$$



Boundary Conditions : $\sigma_{ij} n_j = t_i^*$ on S_2

$$u_i = u_i^* \text{ on } S_1$$

Interface Relations $n_i \sigma_{ij} n_j = T_n$ on Γ_-

$$= -T_n \text{ on } \Gamma_+$$

$$e_i^\alpha \sigma_{ij} n_j = T_\alpha \text{ on } \Gamma_-$$

$$= -T_\alpha \text{ on } \Gamma_+$$

Constitutive eqs for cohesive zone $(T_n, T_\alpha) = f(\Delta_n, \Delta_\alpha)$

We need a new weak form for equilibrium

Let η_i be a kinematically admissible displacement field

Note η_i may be discontinuous across Γ

Let η_i^+, η_i^- denote η_i on two sides of Γ

Weak form of equilibrium

$$\int_{R} \frac{\partial \sigma_{ij}}{\partial x_j} \eta_i \, dV = 0 \Rightarrow \int_{R} \left\{ \underbrace{\frac{\partial}{\partial x_j}(\sigma_{ij} \eta_i)}_{\text{Apply divergence theorem}} - \sigma_{ij} \frac{\partial \eta_i}{\partial x_j} \right\} dV = 0$$

$$\Rightarrow \int_{\Gamma_-} \sigma_{ij} n_j \eta_i^- \, dA - \int_{\Gamma_+} \sigma_{ij} n_j \eta_i^+ \, dA + \int_S \sigma_{ij} n_j \eta_i \, dA$$

$$- \int_R \sigma_{ij} \frac{d\eta_i}{dx_j} \, dV = 0 \quad \forall \text{ admiss } \eta_i$$

Note on Γ $\sigma_{ij} n_j = T_n n_i + T_\alpha e_i^\alpha$

$$\Rightarrow \int_R \sigma_{ij} \frac{\partial \eta_i}{\partial x_j} \, dV + \int_\Gamma T_n (\eta_i^+ - \eta_i^-) n_i \, dA$$

$$+ \int_\Gamma T_\alpha (\eta_i^+ - \eta_i^-) e_i^\alpha \, dA$$

$$- \int_{S_2} t_i^* \eta_i \, dA = 0 \quad \forall \text{ admiss } \eta_i$$

This looks like usual PVW but with some additional terms on interface

Introduce usual interpolations inside volume

$$u_i = N^a u_i^a \qquad \eta_i = N^a \eta_i^a$$

Account for displacement discontinuity using cohesive zone elements

Generate meshes inside two solids joined by interface with coincident nodes on Γ