

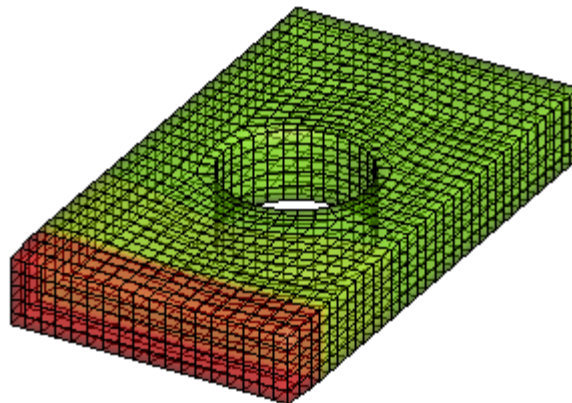


School of Engineering

**EN 2340**  
**COMPUTATIONAL METHODS**  
**IN STRUCTURAL MECHANICS**

A.F. Bower

Fall



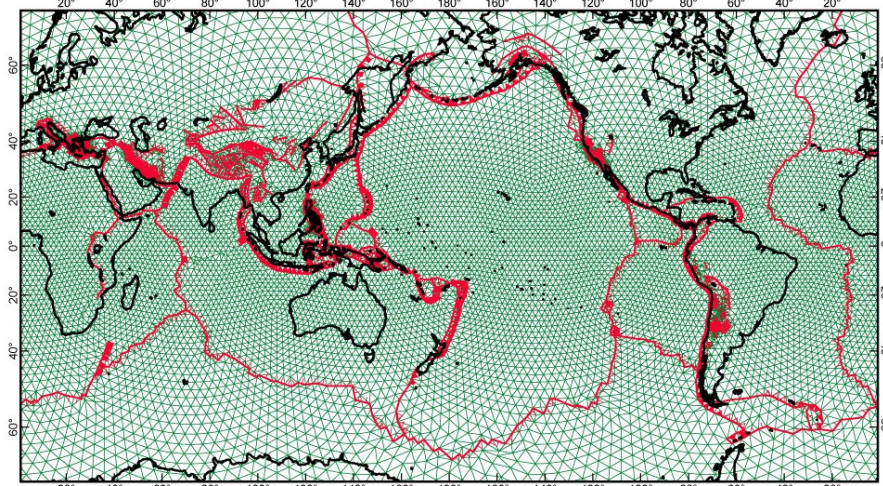
2017

# Applications of FEA

## Plate Tectonics

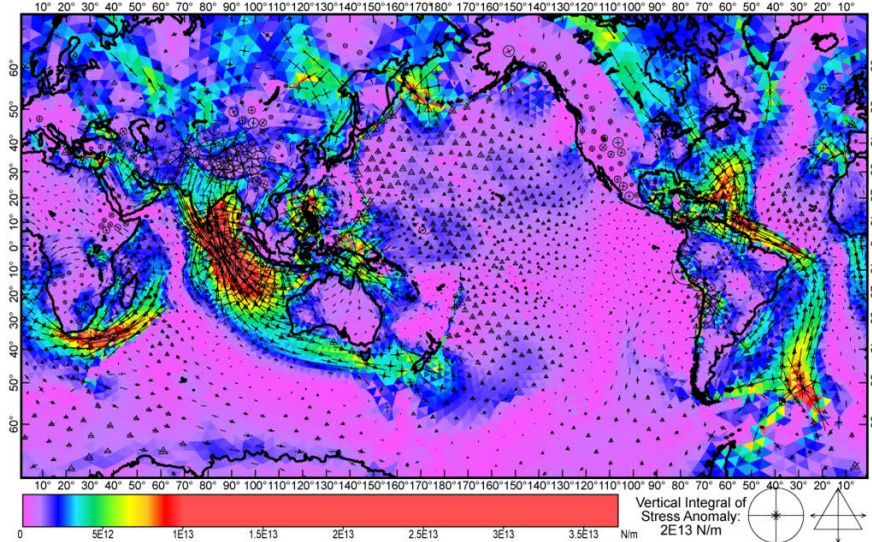
Finite Element Grid

Earth5N.feg: 2nd with new OrbData5; after manual Q reductions from Earth5M.feg



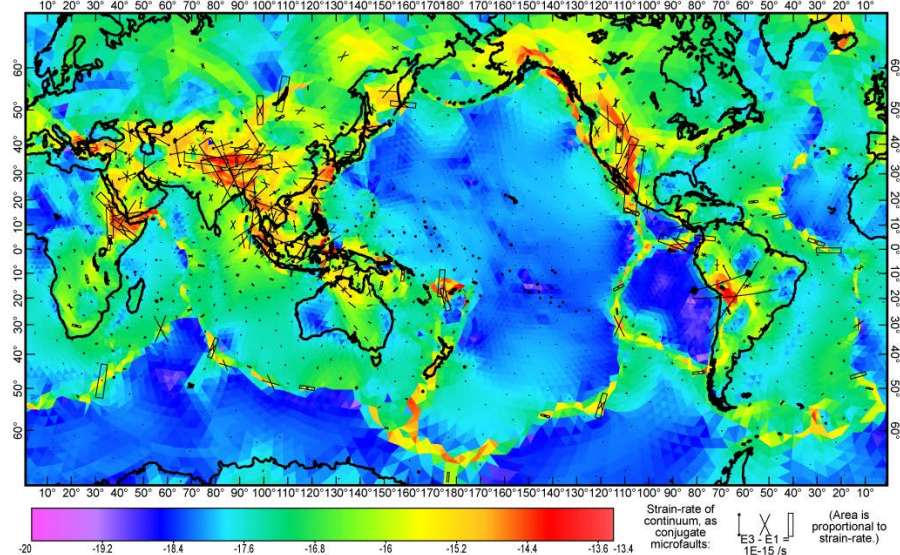
Vertical Integrals of Shear Stress and Stress Anomaly

iEarth5-049.in: FFRIC 0.1, TAUMAX 2.0E12, ICONVE6#3, type-4A slabs; no internal



Common Log of [Largest (Absolute Value) Principal Strain-Rate \* 1 s]

iEarth5-049.in: FFRIC 0.1, TAUMAX 2.0E12, ICONVE6#3, type-4A slabs; no internal

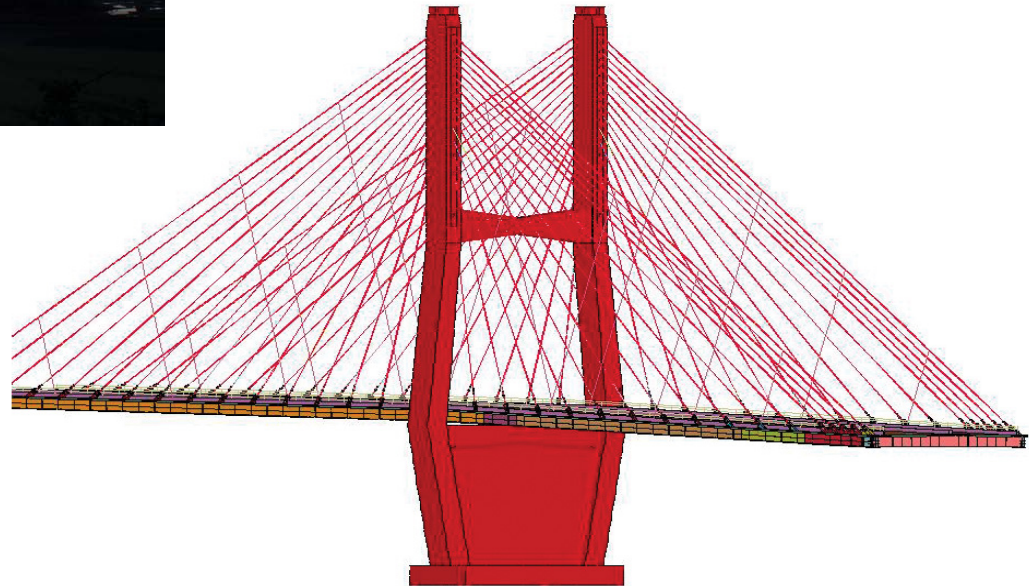




# Applications of FEA



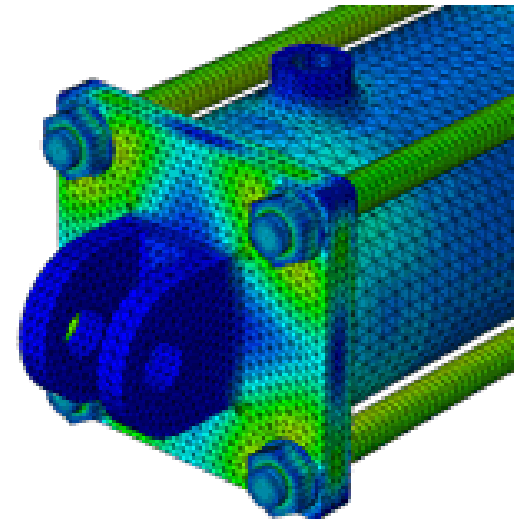
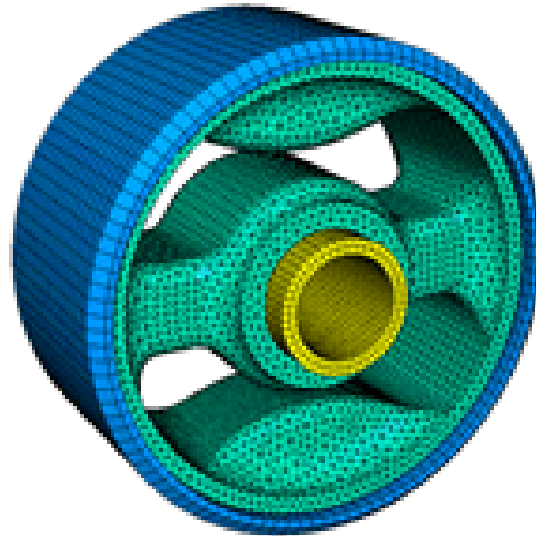
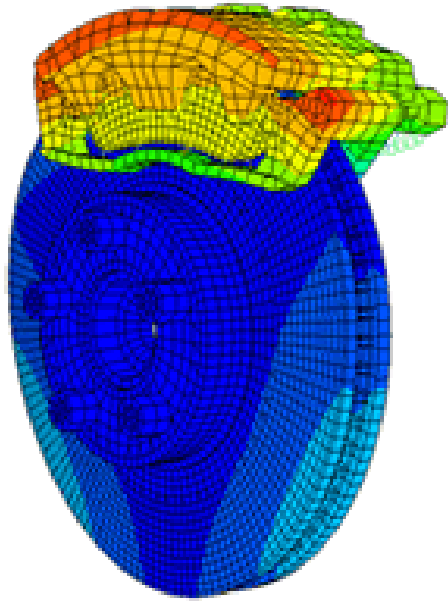
## Structural Analysis



[www.anl.gov/.../bridge\\_conditions.html](http://www.anl.gov/.../bridge_conditions.html)

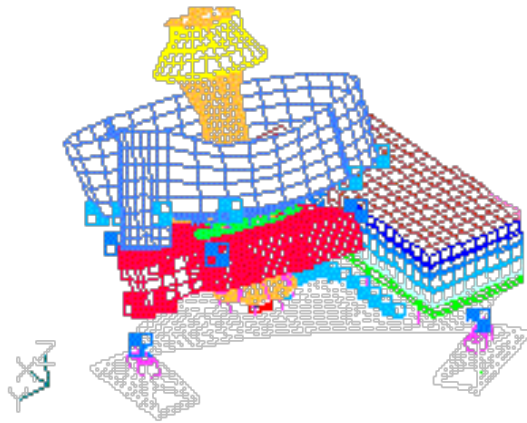
# Applications of FEA

## Machine Design

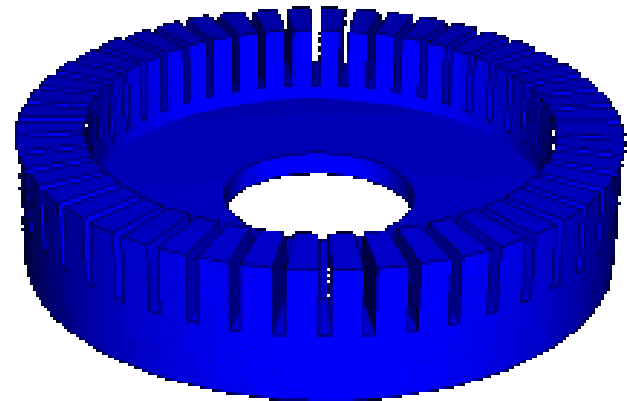


# Applications of FEA

## Dynamics and Vibrations

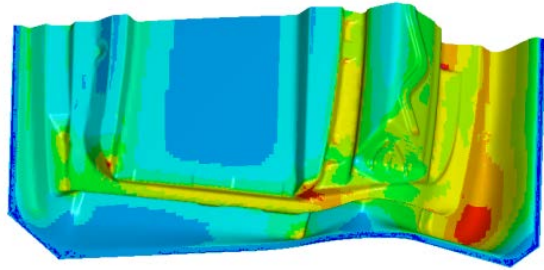


NASA

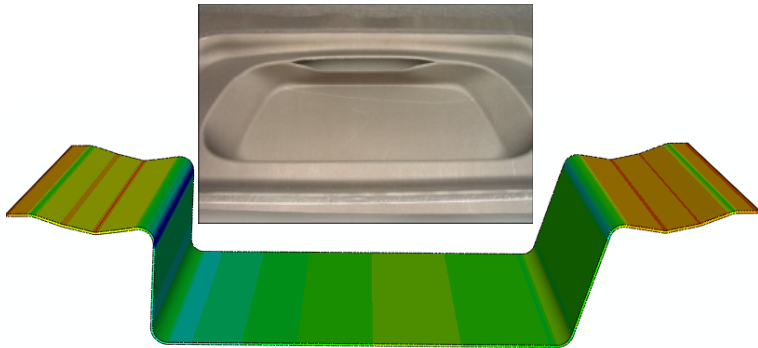


Simulia

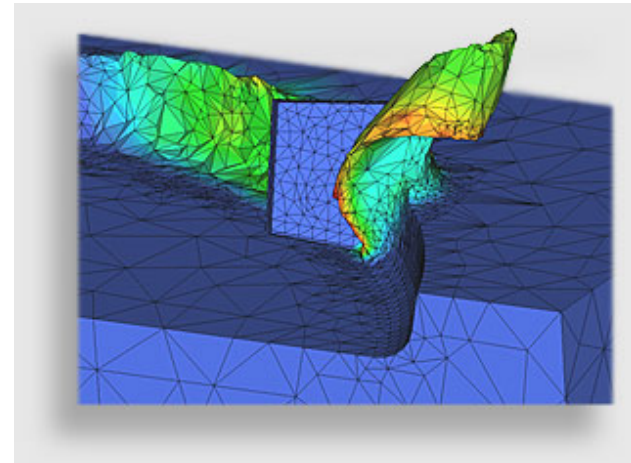
# Applications of FEA



## Process Modeling



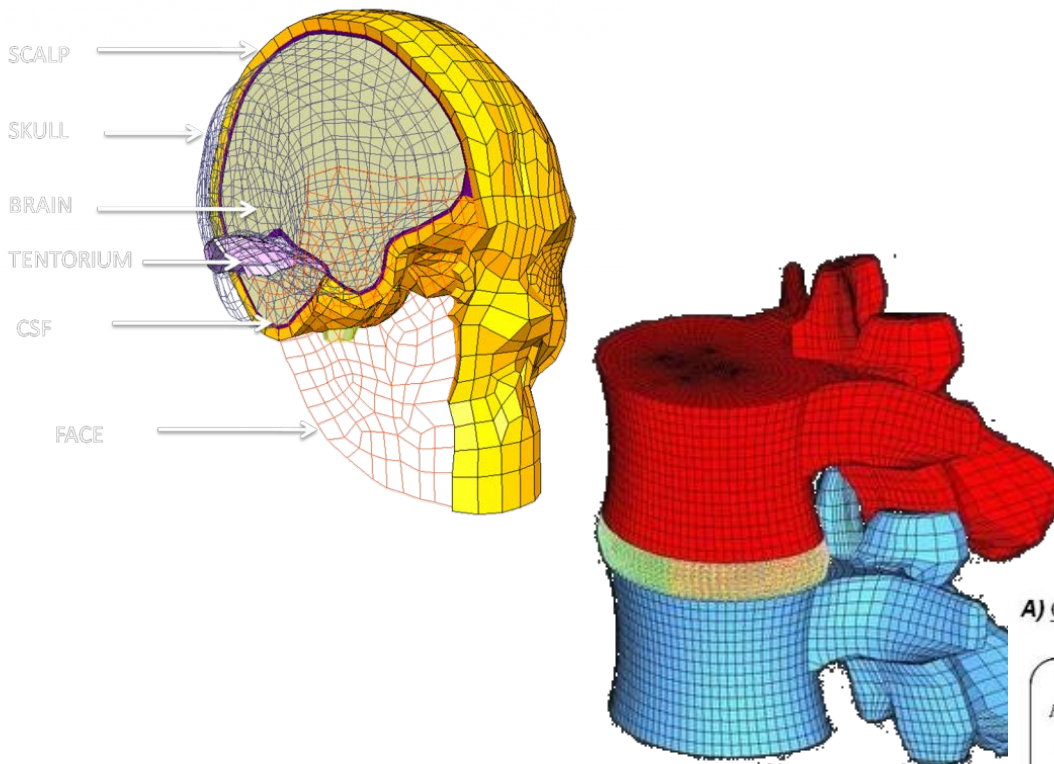
**GM**



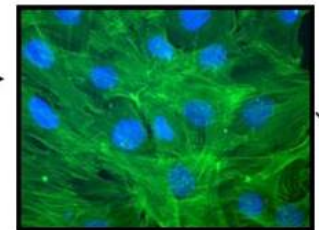
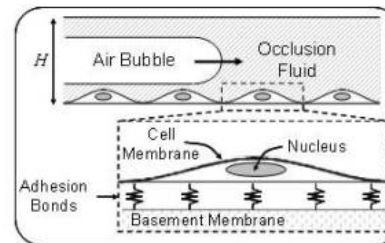
Third Wave Systems

# Applications of FEA

## Biomechanics

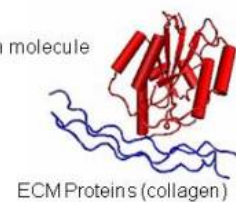


**A) CFD Simulation of Airway reopening**   **B) Optical Measurement of Cytoskeletal Structure and Mechanics:**

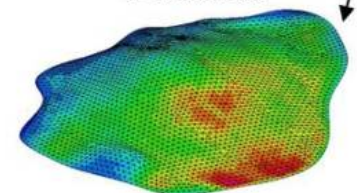


**C) Molecular model of adhesion proteins**

$\alpha 1 \beta 2$  Integrin molecule

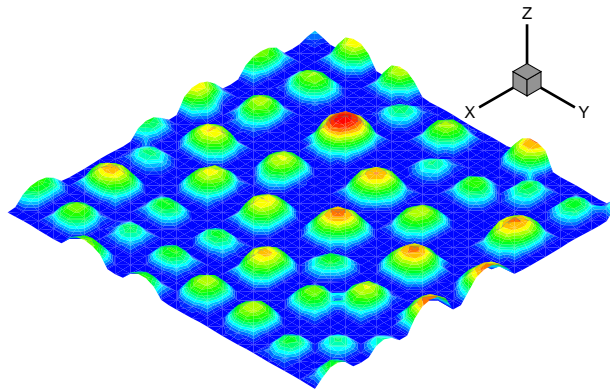
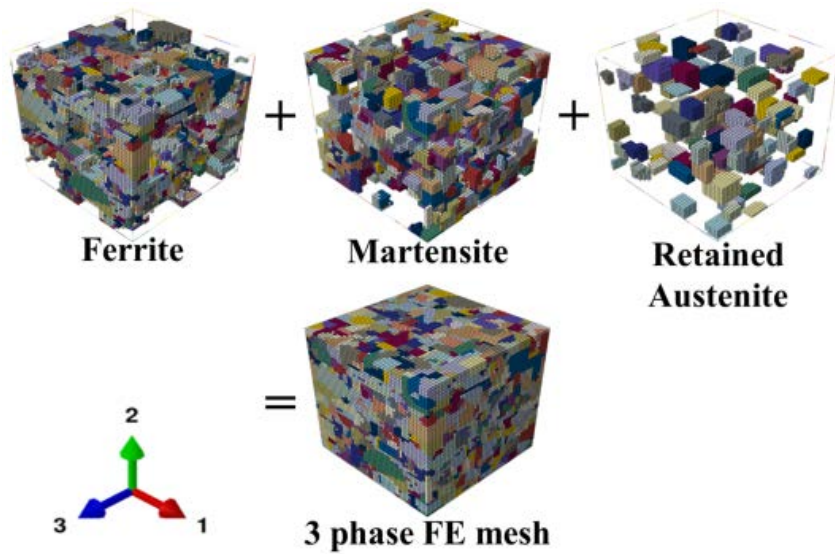


**D) Finite Element Model of Cell Deformation:**

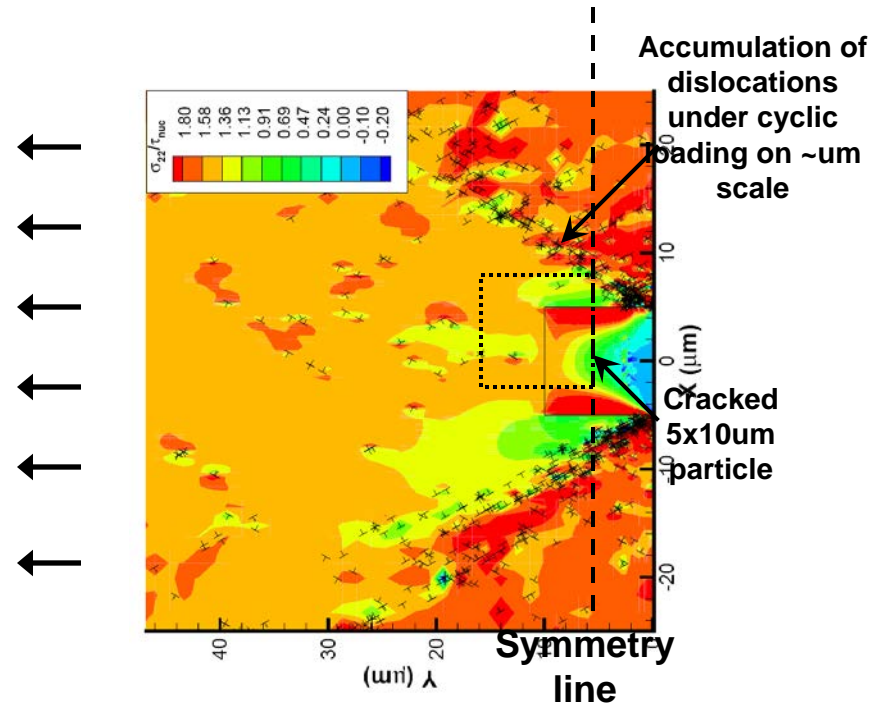




# Applications of FEA



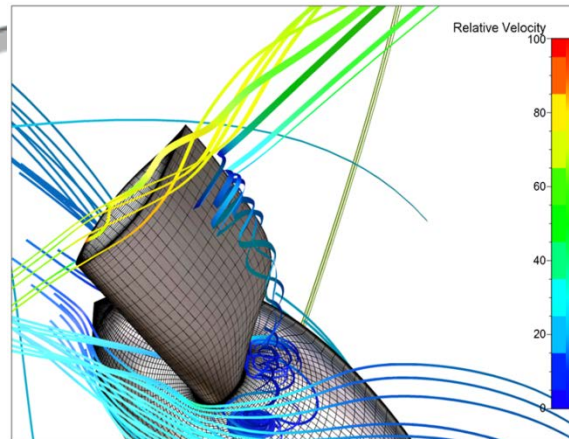
## Materials Design



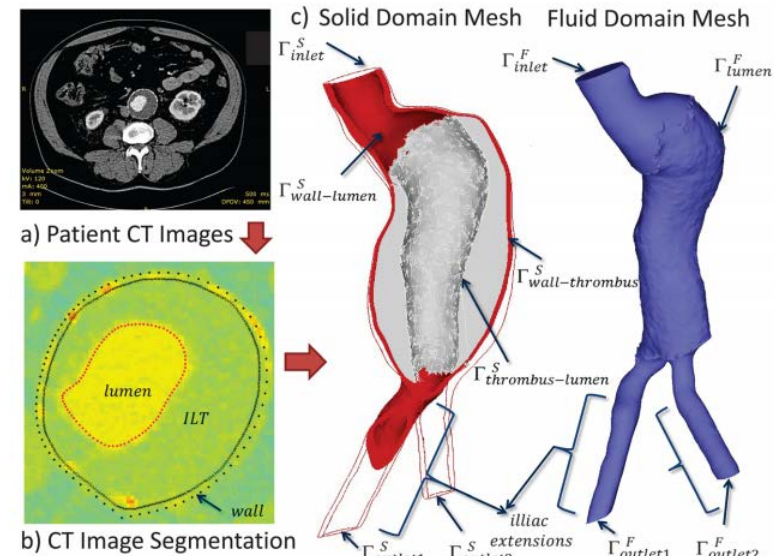


## Coupled problems (multiphysics)

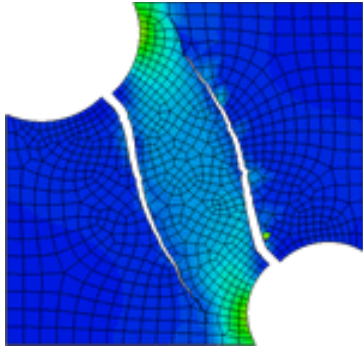
Simulia



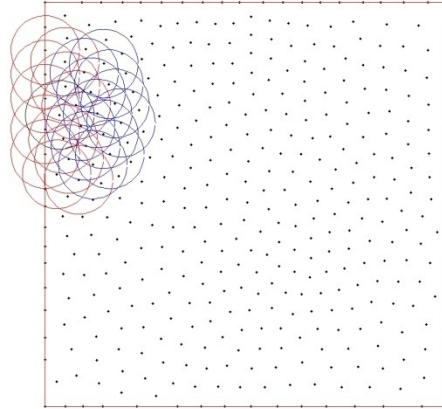
Coupled ABAQUS/Numerical  
FSI



# Applications of FEA



XFEM



Meshfree methods



Asynchronous explicit dynamics parallelization (static and dynamic)



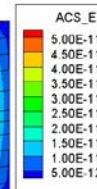
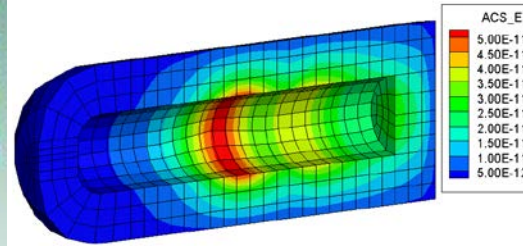
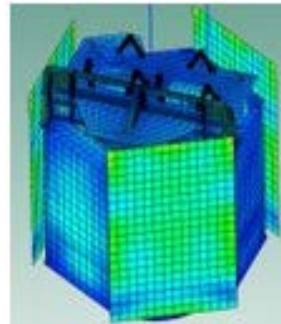
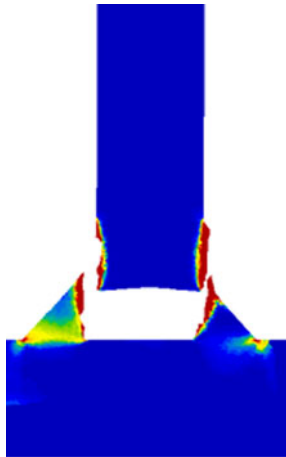
$t=0.3$



$t=1.5$

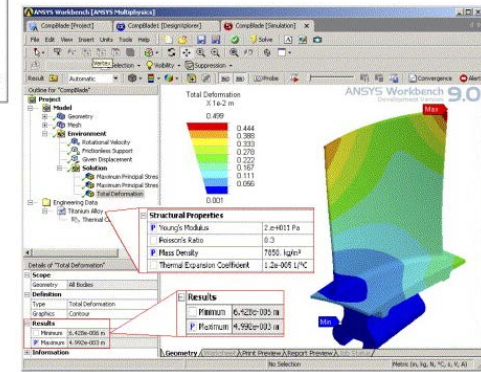


$t=2.7$



Enhanced element formulations,  
Constitutive laws, multiphysics  
Mesh adaption/remeshing...

Special procedures – eg acoustics  
(SEA, ESEA) Design optimization,  
probabilistic FEA



# Selected commercial FEA codes

- ABAQUS

 SIMULIA

**ABAQUS UNIFIED FEA**

COMPLETE SOLUTIONS FOR REALISTIC SIMULATION.

- ANSYS



- ADINA



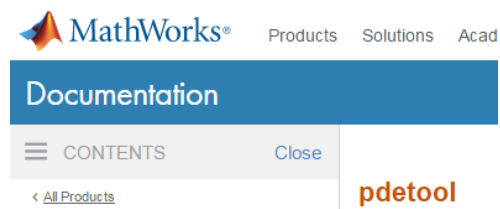
- LS-DYNA



- COMSOL



- MATLAB Pdetool or Mathematica works well for simple problems

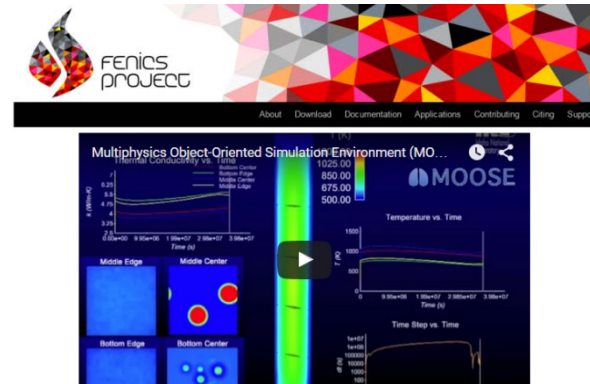


# Selected open-source FEA codes

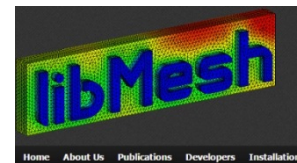
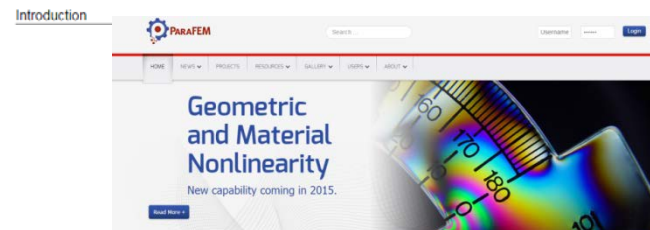
<http://mechanical-engg.com/forum/blogs/entry/704-finite-element-analysis-fea-list-of-fea-software%E2%80%99s-list-of-open-source-software%E2%80%99s-list-of-commercial-software%E2%80%99s/>

These are mostly focused on solving general PDEs. Often good choice for multi-physics. Solid mechanics can be fairly rudimentary, but is improving.

- Fenics – Imperial College
- MOOSE – Idaho National Laboratory
- FreeFem++
- Parafem
- Code Aster
- Lib Mesh
- Get fem++



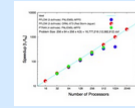
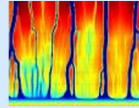
**FreeFem++ v 3.38-1** (June 29 2015 15:56:02.)





# Other useful open-source software

**PETSc**

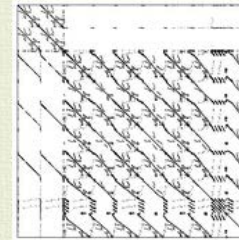


**Portable, Extensible Toolkit for  
Scientific Computation**

## **MUMPS : A PARALLEL SPARSE DIRECT SOLVER**

### MUMPS MAIN FEATURES

- Solution of large linear systems with  
symmetric positive definite matrices  
general symmetric matrices  
general unsymmetric matrices
- Version for complex arithmetic
- Parallel factorization and solve phases  
(uniprocessor version also available)
- Iterative refinement and backward error analysis



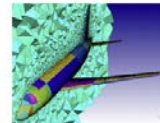
Gmsh: a three-dimensional finite element mesh generator with  
built-in pre- and post-processing facilities

Christophe Geuzaine and Jean-François Remacle

[Download](#) | [Documentation](#) | [Authors and credits](#) | [Licensing](#) | [Screenshots](#) | [Links](#) | [References](#)

Gmsh is a 3D finite element grid generator with a built-in CAD engine and post-processor. Its design goal is to provide a fast, light and user-friendly meshing tool with parametric input and advanced visualization capabilities. Gmsh is built around four modules: geometry, mesh, solver and post-processing. The specification of any input to these modules is done either interactively using the graphical user interface or in ASCII text files using Gmsh's own scripting language.

See the screencasts for a quick tour of Gmsh's graphical user interface, or the reference manual for a more thorough overview of Gmsh's capabilities and some frequently asked



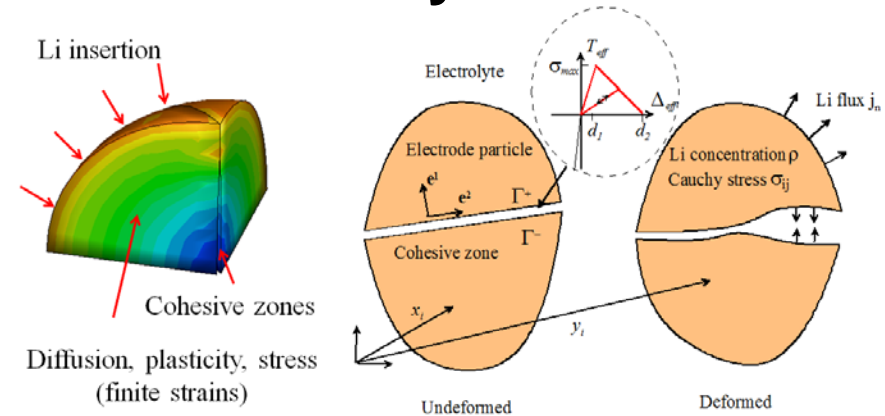
[http://www.cg.its.tudelft.nl/~matthijss/oss\\_meshing\\_software.html](http://www.cg.its.tudelft.nl/~matthijss/oss_meshing_software.html)

## Transport, deformation and failure in battery electrodes

### Deformation/fracture in electrode microstructures

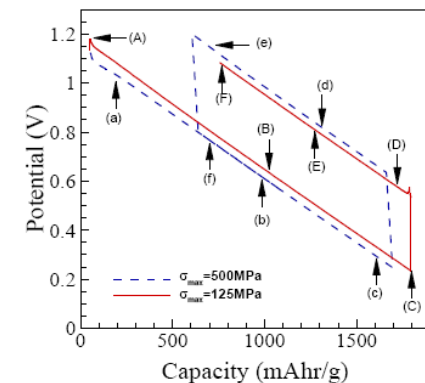
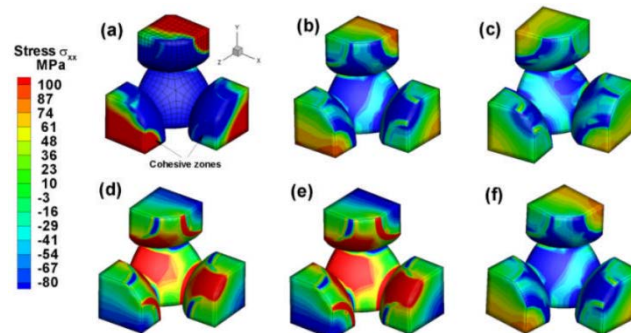
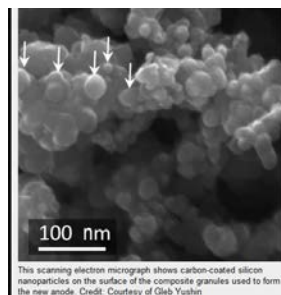
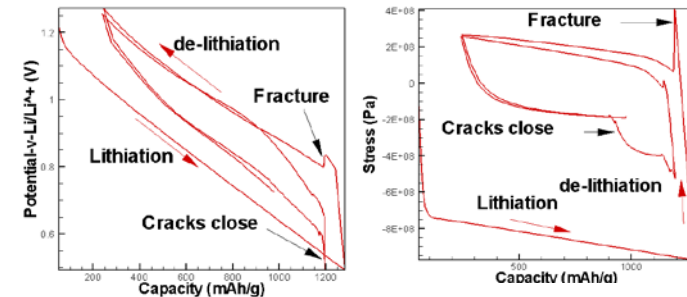
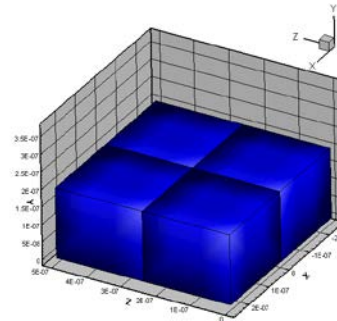
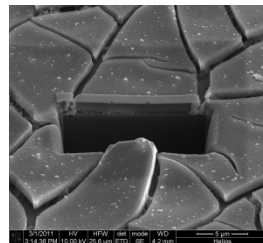
Bower/Guduru MSMSE 2012

- Material model / surface electrochemistry model are taken from thin film studies
- Model also incorporates *fracture* using cohesive zones. Li insertion through new crack surfaces is included.
- Numerical methods to handle coupled diffusion/deformation, finite strains, electrochemical boundary conditions
- Now implemented as user subroutines in ABAQUS for sharing



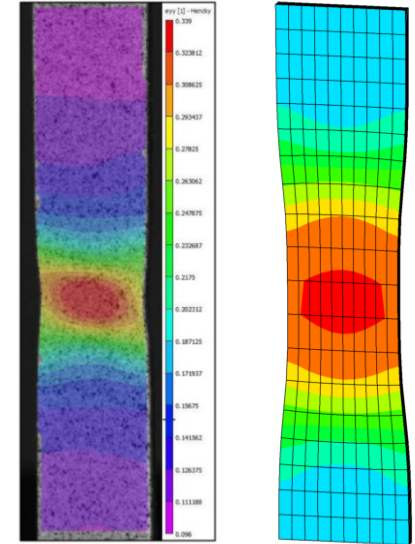
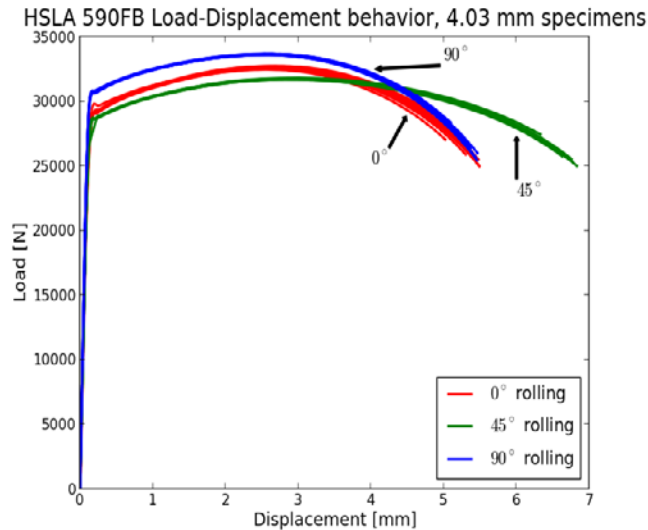
### Applications

- Fracture in thin Si film
- Fracture in Si nanoparticle electrode



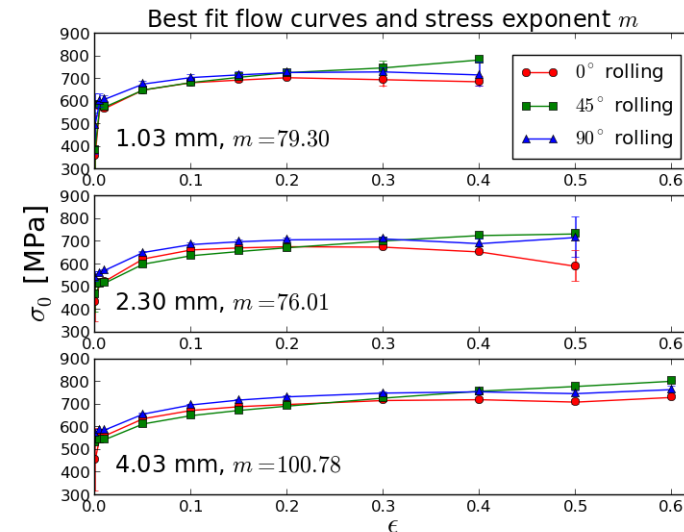
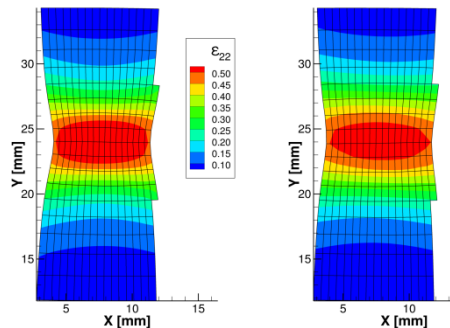
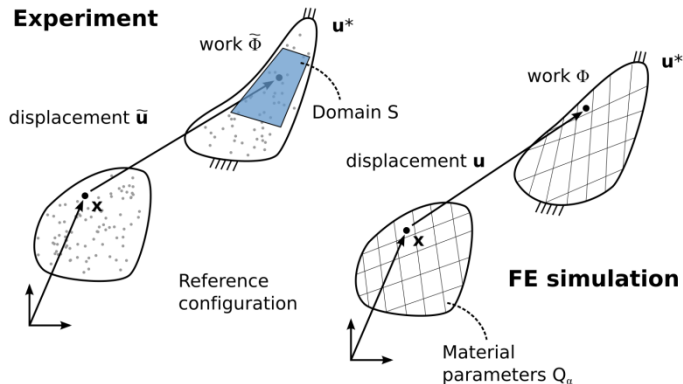
## FEA – DIC for measuring material properties

- We want to find material properties that best fit experimental displacements (from DIC) and load-displacement curves for a specimen of some sort.
- Use an objective function that penalizes differences between FEA simulations and experimental results.
- Minimizing the objective function can be done analytically alongside a normal finite element computation



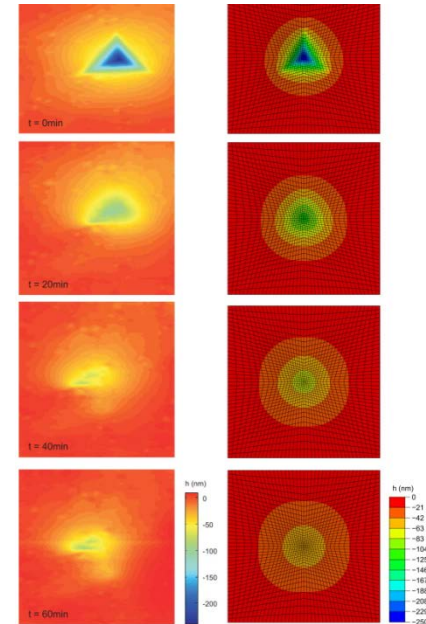
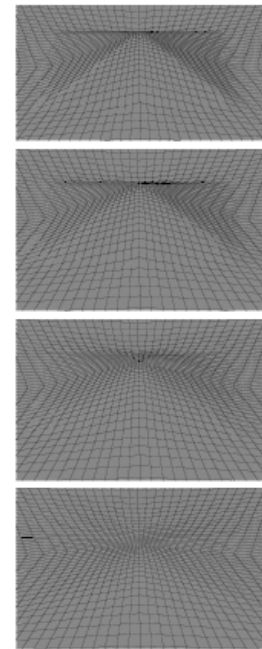
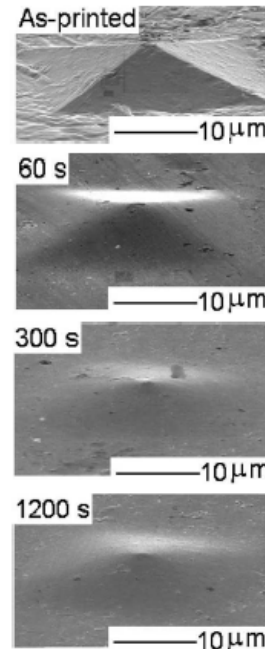
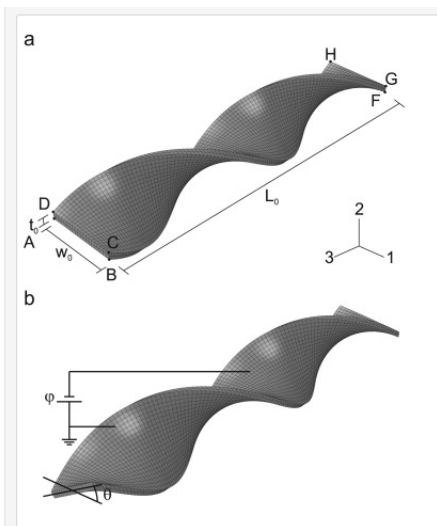
Goal: find material parameters  $Q_a$  that minimize

$$\Pi = \frac{\beta}{\bar{\Phi}^2 \Delta t} \sum (\Delta \Phi - \Delta \bar{\Phi})^2 + \frac{(1-\beta)}{|\bar{\mathbf{u}}|^2} \sum \int_S (\Delta \mathbf{u} - \Delta \bar{\mathbf{u}}) \cdot (\Delta \mathbf{u} - \Delta \bar{\mathbf{u}}) dA$$



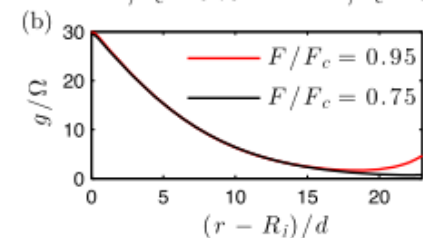
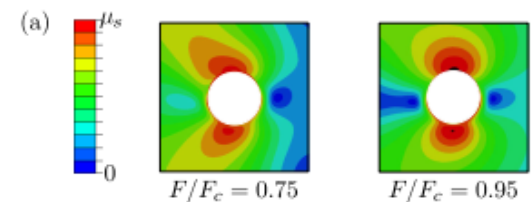
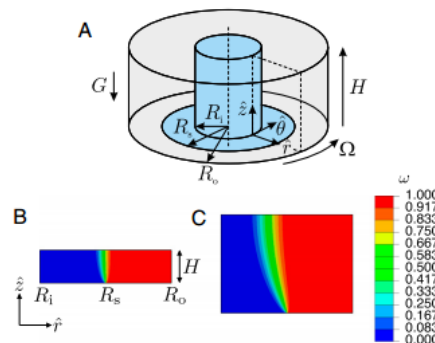
## Metallic glasses, dielectric actuators, granular materials - David Henann

- Large strain constitutive models for metallic glasses
- Constitutive models for electro-active polymers
- Constitutive models for granular materials
- Implementation in ABAQUS



(a)

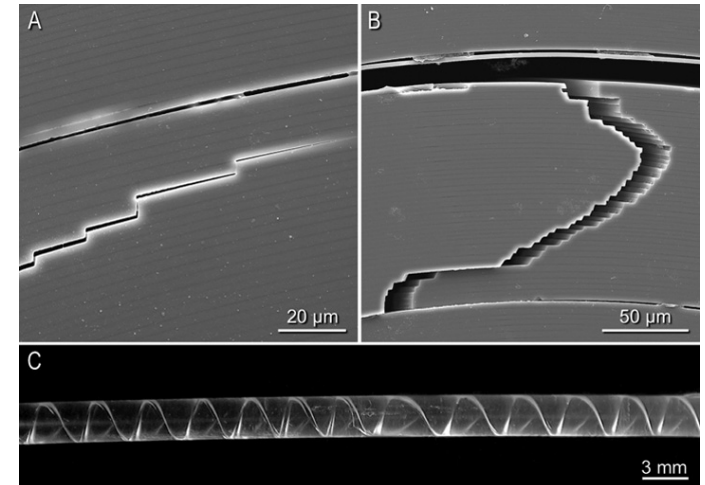
(b)





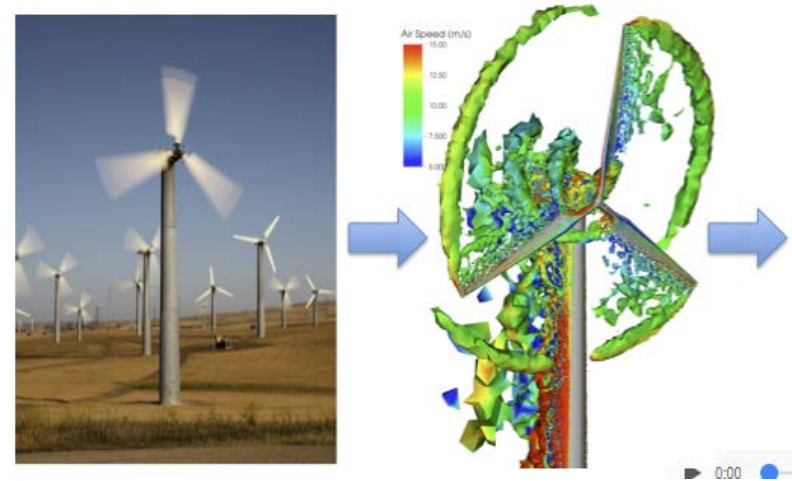
## Phase field fracture modeling (Haneesh Kesari)

- Phase field models are relatively new for studying fracture mechanics and damage evolution problems. However, they show great promise for tackling fracture problems that have been difficult to handle using standard finite element techniques. They are especially suitable for handling fracture problems which involve topology changes, such as crack branching, merging, etc. They, however, require further development for studying crack evolution in materials with spatial heterogeneity in mechanical properties and architecture.

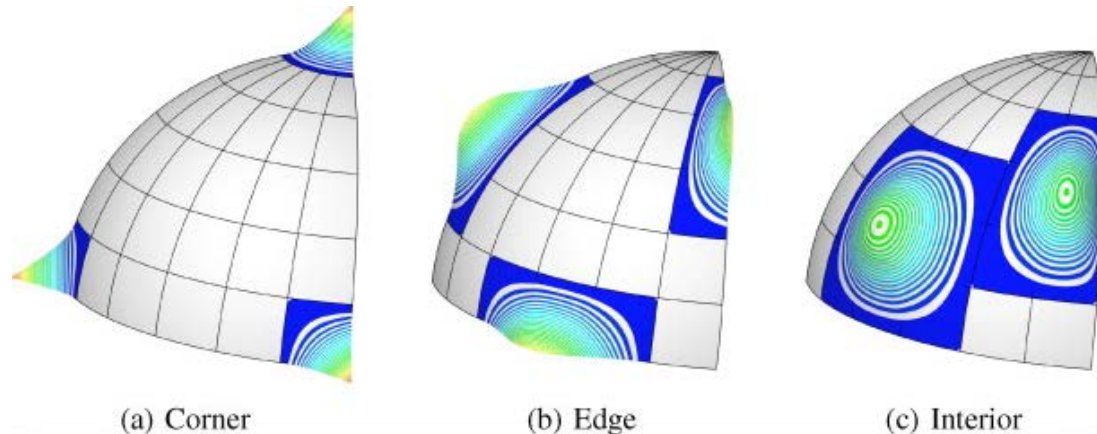


## Isogeometric Analysis, FSI (Yuri Bazilevs)

A numerical formulation for stratified incompressible flows, which is based on ALE-VMS methodology, is applied to simulate multiple 5MW horizontal-axis wind turbines (HAWT) at full scale interacting with Atmospheric Boundary Layer (ABL) flow. A multi-domain modeling (MDM) approach is adopted for computational efficiency.



A Reissner–Mindlin shell formulation based on a degenerated solid is implemented for NURBS-based isogeometric analysis. The performance of the approach is examined on a set of linear elastic and nonlinear elastoplastic benchmark examples. The analyses were performed with LS-DYNA



# Course Goals

Fundamentals of the finite element method of structural analysis. Nodal points, element design, and consistent formulation for assumed functions. Principle of virtual work, formulation of element stiffness and master stiffness matrices. Relation to variational and minimum principles. Linear elastic analysis for static problems; direct and iterative procedures. Nonlinear static analysis with piecewise linearization; elastic-plastic behavior, large geometry changes. Time dependent behavior.

## **After completing EN2340, you should**

- Be familiar with the theoretical basis for FEA in solid mechanics
- Be able to set up and solve problems using ABAQUS (or matlab/mathematica for simple problems)
- Be able to develop user elements and materials for ABAQUS (or use open source codes)
- Be able to use an IDE/Version control system to write and share code, and document revisions

# Related Courses

## APMA2230-S01 Partial Differential Equations

CRN:15807


980 of 999 Seats Available

Primary Meeting: T R 10:30 am - 11:50 am

Barus & Holley 163

The theory of the classical partial differential equations, as well as the method of characteristics and general first order theory. Basic analytic tools include the Fourier transform, the theory of distributions, Sobolev spaces, and techniques of harmonic and functional analysis. More general linear and nonlinear elliptic, hyperbolic, and parabolic equations and properties of their solutions, with examples drawn from physics, differential geometry, and the applied sciences. Generally, semester II of this course concentrates in depth on several special topics chosen by the instructor.

### Instructor(s)

Constantine M. Dafermos (P) 

## APMA2550-S01 Numerical Solution of Partial Differential Equations I

CRN:15809


29 of 40 Seats Available

Primary Meeting: W 03:00 pm - 05:30 pm

Barus & Holley 165

Finite difference methods for solving time-dependent initial value problems of partial differential equations. Fundamental concepts of consistency, accuracy, stability and convergence of finite difference methods will be covered. Associated well-posedness theory for linear time-dependent PDEs will also be covered. Some knowledge of computer programming expected.

### Instructor(s)

Chi-Wang Shu (P) 

## APMA 2570 - Numerical Solution of Partial Differential Equations III

No description available.

1.000 Credit hours

1.000 Lecture hours

**Levels:** Graduate, Undergraduate

**Schedule Types:** Do not Schedule

Applied Mathematics Department

## APMA 2580A - Computational Fluid Dynamics

The course will focus on finite element methods for viscous incompressible flows. We first find discretization for Stokes equation and in the process discuss inf-sup stability and other stability techniques. We also consider different time discretizations as well as splitting methods. We then consider Navier-Stokes equations. We will write code and use software (e.g. Fenics) to run numerical experiments. Pre-requisites: APMA 2550, basic knowledge of finite element methods would be helpful but not necessary.

1.000 Credit hours

1.000 Lecture hours

**Levels:** Graduate, Undergraduate

**Schedule Types:** Primary Meeting

Applied Mathematics Department

## APMA 2240 - Partial Differential Equations

The theory of the classical partial differential equations, as well as the method of characteristics and general first order theory. Basic analytic tools include the Fourier transform, the theory of distributions, Sobolev spaces, and techniques of harmonic and functional analysis. More general linear and nonlinear elliptic, hyperbolic, and parabolic equations and properties of their solutions, with examples drawn from physics, differential geometry, and the applied sciences. Generally, semester II of this course concentrates in depth on several special topics chosen by the instructor.

1.000 Credit hours

1.000 Lecture hours

**Levels:** Graduate, Undergraduate

**Schedule Types:** Primary Meeting

## APMA2560-S01 Numerical Solution of Partial Differential Equations II


CRN:24789

999 of 999 Seats Available

Primary Meeting: M W F 02:00 pm - 02:50 pm

An introduction to weighted residual methods, specifically spectral, finite element and spectral element methods. Topics include a review of variational calculus, the Rayleigh-Ritz method, approximation properties of spectral and finite element methods, and solution techniques. Homework will include both theoretical and computational problems.

### Instructor(s)

Mark Ainsworth (P) 

## APMA 2570A - Numerical Solution of Partial Differential Equations III

We will cover spectral methods for partial differential equations. Algorithm formulation, analysis, and efficient implementation issues will be addressed.

Prerequisite: APMA 2550 or equivalent knowledge in numerical methods.

1.000 Credit hours

1.000 Lecture hours

**Levels:** Graduate, Undergraduate

**Schedule Types:** Primary Meeting

Applied Mathematics Department



## What is special about FEA for solid mechanics?

- We always solve the same PDE ( $\mathbf{F}=\mathbf{0}$  or  $\mathbf{F}=\mathbf{ma}$ )
- EOM (for dynamics) is second order in time
- Problems are nearly always nonlinear
- Describing geometry changes, and designing elements that capture these changes properly, can be tricky
- We are usually using a complex history and/or time dependent material model
- We often need to handle challenging boundary conditions (eg contact)
- We often need special procedures only relevant to solids – buckling; fracture; etc

# Course Outline

1. Overview of FEA, introduction to ABAQUS and ABAQUS/CAE
2. FEA for static linear elasticity
3. Advanced Element Formulations – shear locking; pressure locking; hybrid elements
4. FEA for nonlinear materials (hypoelasticity; small strain plasticity)
5. FEA with finite deformation problems (hyperelasticity; large strain plasticity)
6. FEA for time dependent problems: diffusion, dynamic linear elasticity
7. Structural elements (beams, plates, shells)
8. Cohesive Zones
9. Contact

# Organization

- Course Web Site  
<http://www.brown.edu/Departments/Engineering/Courses/En2340/>
- <http://solidmechanics.org> – online solids text – see FEA codes section for demo MATLAB codes

You will need the following software:

- MATLAB/Mathematica (download from CIS website)
- ABAQUS and ABAQUS/CAE (available on instructional computer facility and via remote desktop)
- ABAQUS is available for windows/linux through engineering software server (no mac version).
- Can run ABAQUS on CCV with a free exploratory account
- For writing code (these work on Win, Mac, Linux):
  - Either: Gnu Fortran/Eclipse IDE, or: Intel parallel studio
  - A GitHub account
  - EN234FEA (on GitHub)
- TECPLOT (download from CIS website)

# Assignments

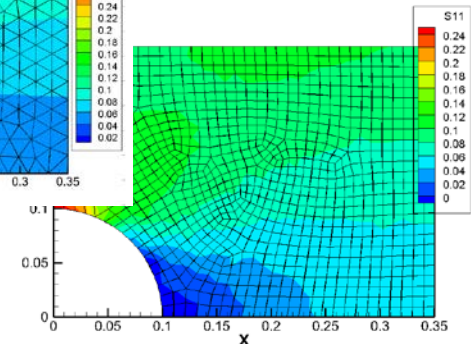
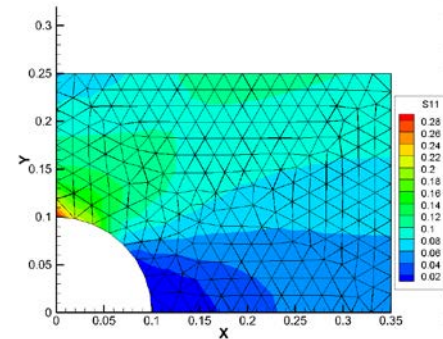
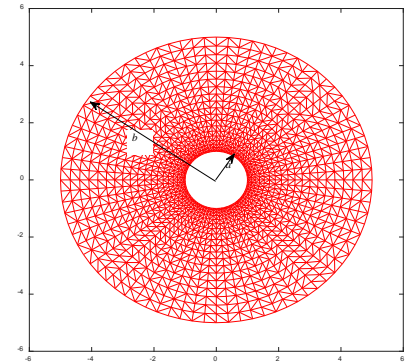
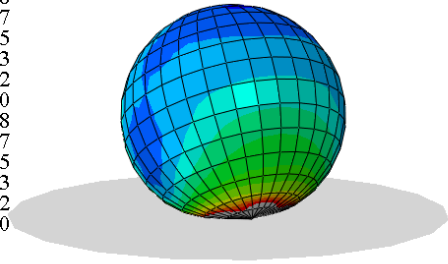
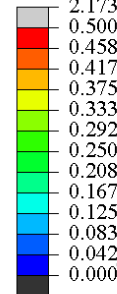
- ABAQUS simulations
  - MATLAB coding
  - FEA with Mathematica
  - Implementing elements/materials in ABAQUS
  - A final project
- 
- Assignments 1 week long
  - Due Fridays
  - Assignments submitted via Canvas / GitHub



# Assignments

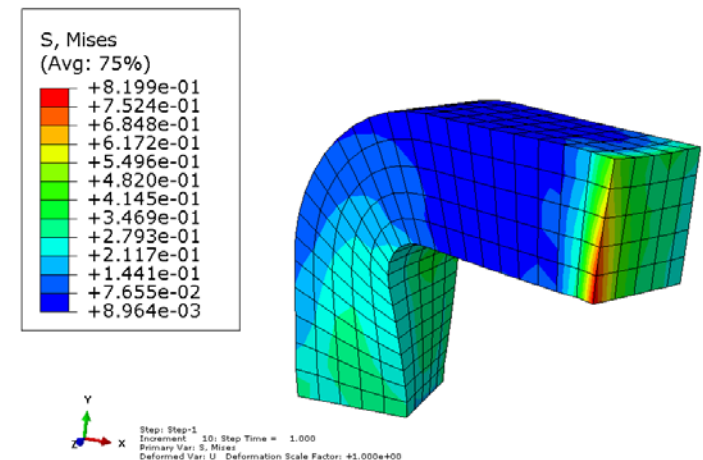
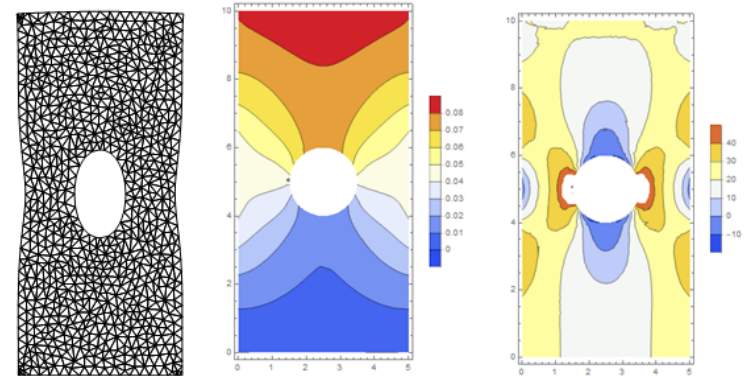
- HW1: ABAQUS/CAE simulation
- HW2: Basic FEA – write a MATLAB code to solve a simple incompressible flow problem
- HW3: A Basic ABAQUS user element – implement 2D plane elasticity

S, Mises  
(Avg: 75%)



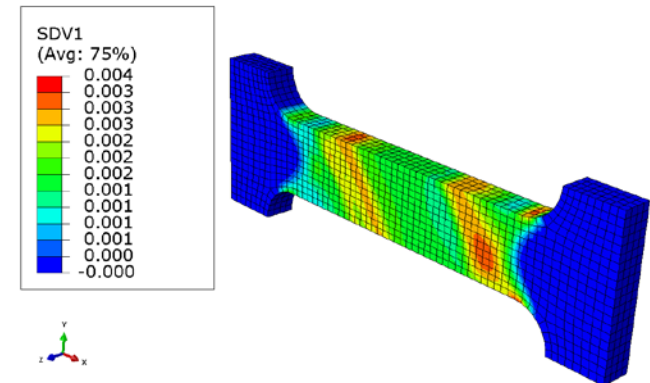
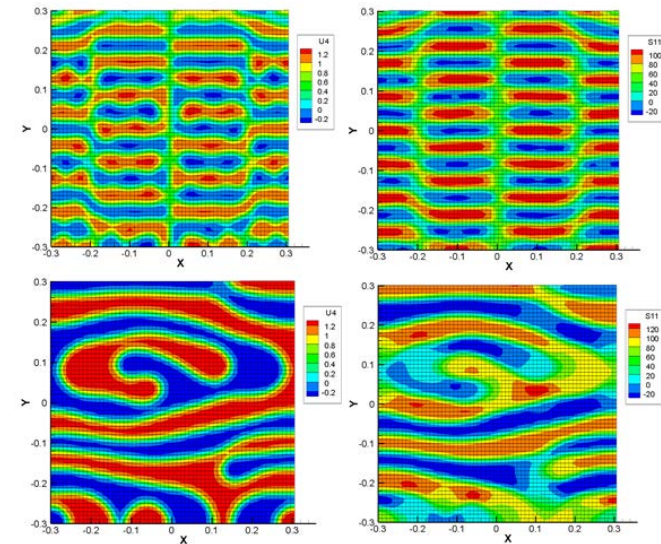
# Assignments

- HW4: FEA with Mathematica
- HW5: Special elements – incompatible mode elements
- HW6: Nonlinear materials: ABAQUS UMAT for a simple porous elasticity model
- HW7 Large deformations (anisotropic hyperelasticity)



# Assignments

- HW8: Phase field simulations
- HW9: Dynamic plasticity (modeling PLC bands)
- HW10: Continuum beam elements



# Grading

- Completed all phases of the course - (use of ABAQUS; Matlab coding exercises; ABAQUS user element and material subroutine development); no more than two assignments submitted late; solutions were correct, well presented and organized; evidence of initiative and significant effort in most assignments and in final project - A
  - Completed 8 or more homeworks and a final project; no more than four assignments were submitted late; assignments well presented and organized; final project is extensive and shows initiative; B
  - Completed 6 or more homeworks; completed a final project - C
  - Anything else – NC
- 
- If you have a very busy semester and might find it hard to meet the HW deadlines consider S/NC