Odysseas Skartsis

ENGN2340

Project

LEFM Convergence Study

12/11/13

Thanks to Tianyang Zhang, Davide Grazioli, and Ruike Zhao for their helpful tips on using ABAQUS.

Structure of this report

Main body: Results and discussion on:

1) Mesh Refinement
2) Extrapolation of SIF
3) Comment on the value of radius used

Appendix: K-graphs showing the:

1) Effect of refinement
   - Free-Free-Free meshing algorithms
   - Sweep-Sweep-Free meshing algorithms
   - Sweep-Structured-Free meshing algorithms
   - Structured-Structured-Free meshing algorithms
2) Effect of choice of data points
3) Effect of radius used

Figures of different mesh configurations (undeformed and deformed contour plots)

Preliminary Info

The ABAQUs element for plane strain with the default settings (standard, linear, quad-dominated, reduced integration, etc.) is used for all the analyses below.

1) Effect of Level of Mesh Refinement and Algorithm in the Estimation of the Stress Intensity Factor

The procedure and formulas described in the proposal were followed to estimate by extrapolation the stress intensity factor by the stress and displacement field near the crack tip. The results are summarized in the following table.

METHODOLOGY

In the beginning different refinement levels of mesh are used and the whole solid is meshed with the "free" algorithm. Then, the solid where partitioned close to the crack tip with two circular regions and different meshing algorithms meshed those partitions. More on how to obtain the Stress Intensity Factor (SIF) is discussed in the following section.

DISCUSSION OF RESULTS

Concerning the free-free meshing we observe that by increasing the refinement close to the crack (see sample screenshots of the mesh in the Appendix, Fig. 1) the prediction of the SIF is not improved. This disagrees with the results shown on Anderson's Fracture Mechanics book.

Table 1: Summary of analyses results for different meshing algorithms and refinement levels.

		Mesh Type in the Two Inner Partitioned Regions							
		Free-Free		Sweep-Sweep		Sweep-Structured		Structured-Structured	
		Ku/K	Ks/K	Ku/K	Ks/K	Ku/K	Ks/K	Ku/K	Ks/K
Refinement Level	Baseline	1.020	0.990	0.906	0.770	0.942	0.750	-	-
	2x	0.950	0.925	-	-	-	-	-	-
	7x	-	-	0.919	0.812	-	-	-	-
	10x	0.950	0.921	-	-	-	-	-	-
	14x	0.930	0.900	-	-	-	-	-	-
	20x	0.940	0.960	-	-	-	-	-	-
	100x	0.900	0.905	-	-	0.980	0.880	0.980	0.920
	1000x	-	-	0.958	0.925	-	-	-	-

Note: Absences are due to either inability of the ABAQUS algorithm to mesh the defined partitioned geometry at the corresponding refinement level, or deliberately omitted as unnecessary.

However, when the sweep algorithm is used for the inner circular region at the crack-tip the evolution of the prediction of the SIF does indeed improve with refinement. Additionally, the distribution of the normalized SIF for those types of meshing matches the author's drawings (graph is sinking for r->0, see K-graphs in Appendix).

Also, for all the analyses carried out the estimate of the SIF based on the displacement field is better than that of the stress field (due to the singularity of the latter field), again in agreement with the author's results.

The model that agrees most with the author's analysis is the sweep-structured, although in our analysis the SIF prediction even for crude mesh is very close to the analytical value (and better than the author's). This result is probably expected by looking at the author's pictures of his mesh around the crack tip (he seems to be using a sweep-structured algorithm).

The interesting result is that the free-free meshing algorithm predictions for a very crude mesh give same (for the displacement field) or better results (for the stress field) than the author's choices or the other algorithms at much higher refinement, although the data points are significantly fewer and the path extent is 6 times increased (origin is the crack tip: from 0.1 -> 0.6, see K-graphs in Appendix)!

For the free-free meshing, the refinement did not yield better results; in trying to explain this deformed shapes and σ22 contour plots are shown in figures 2-6. The crude mesh elements are not so badly distorted (during deformation) close to the crack tip as in the very refined mesh and this might contribute to this divergent behavior. But, again, the structured-structured mesh yields elements with significant distortion at the crack tip (see Fig. 7), alas the results are much better.

In conclusion, the normalized SIF distribution around the crack tip and its convergence with increasing refinement for sweep-structured meshing of the nearest to the crack tip partitions agrees with Anderson's Fracture Mechanics (Chapter 12) results. Convergence is observed for the other algorithms also, except for the free, which seems to be the most efficient (best results with much less elements).

As a final note, the element used by Anderson is not the standard element that was used in this analysis, rather a degenerated element with coincident nodes at the crack tip.

2) Effect of data used for the SIF extrapolation

METHODOLOGY

To obtain an estimate of the asymptotic SIF crack tip the data points of the stress- and displacement-field based SIF are extrapolated for r->0 (see K-graphs in the Appendix).

DISCUSSION

As shown in the graphs---and also mentioned by Anderson---the choice of the points taken into consideration when performing the extrapolation for the SIF can affect the outcome (see Sec. 2 in Appendix for an example). Since the author's distribution is similar to the ones shown in section 1-c of the Appendix, where omitting points too close to the crack tip improves the agreement, he states that some initial will have to be omitted from the fitting.

However, for the other types of meshing algorithms, it can be seen that including those points in the fitting brings the estimate closer to the analytical solution for SIF.

3) Effect of radius used

When selecting the path along which results will be output ABAQUS has the ability (default choice) to give the "true distance", thus taking into account the displacement of the mesh nodes during loading (even when the nonlinear geometries option is not selected). Since the order of displacements here is very small the effect of taking the true distance instead of the X distance (initial projected X coordinate) is inconsequential (see plots in Sec. 3 of Appendix). In the graphs, unless otherwise specified, the "true distance" is used as the x coordinate of the graphs (radius in the formulas of SIF).

In the raw data of the displacements we see, for example, that the node of X- coordinate 0.004013 is vertically displaced by 2.63E-05 (consistent but arbitrary units), or the one at 0.1 by 1.31E-04. The two options yield the same result for the x coordinate at the first 6 decimal digits (maybe because in the distance formula the coordinates and their displacements are squared under the square root).

In the case, however, that mesh deformations comparable to the mesh dimensions and the path does indeed change that might suggest that the formulas---of SIF here---should be derived taking into account large deformations.

APPENDIX

OUTLINE

1) Effect of refinement*

   a) Free-free-free meshing (i.e., free type of mesh is used in all partitions)

   b) Sweep-sweep-free meshing (from the crack-tip to the outer boundaries)

   c) Sweep-structured-free meshing

   d) Structured-structured-free meshing

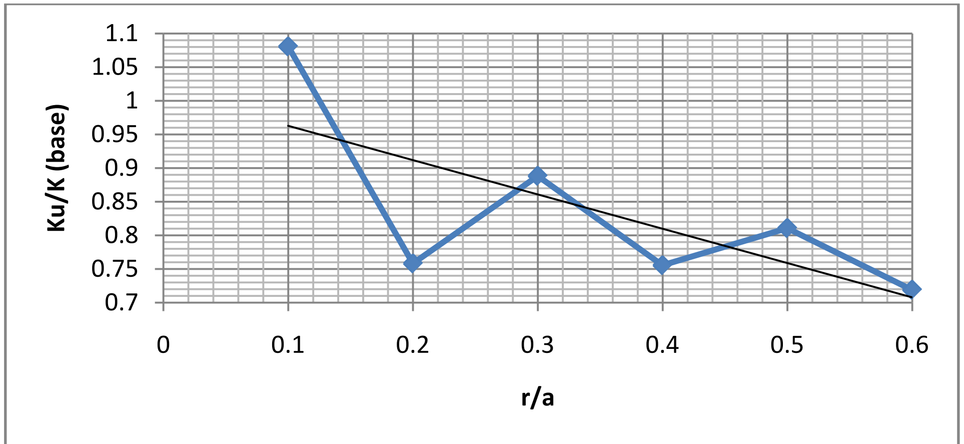
2) Effect of source of linear extrapolation

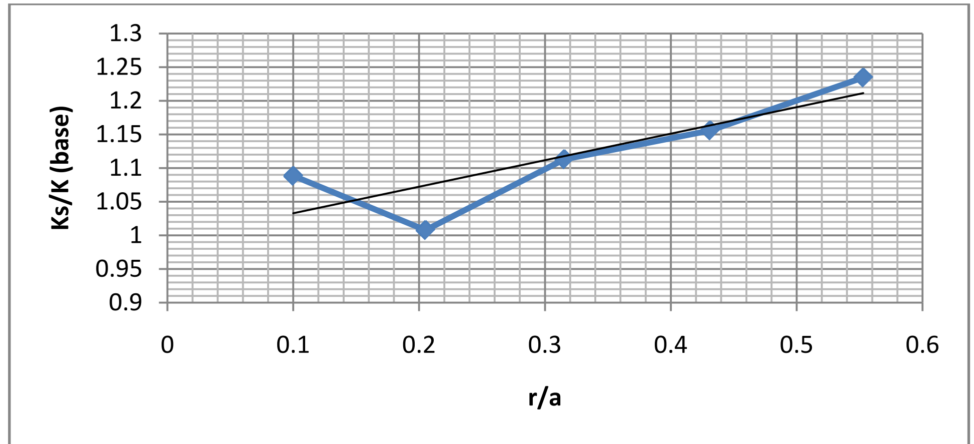
3) Effect of distance projection

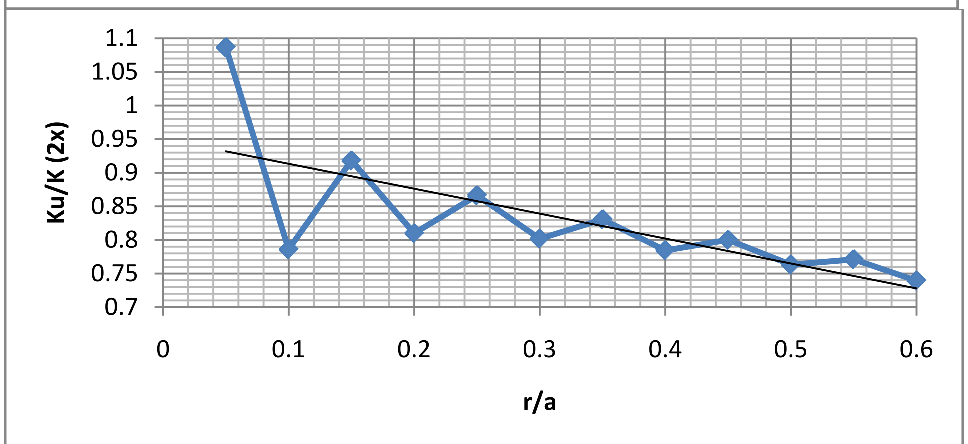
* The left and right columns show the stress intensity factor (Ku, Ks) distribution and its extrapolation using the displacement field u2 (crack line, $\theta = \pi$) or the stress field $\sigma 22$ (crack front, $\theta = 0$) respectively. The formulas used are given in the project proposal.

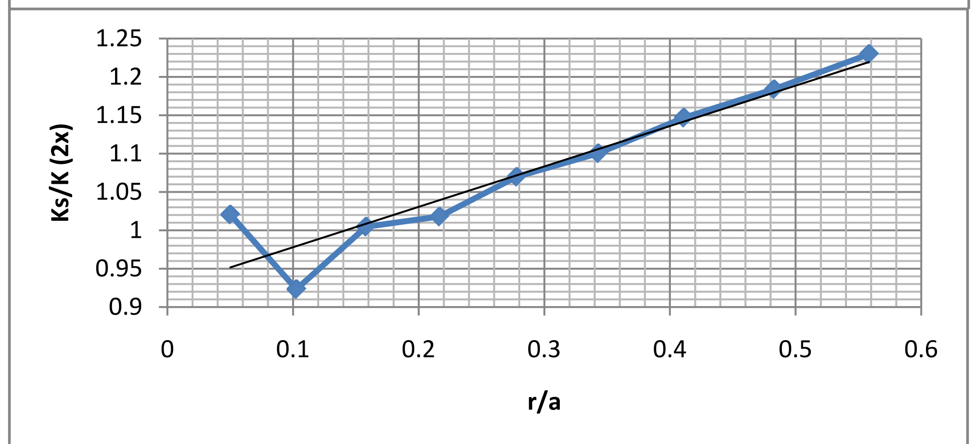
1) Effect of refinement

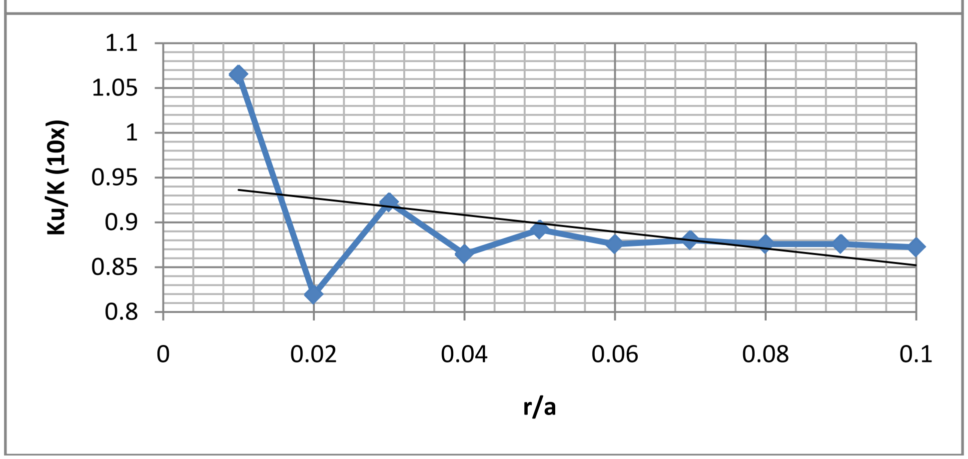
a) Free-free-free meshing (i.e., free type of mesh is used in all partitions)

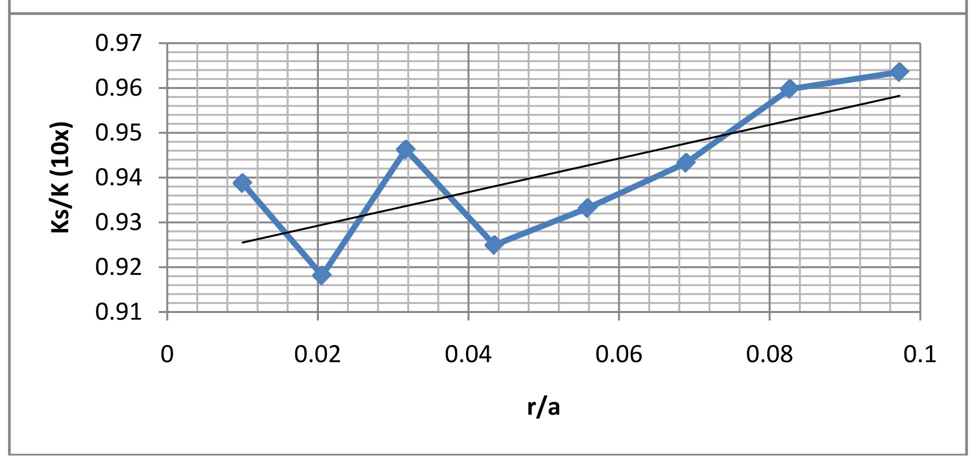


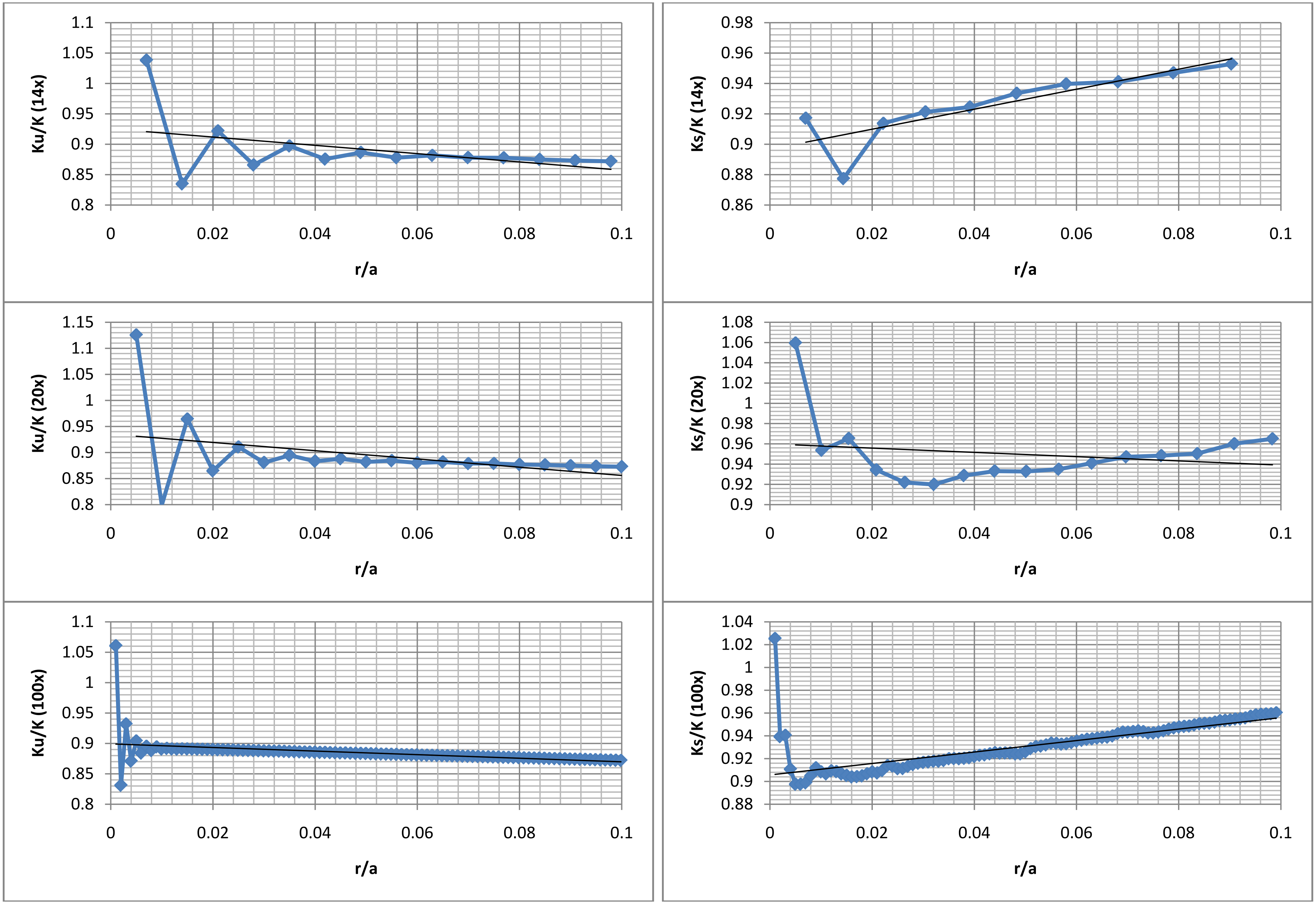




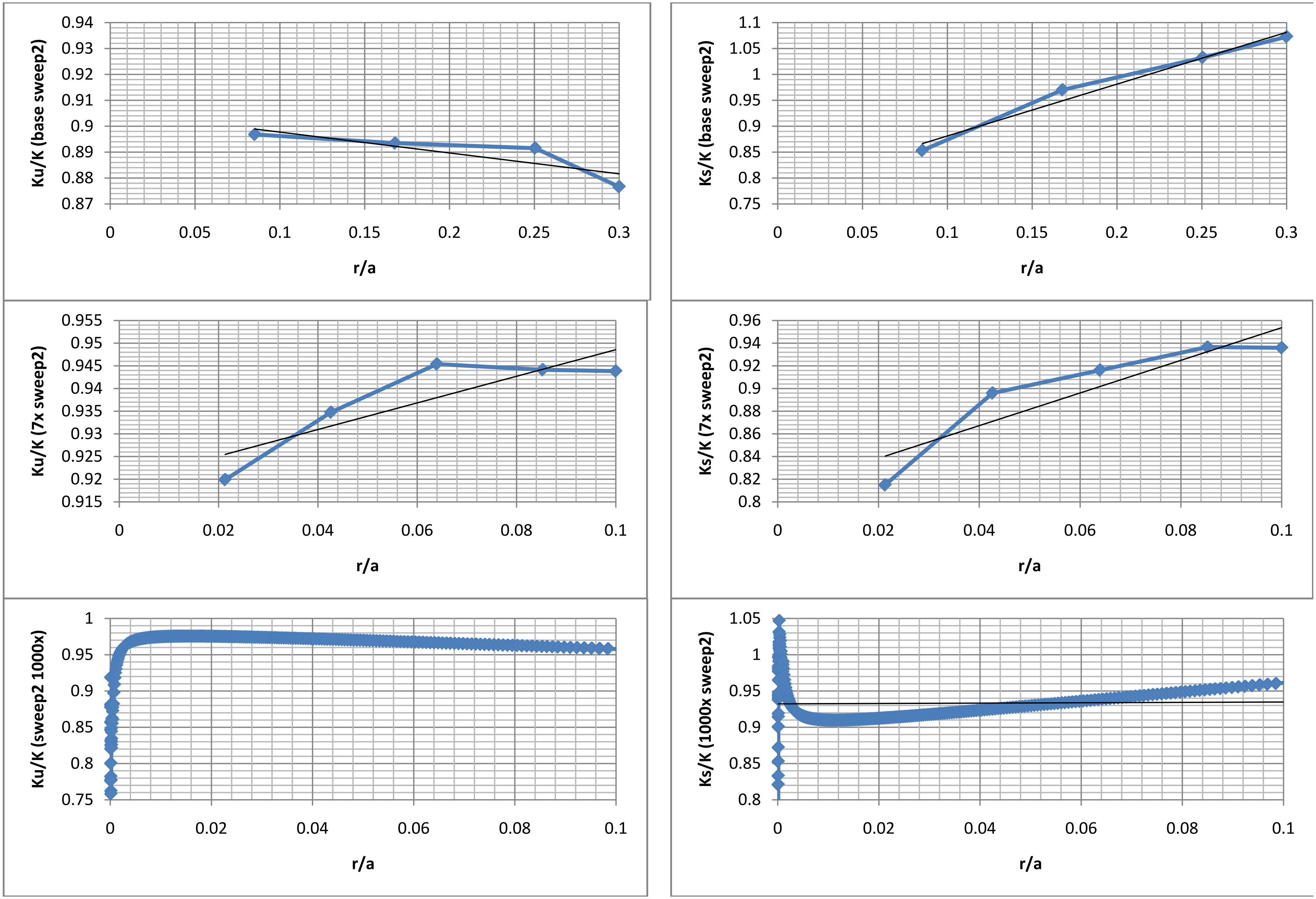




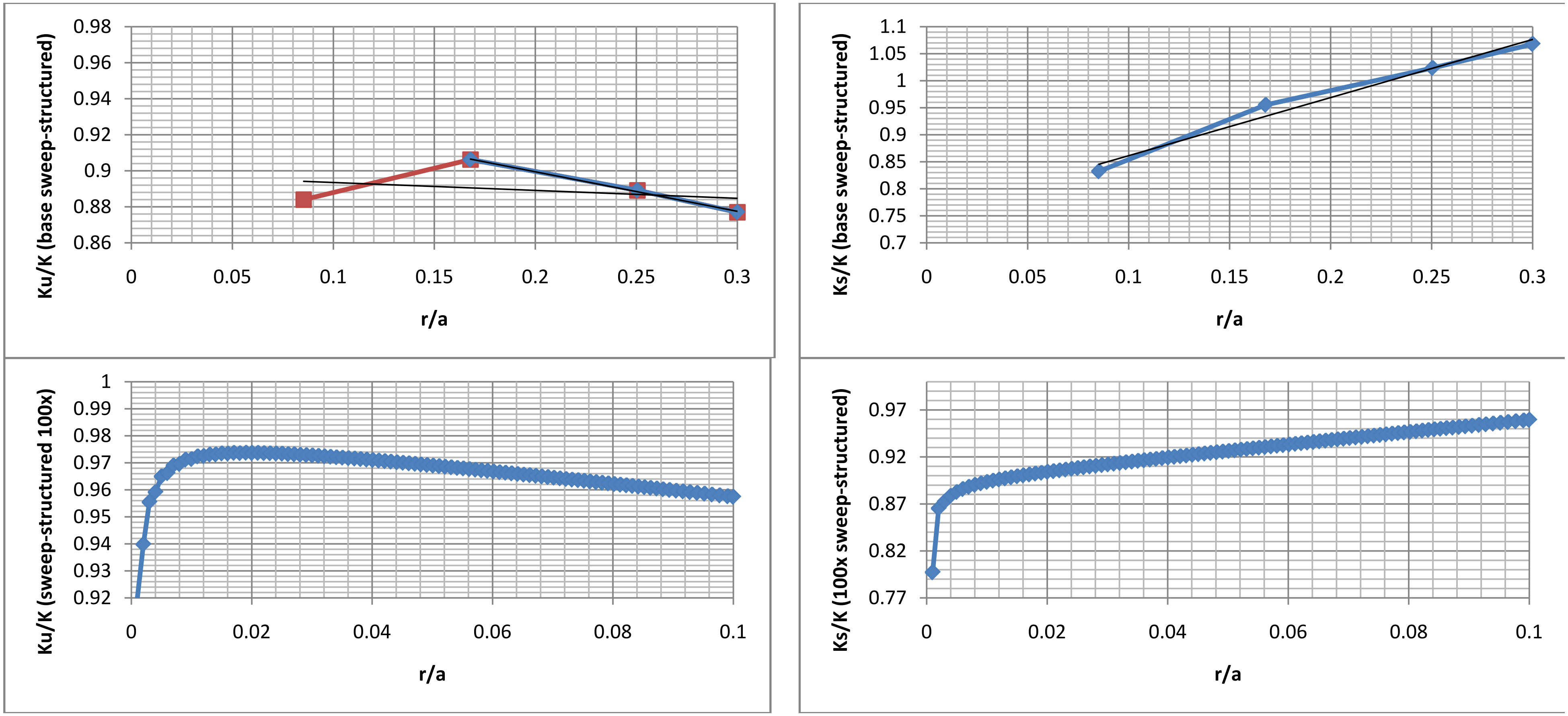



Ku/K (14x)
r/a
Ks/K (14x)
r/a
Ku/K (20x)
r/a
Ks/K (20x)
r/a
Ku/K (100x)
r/a
Ks/K (100x)
r/a

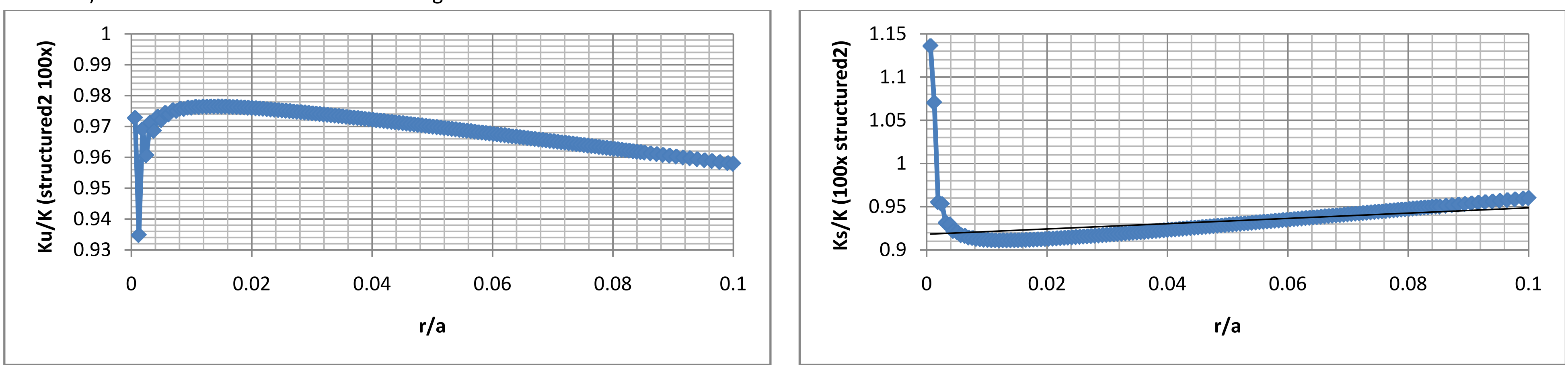
b) Sweep-sweep-free meshing (from the crack-tip to the outer boundaries)



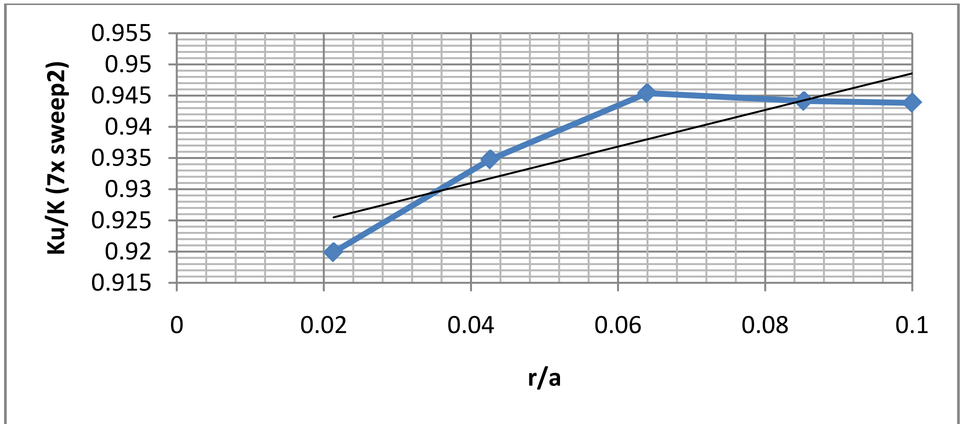
c) Sweep-structured-free meshing

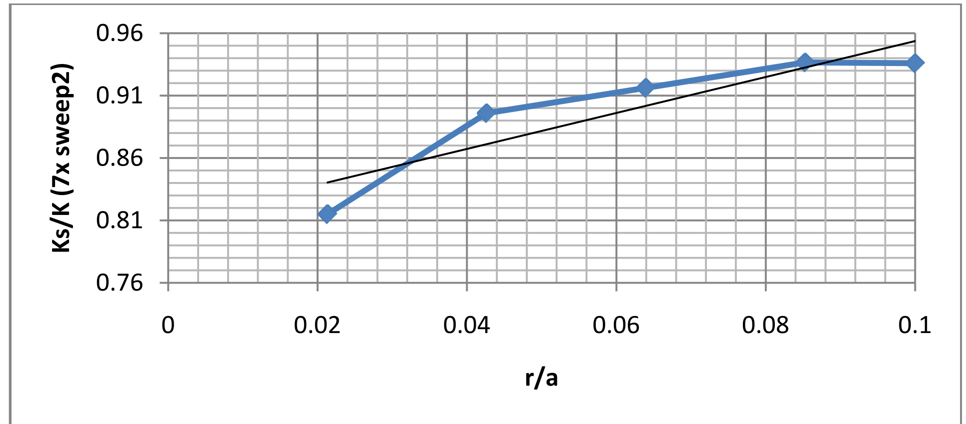


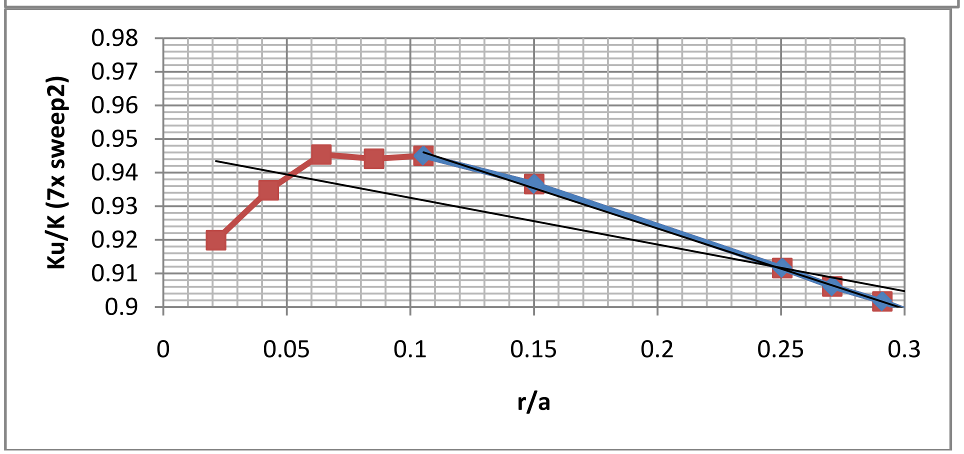
d) Structured-structured-free meshing

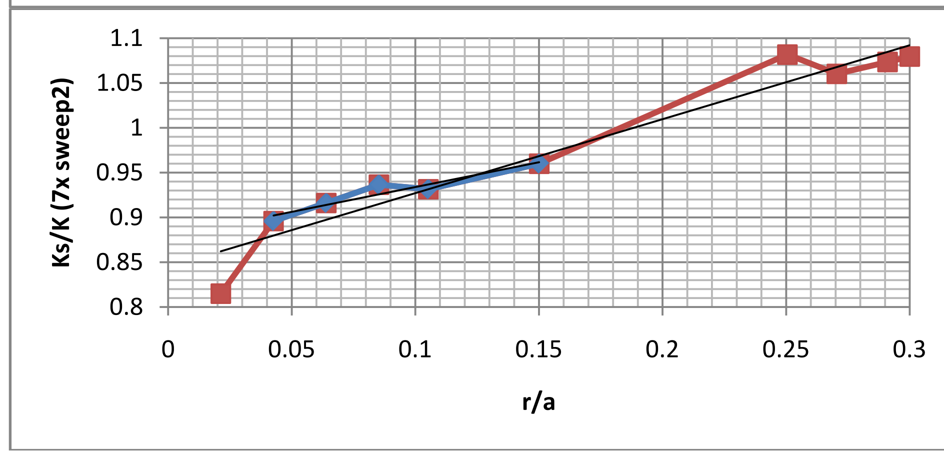


2) Effect of source of linear extrapolation

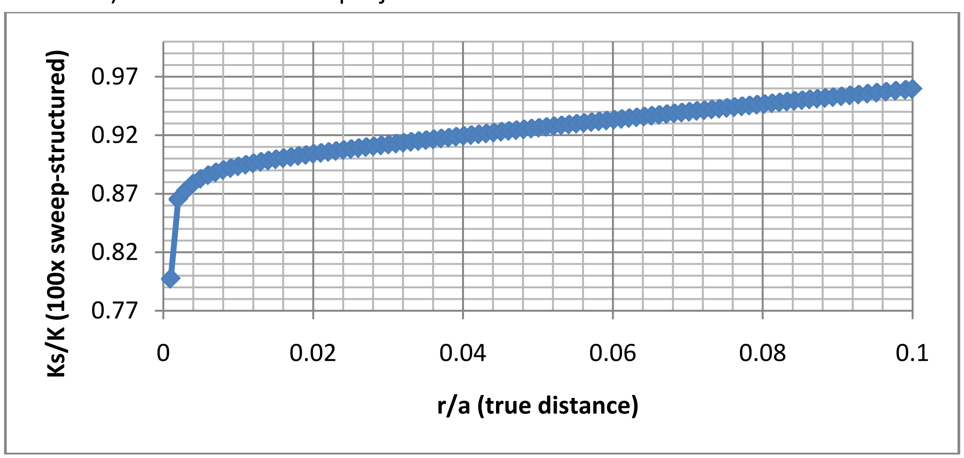


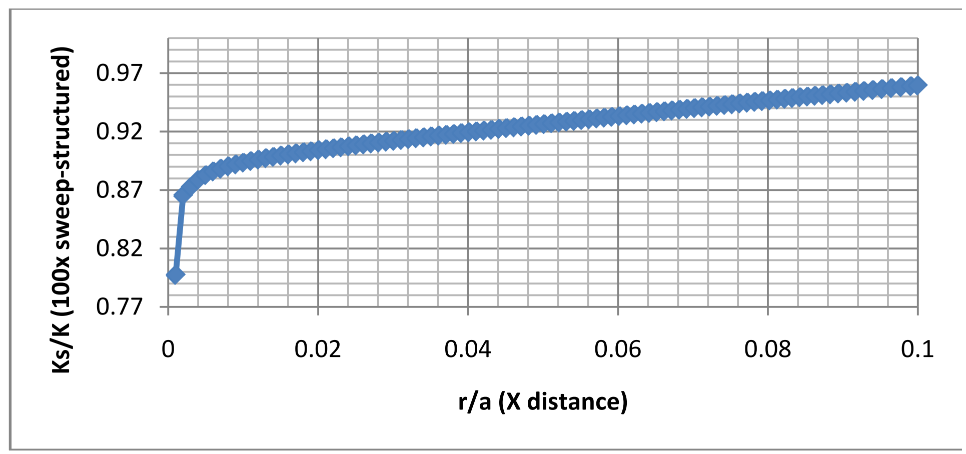






3) Effect of distance projection





Free-free-free meshing

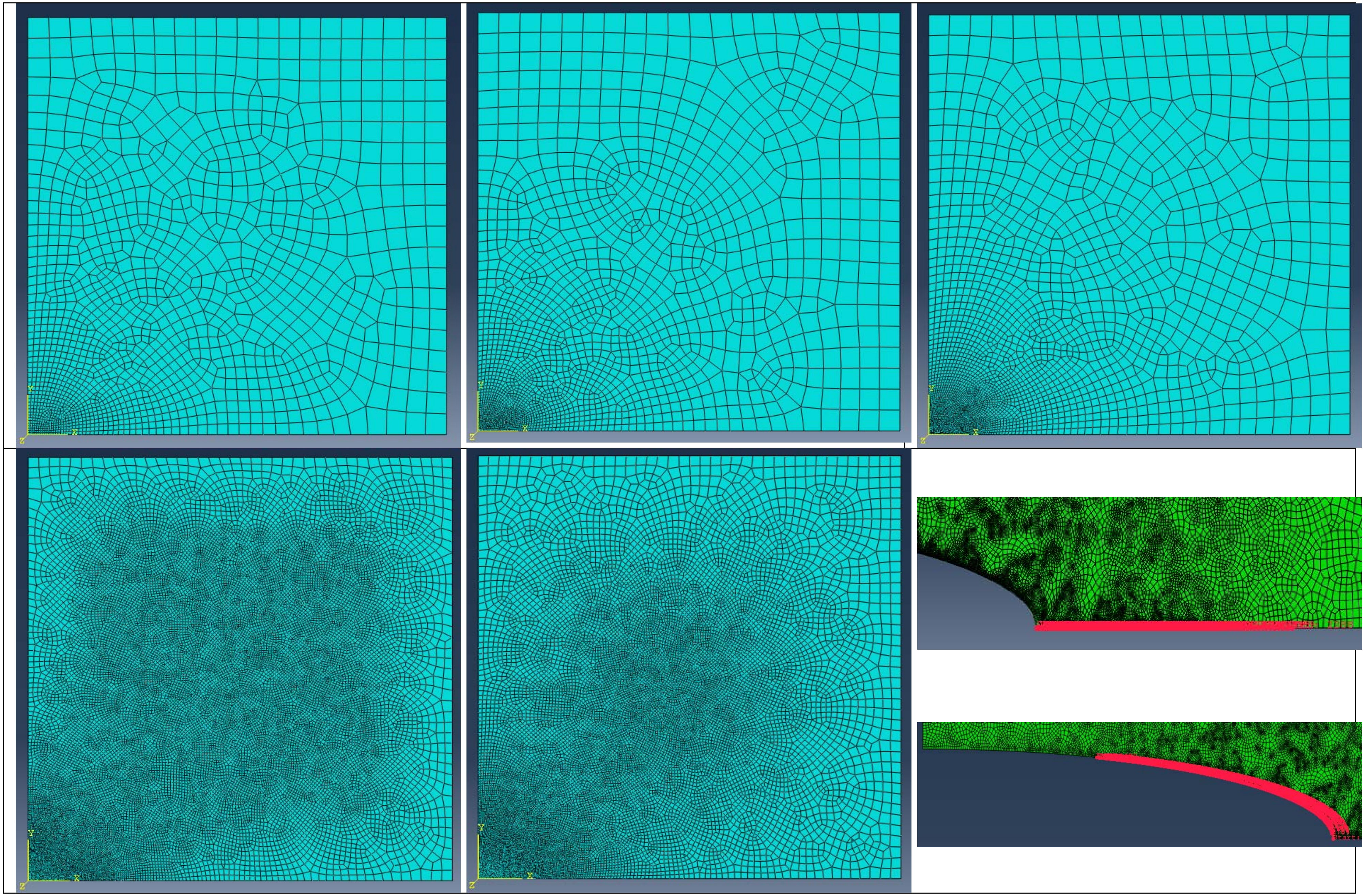


Figure 1: From top-left corner to bottom right: The complete mesh of the body for gradual refinements: Baseline Model, and 2x, 10x, 20x, 100x Refinements (14x Refinement not shown). Here only the edges are partitioned (not the faces) and seeded separately. The mesh type is free.

Free-free-free meshing

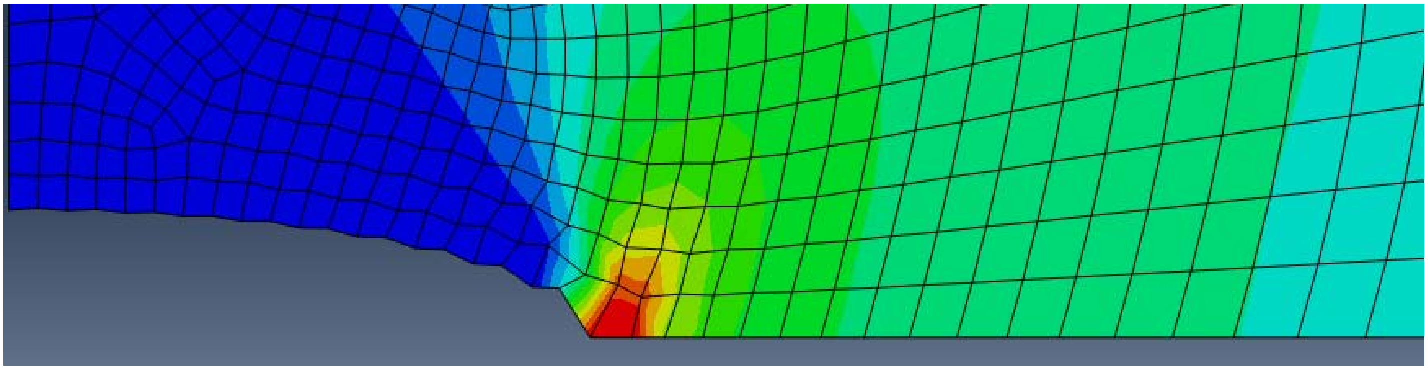


Figure 2: Mesh, u_2, and σ_{22} around the crack for 2x Refinement

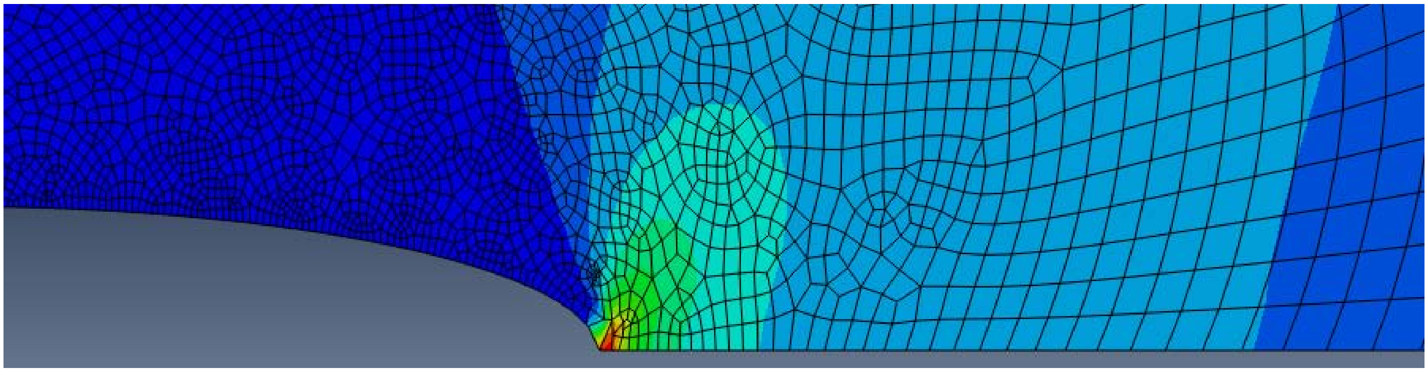


Figure 3: Mesh, u_2, and σ_{22} around the crack for 10x Refinement

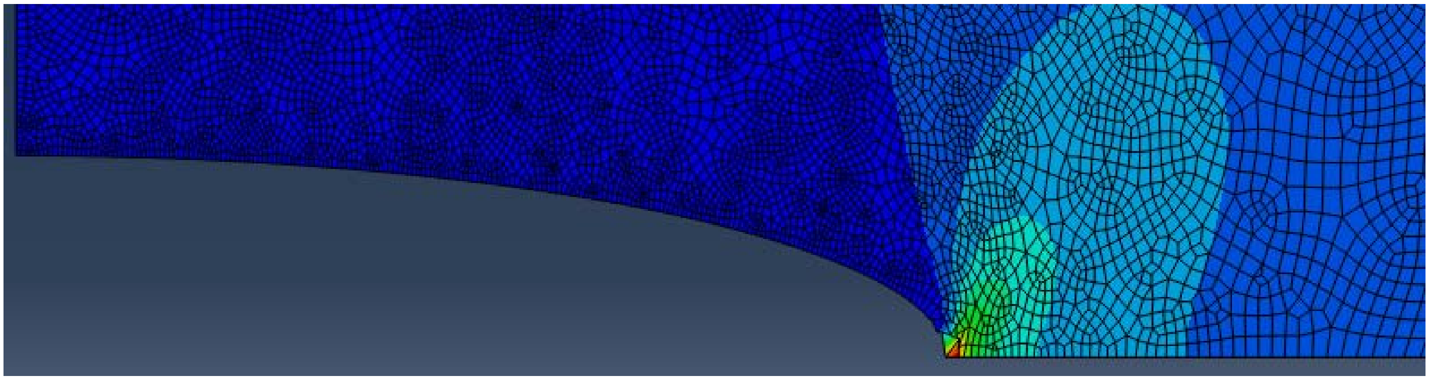


Figure 4: Mesh, u_2, and σ_{22} around the crack for 20x Refinement

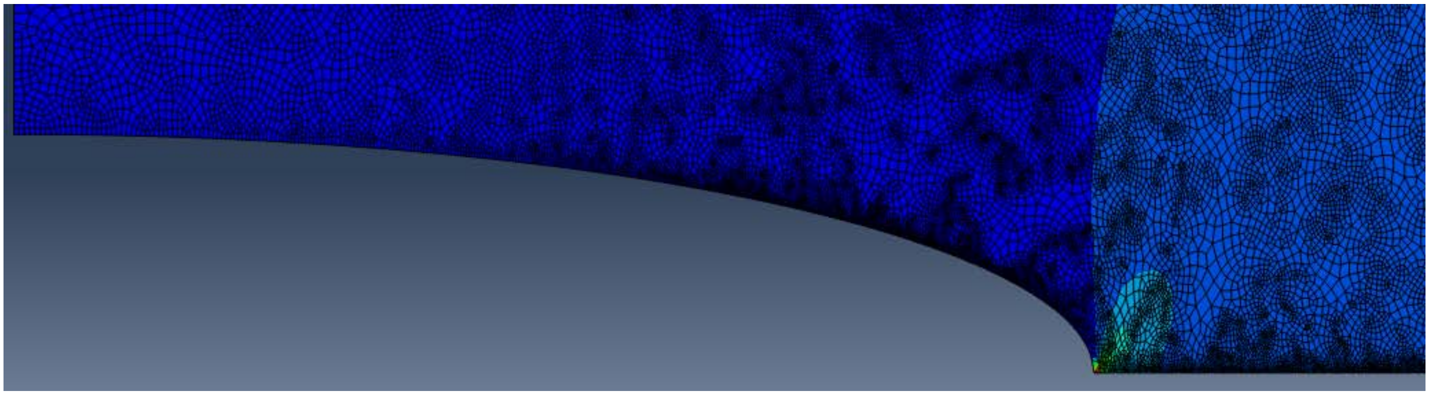


Figure 5: Mesh, u_2, and σ_{22} around the crack for 100x Refinement

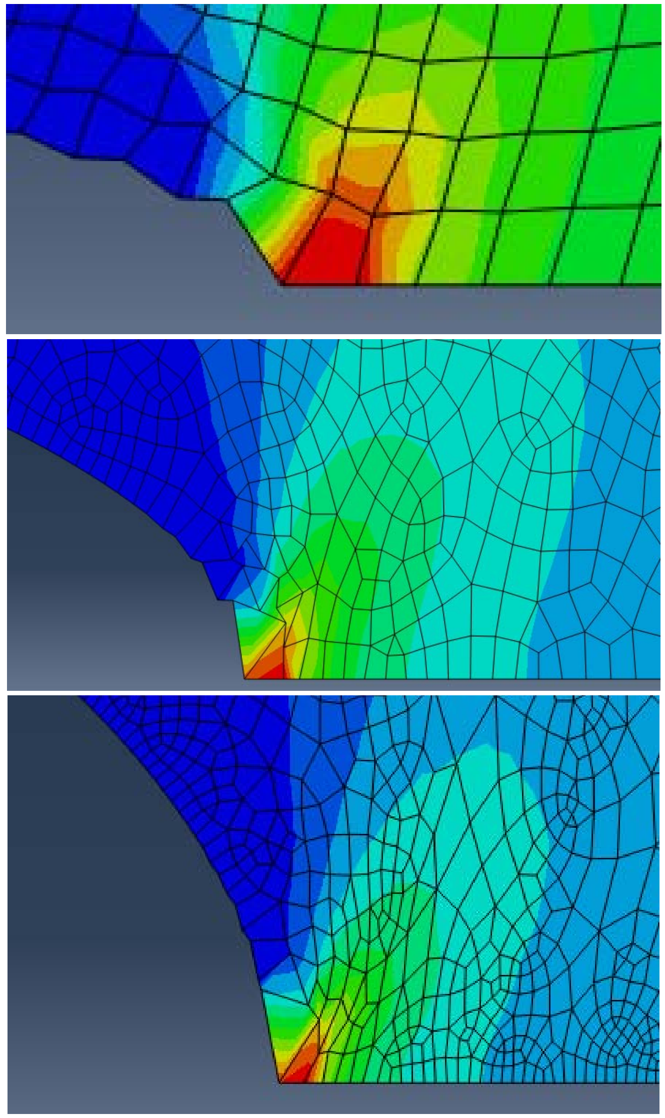


Figure 6: Mesh, u_2, and σ_{22} around the crack-tip for 2x, 20x, and 100x Refinement (top to bottom).

Structured-structured-free meshing

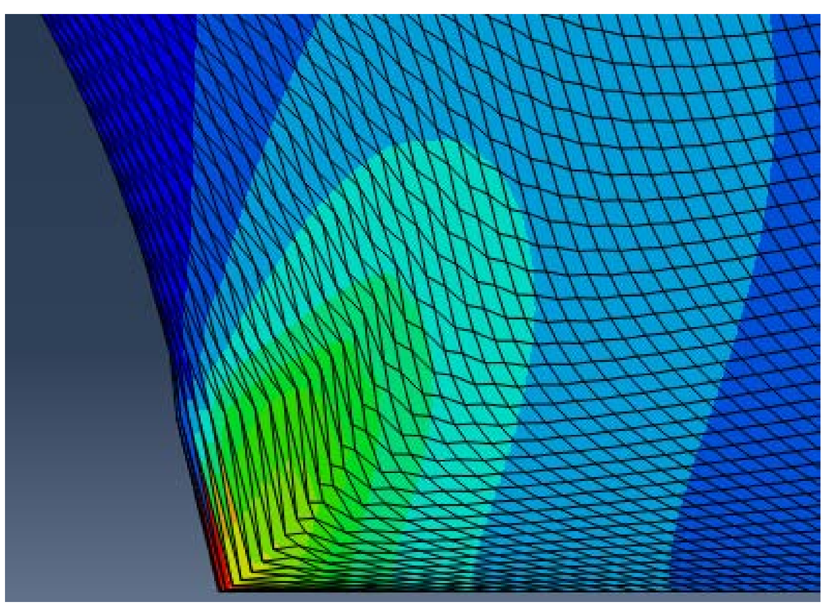


Figure 7: Detail of the deformation of the elements (structured mesh) at the crack tip. The contours show levels of σ22.

Sweep-structured-free meshing

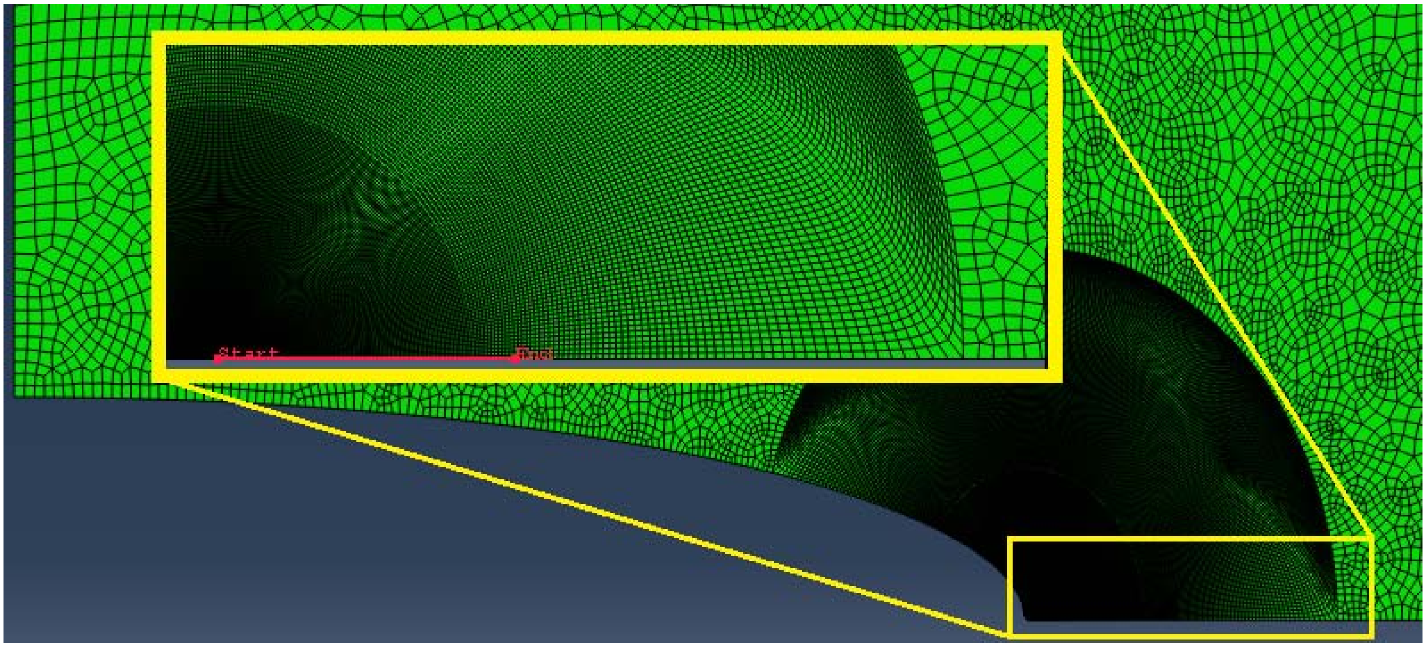


Figure 8: The three partitions around the crack tip. The mesh is increased close to the crack tip (100x). The crack front path for data extraction is also shown. Scale: the total width of the solid is 20, the crack length is 1, and the red path-line is 0.1. Note that the inset is in the undeformed state and the region is approximately mapped for illustration purposes.

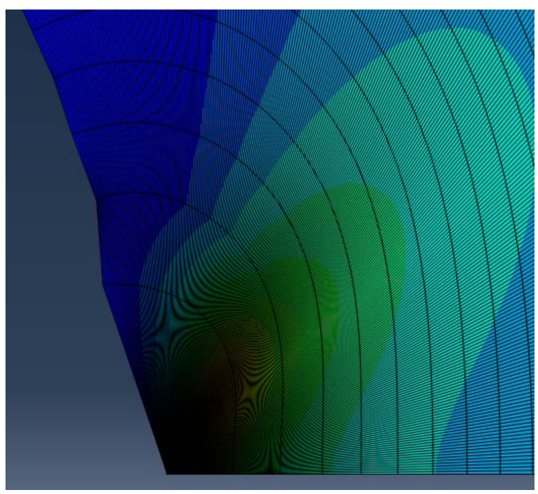


Figure 9: Detail of the deformation of the elements (sweep mesh) at the crack tip. The contours show levels of σ22.

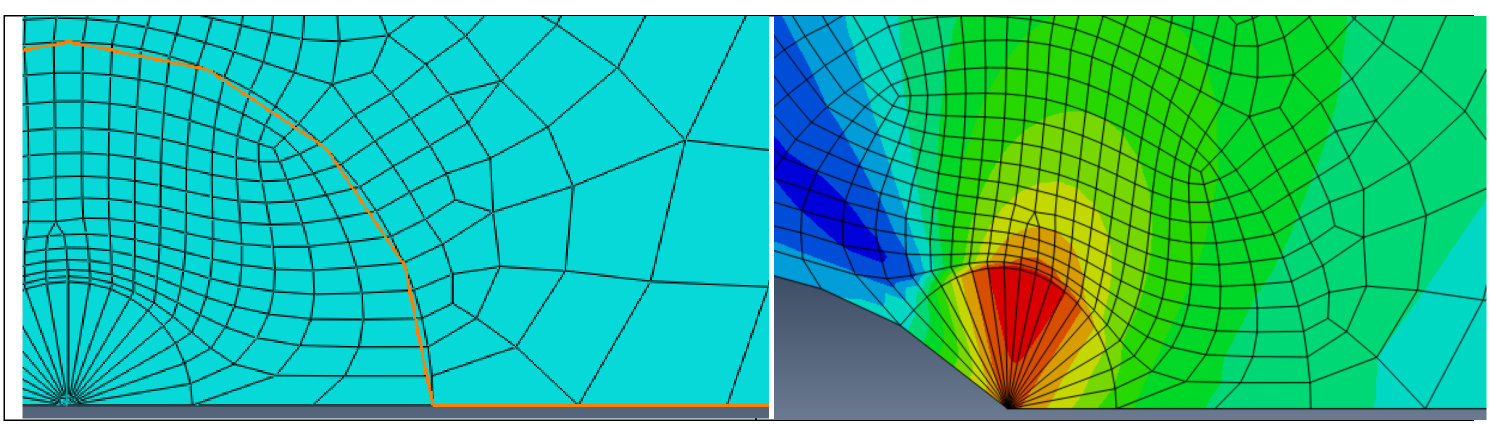


Figure 10: The coarse mesh (Baseline) for sweep-structured-free meshing. *Left:* undeformed, *right:* deformed.

Sweep-sweep-free meshing

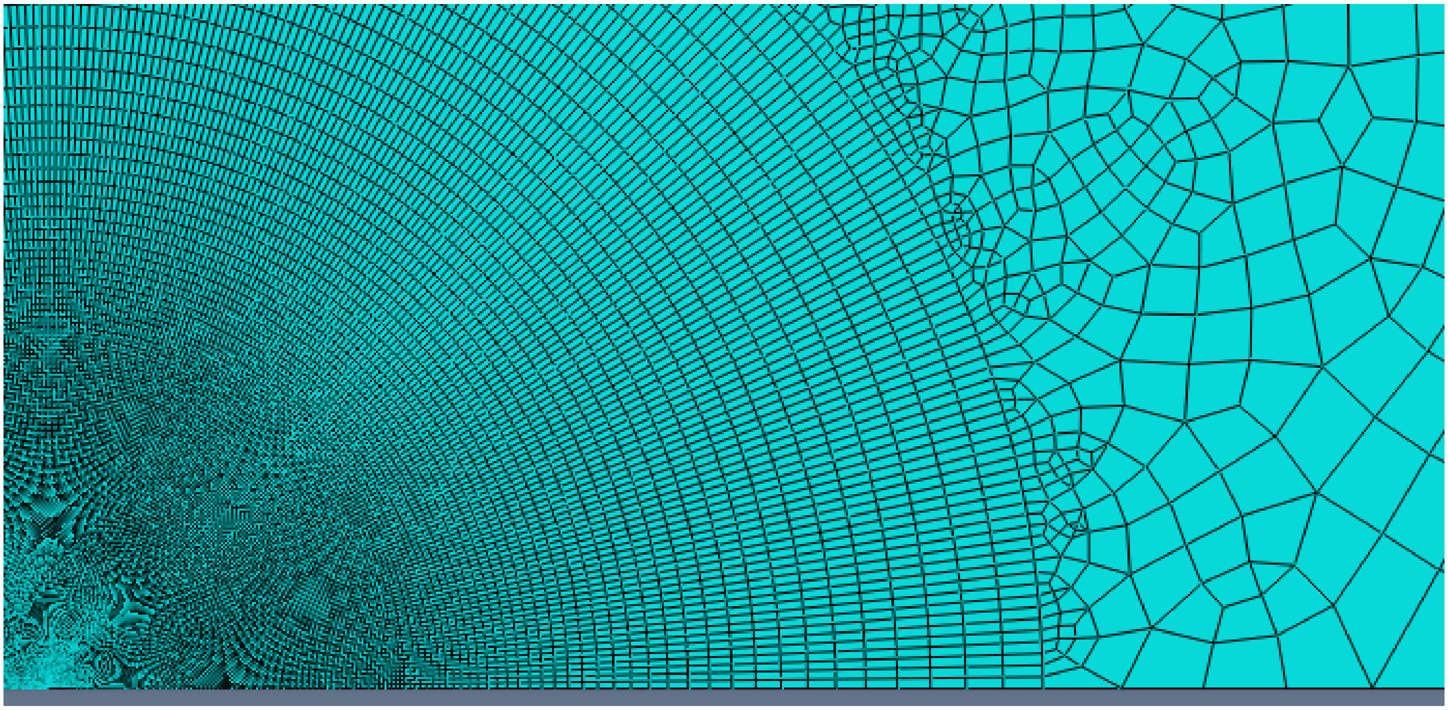


Figure 11: The three partitions of the mesh around the crack tip. From inside out: sweep, sweep, free. The deformation state around the crack tip is similar to that in figure 10. Here, the 1000x Refinement Model is shown.

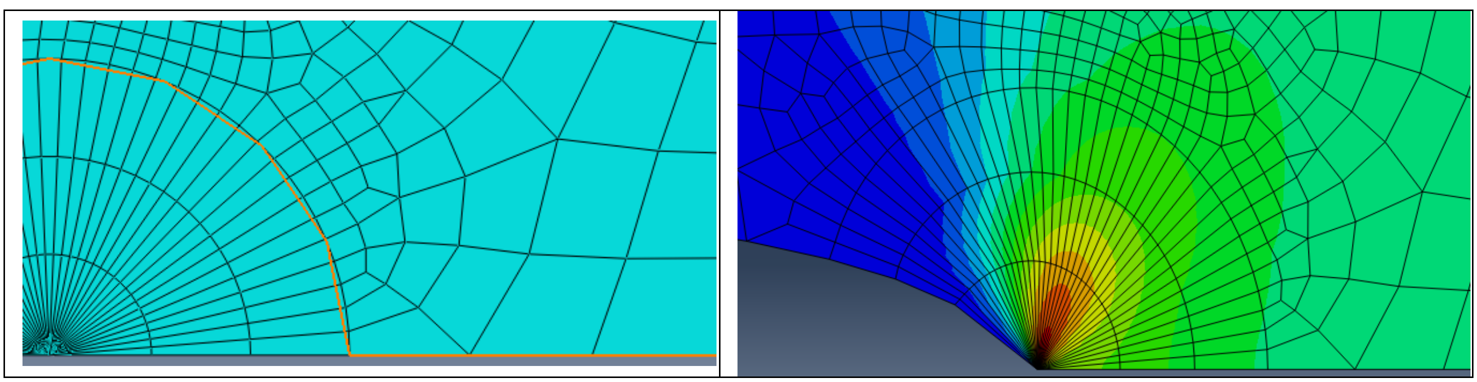


Figure 12: The coarse mesh (Baseline) for sweep-structured-free meshing. *Left:* undeformed, *right:* deformed.