**Github Clone URI:**`https://github.com/EnetBan/EN234_FEA.git`

# 1 Testing

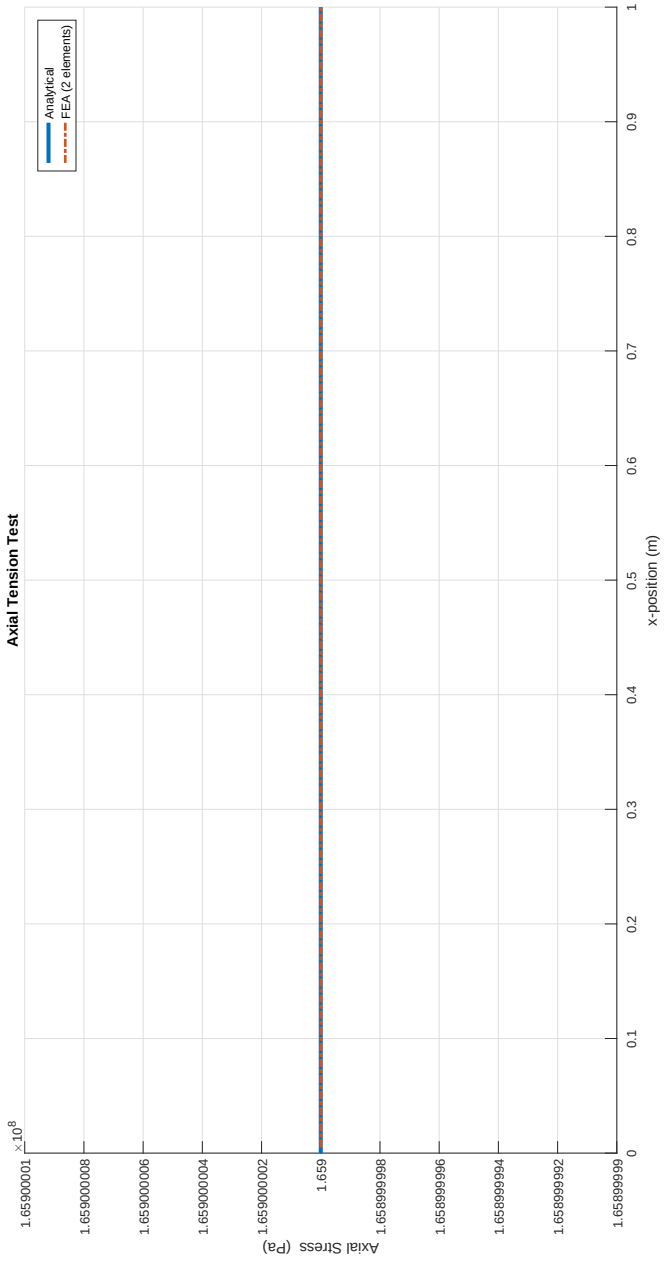
The Timoshenko beam element was tested under three conditions – axial tension, pure torsion, and as a cantilever beam under end loading. In each case, results were compared to analytical results for the same loading, material properties, and dimensions.

All tests were run on a 1m long beam with a 10cm diameter solid cross-section. Material used was Steel, with corresponding young's modulus of 210GPa and shear modulus of 77GPa

## 1.1 Axial Tension

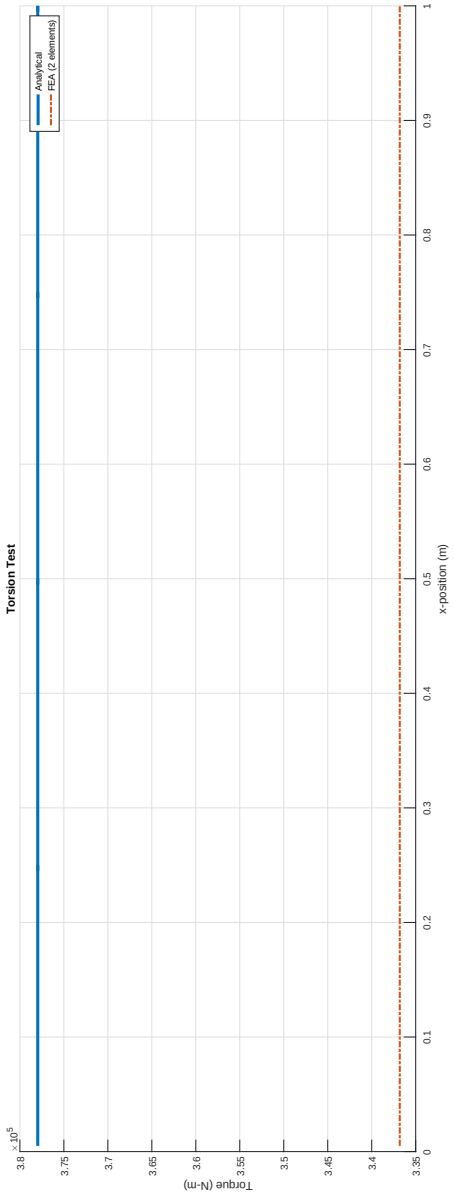
A displacement of .1m was applied to the end node of the beam, with the opposite end being fixed in all degrees of freedom. Since displacement was constrained, axial stress was compared to the analytical value.

The stress should be constant throughout the cross-section, and was. The average error was much less than 1%.


Axial Tension Test
Analytical
FEA (2 elements)
Axial Stress (Pa)
x-position (m)
×10^8
1.65900001
1.659000008
1.659000006
1.659000004
1.659000002
1.659
1.658999998
1.658999996
1.658999994
1.658999992
1.658999999
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

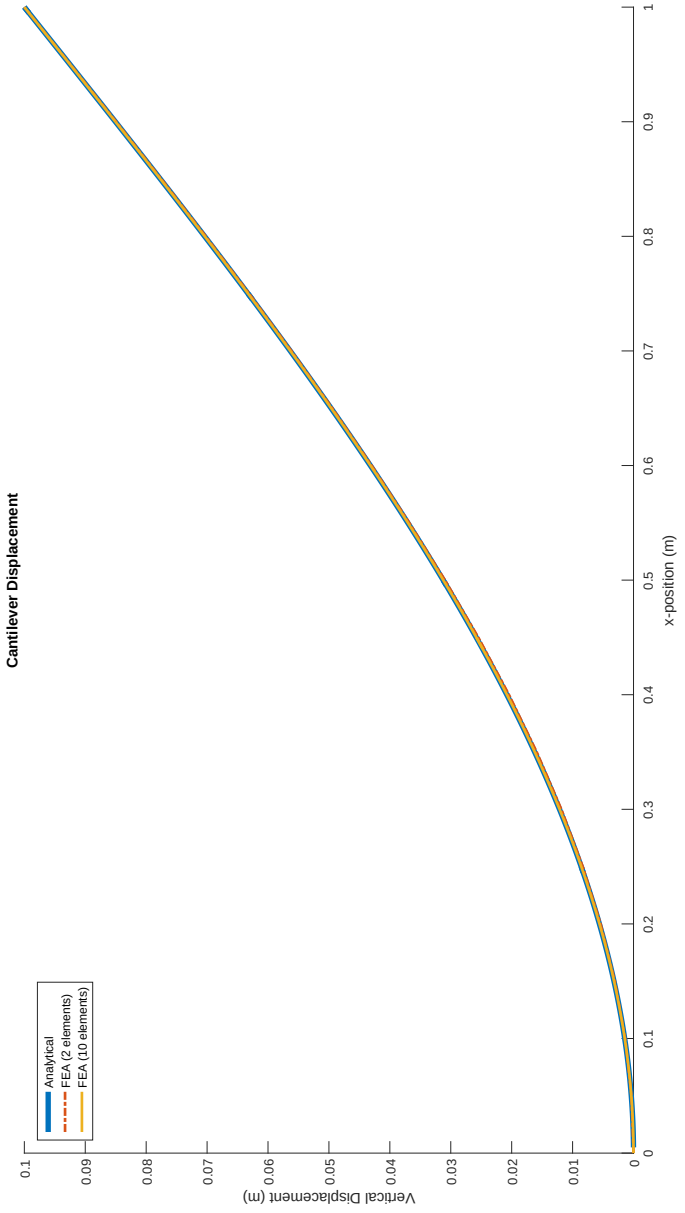
### 1.2 Torsion

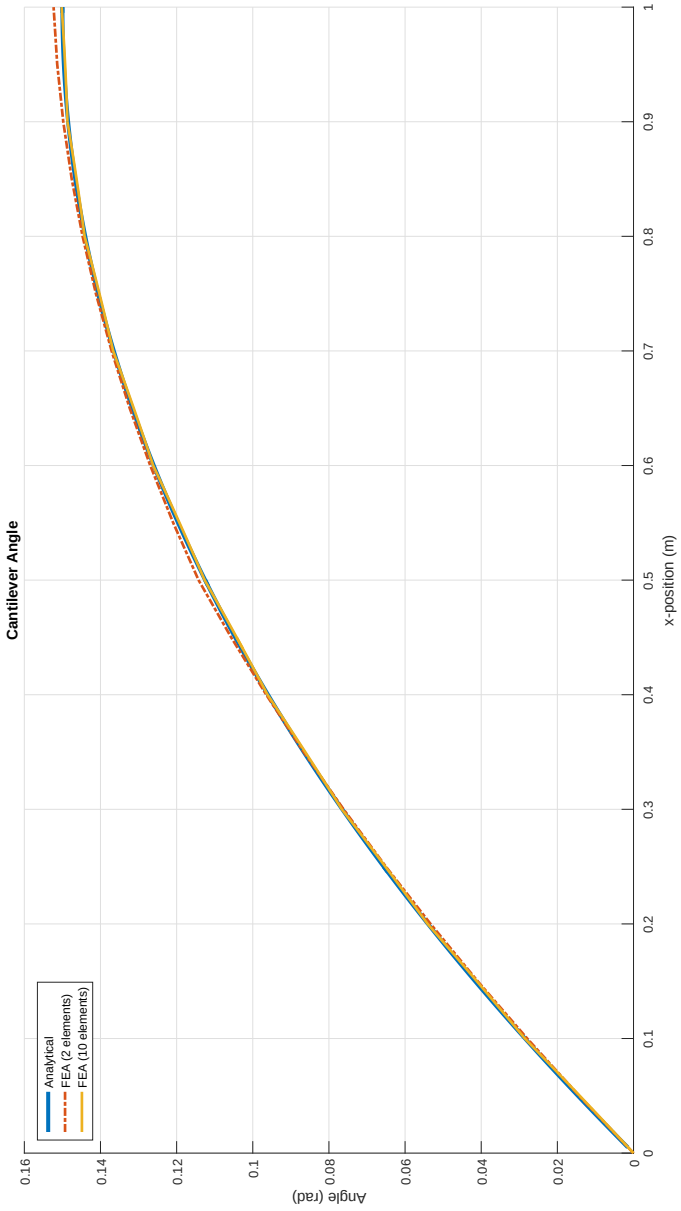
An axial rotation of .5 radians was applied at the end of the beam. As expected, the internal torque was constant throughout the length of the beam. Due to the shear coefficient however, they don't match up exactly with the analytical results – the average error was 11%.

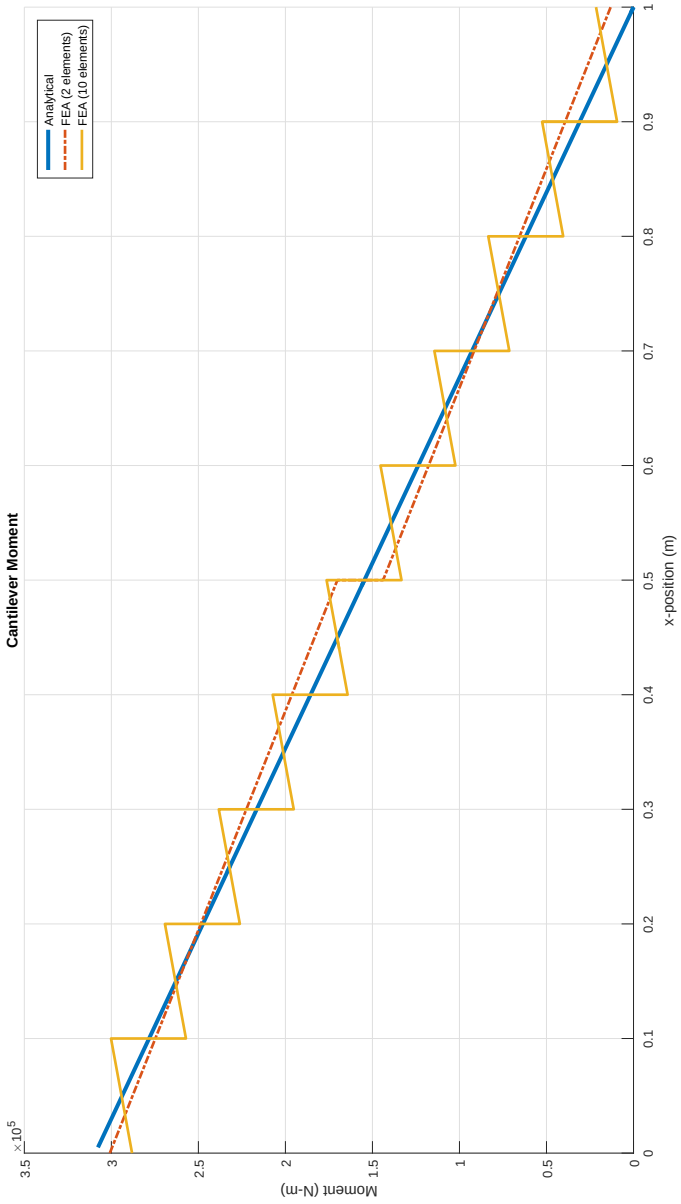

Torsion Test
Analytical
FEA (2 elements)
Torque (N-m)
x-position (m)
×10^5
3.8
3.75
3.7
3.65
3.6
3.55
3.5
3.45
3.4
3.35
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

### 1.3 Transverse Loading

The beam was completely fixed at one end and a displacement of .1m was applied vertically at the other end. Displacement, angle, and internal moment plots were compared with the theoretical values. Both the displacement and angle plots matched up with theory quite well, increasingly so with a greater number of elements. Torsion did not, however. It may be a bug in my code, or it may have something to do with the model itself – either the derivation of the shape functions was incorrect, or something else is going on. I'm not sure.


Cantilever Displacement
Analytical
FEA (2 elements)
FEA (10 elements)
Vertical Displacement (m)
x-position (m)
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


Cantilever Angle
Angle (rad)
x-position (m)
Analytical
FEA (2 elements)
FEA (10 elements)
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


Cantilever Moment
Analytical
FEA (2 elements)
FEA (10 elements)
Moment (N-m)
x-position (m)
×10^5
3.5
3
2.5
2
1.5
1
0.5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1