ENGN 2340 Final Project

User Elements for 3D Cohesive Zones Model

By Yu Liu

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1 Motivation

Engineering problems requiring crack growth analysis are receiving increased attention because of the focus on damage tolerance and durability of structures made of advanced materials. Cohesive zone modeling approach has emerged as a popular tool for investigating fracture processes in materials and structures. The motivation of present project is to implement the 3D cohesive zone model through an Abaqus UEL.

2 Theory of Cohesive Zone Model (CZM)

In a cohesive zone model, a narrow-band of vanishing thickness termed the cohesive zone is assumed to exist ahead of a crack tip to represent the fracture process zone. The upper and lower surfaces of the narrow-band are termed as the cohesive surfaces and are acted by the so-called cohesive traction which follows a cohesive constitutive law that relates the cohesive traction to the separation displacement of the cohesive surfaces. Consider a solid in which the weak interface occupies a plane S. Introduce an arbitrary coordinate system on S, and at each point define an orthonormal basis \( \{n, t_1, t_2\} \). Let \( u(x) \) denote the displacement field in the solid, which is continuous everywhere except on S. The displacement discontinuity across S follow as 
\[
\Delta n = \Delta u \cdot n, \quad \Delta t_1 = \Delta u \cdot t_1, \quad \Delta t_2 = \Delta u \cdot t_2.
\]
Let \( T_n, T_{t_1}, T_{t_2} \) denote the traction in three directions. The cohesive interface law relates \( (\Delta n, \Delta t_1, \Delta t_2) \) and \( (T_n, T_{t_1}, T_{t_2}) \). For an ideal elastic interface, the relationship is defined by an elastic potential function \( \phi \) such that
\[
T_n = \frac{\partial \phi}{\partial \Delta n}, \quad T_{t_1} = \frac{\partial \phi}{\partial \Delta t_1}, \quad T_{t_2} = \frac{\partial \phi}{\partial \Delta t_2}.
\] (1)

Various forms of \( \phi \) have been use in simulations. Here, I use one developed by Xu and Needleman.
\[
\phi(\Delta n, \Delta t_1, \Delta t_2) = \phi_n + \phi_n exp(-\frac{\Delta n}{\delta_n})\{[1 - r + \frac{\Delta n}{\delta_n}]\frac{1 - q}{r - 1}
\]
\[-[q + (\frac{r - q}{r - 1})\frac{\Delta n}{\delta_n} exp(-\frac{\Delta t_1^2 + \Delta t_2^2}{\delta_t^2})]\] (2)

where \( \phi_n, \Delta n, \delta_n, q \) and \( r \) are constitutive parameters. Under normal loading the interface has a work of separation \( \phi_n \), and the normal traction reaches a maximum
value $\sigma_{\text{max}}$ at an interface separation $\Delta_n = \delta_n$.

By adding an additional viscous dissipation to the interface model to avoid convergence problems, the traction-displacement relation for the interface is

\[
T_n = \sigma_m \exp\left(1 - \frac{\Delta_n}{\delta_n}\right) \left\{ \frac{\Delta_n}{\delta_n} \exp\left(-\frac{\Delta_{t1}^2 + \Delta_{t2}^2}{\delta_t^2}\right) \right\} + \frac{1 - q}{r - 1} \left[1 - \exp\left(-\frac{\Delta_{t1}^2 + \Delta_{t2}^2}{\delta_t^2}\right)\right] \exp\left(-\frac{\Delta_n}{\delta_n}\right) + \zeta_n \frac{d}{dt}\left(\frac{\Delta_n}{\delta_n}\right) \tag{3}
\]

\[
T_1 = 2\sigma_m \left(\frac{\delta_n}{\delta_t}\right) \frac{\Delta_{t1}}{\delta_t} \left\{ \frac{r - q}{r - 1} \Delta_n \right\} \exp\left(1 - \frac{\Delta_n}{\delta_n}\right) \exp\left(-\frac{\Delta_{t1}^2 + \Delta_{t2}^2}{\delta_t^2}\right) + \zeta_{t1} \frac{d}{dt}\left(\frac{\Delta_{t1}}{\delta_t}\right) \tag{4}
\]

\[
T_2 = 2\sigma_m \left(\frac{\delta_n}{\delta_t}\right) \frac{\Delta_{t2}}{\delta_t} \left\{ \frac{r - q}{r - 1} \Delta_n \right\} \exp\left(1 - \frac{\Delta_n}{\delta_n}\right) \exp\left(-\frac{\Delta_{t1}^2 + \Delta_{t2}^2}{\delta_t^2}\right) + \zeta_{t2} \frac{d}{dt}\left(\frac{\Delta_{t2}}{\delta_t}\right) \tag{5}
\]

where $\zeta_n$, $\zeta_{t1}$, and $\zeta_{t2}$ are parameters that govern energy dissipation under loading.

3 Tests and Results

3.1 Simple test with two elements

A simple example of a problem involving two element and one cohesive zone element has been tested. Two element with young’s modulus $E$, height $a=1$, length $L=1$, and width $w=1$ are connected by a weak interface in between. We can add relative small gap 0.001 between two element for the easy calculation of unit vector. This small gap won’t influence the calculation result. The deformation is loaded by displacing the top boundary to 0.1 while holding the bottom boundary. The interface is modelled using cohesive zone law with simple parameter values $q=1, r=0$. Stress field during deformation shows in Figure 1. Stress increase with small deformation and start decrease when cohesive element debonded. Relation between strain energy of whole model and time presents in figure 2.
Figure 1: Stress field of test with two elements
(a) 0 step (b) 5 step (c) 6 step (d) 8 step

Figure 2: Relation between Stress field and time
3.2 Influence of viscosity

To investigate the influence of viscosity on the separation of the interface, different viscosity-like parameters have been selected for tests. With this viscous term, there is no convergence problem for finite element analysis. Figure 3 and figure 4 show the stress and interface separation predicted by ABAQUS as a function of normalized displacement of top surface. Evidently, the equilibrium iterations have no difficulty converging.

Figure 3: $\sigma/\sigma_m$ as a function of $U/\delta_n$
4 Conclusions and Future Work

The present project successfully implement the 3D cohesive zone model through an Abaqus UEL and test it with a simple two elements 3D model. Meanwhile, investigate the influence of viscosity on the cohesive zone model. In future, more complicated 3D model can be set up and calculate fracture toughness with cohesive zone elements. Hope new version of ABAQUS can set up 3D cohesive zone model and modify input file more easier.

References