The Behavior of Shear Band Under Impact

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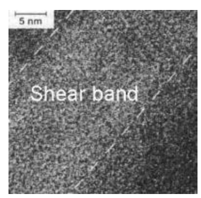
1. Introduction

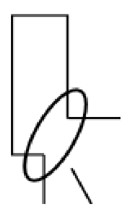
1) Define the shear-band

Under the effect of the impact loads such as high-speed collisions, the target material is subjected to the impact loads. In areas where the target material has been deformed in large plastic deformation, there exists some white ribbons which is called--Adiabatic Shear Band. This is because of under the extremely high strain rate, the heat produced by local large plastic distortion is too late to transmit.



2) The goal of the project

In the project, the first goal is to find the minimum velocity of impact at which shear band could goes through the entire gauge section.



If the velocity of impact is small, then the shear band will stop in someplace on the sample. (like the picture above, the shear band will stop in somewhere within the circle). Meanwhile, if the velocity of the impact is large enough, then the shear band can go through all the gauge section between the two sharp corners.

The second goal of the project is trying to find the temperature in the shear band. We already know that when object is subjected to the impact, the temperature in the shear band area will be higher than the other places, which called 'soften', and the high temperature is the possible reason that cause the shear band and the fracture. Hence, if we can find the relation between the temperature and the shear band area, then we can probably have some valued solutions.

2. **The Governing Equations** (field equations and constitutive law)

2.1. Kinematics Equations

The local deformation gradient:

$$F = \begin{bmatrix} \lambda(t) & 0 & 0 \\ -\kappa(t) & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The velocity gradient:

$$L = \dot{F}F^{-1} = \begin{bmatrix} \dot{\lambda}\lambda & 0 & 0 \\ -\dot{\kappa}/\lambda & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The Jaumann rate of Eulerian tensor:

$$\dot{\sigma} = \breve{\sigma} - \omega\sigma + \sigma\omega$$

2.2. Johnson-Cook Model

Plasticity Equations:

$$Mises\ stress: \bar{\sigma} = \sqrt{\frac{3}{2}\sigma'\sigma'}$$

$$\sigma' = \sigma - \frac{1}{3}tr(\sigma)I$$

In the Johnson-Cook model, the stress is given as:

$$\sigma_{eq} = \left(A + B\varepsilon_{eq}^{n}\right) * \left(1 + Cln\dot{\varepsilon}_{eq}^{*}\right) * \left(1 - T^{*m}\right)$$

Where A,B,n,C,m is the model parameters; σ_{eq} is the equivalent stress; ε_{eq} is the equivalent plastic strain; $\dot{\varepsilon}_{eq}^{*}$ is the dimensionless equivalent plastic strain rate, $\dot{\varepsilon}_{eq}^{*} = \exp[\frac{1}{C}(\frac{\sigma_e}{A+B\varepsilon_{eq}^{n}} - 1)]$; $\dot{\varepsilon_{eq}}$ is the strain rate in the experiment; $T^{*} = (T - T_r)/(T_m - T_r)$ is the dimensionless temperature; T_r is the reference temperature, which satisfies $T_r = 293K$; T_m is the melting point of the material; T is the temperature in the experiment. The three items on the right of the equation represent the effect of the equivalent plastic strain, strain rate and temperature on the flow stress.

Due to high strain rate, a large part plastic work becomes to thermal energy. Therefore, the temperature increases dT is like below:

$$dT = \frac{\alpha}{\rho C_p}\sigma d\varepsilon$$

α is a constant and C_p is the specific heat capacity.

Also, we need to define the actual plastic strain rate:

$$\varepsilon_{eq}^{\dot{a}} = \frac{\varepsilon_{eq}^{\dot{l}} * \dot{\varepsilon}_{eq}}{\varepsilon_{eq}^{\dot{l}} + \dot{\varepsilon}_{eq}}$$

Where $\varepsilon_{eq}^{\dot{l}}$ is the limiting strain rate.

Johnson-Cook Dynamic Failure Criterion:

$$\omega = \Sigma \frac{\Delta\varepsilon_{eq}}{\varepsilon_f}$$

Where $\Delta\varepsilon_{eq}$ is an equivalent plastic strain increment of integral cycle; ε_f is an effective fracture strain under current time step.

The fracture strain is given by the following formula:

$$\varepsilon_f = [D_1 + D_2 \exp(D_3\sigma^*)] * \left[1 + D_4 ln\dot{\varepsilon}_{eq}^*\right] * [1 + D_5 T^*]$$

Where ε_f is effective fracture strain; $D_i, i = 1, \ldots, 5$ is the input constant; $\sigma^* = \sigma_H/\sigma_{eq}$ is the stress three-axis degree; σ_H is the average stress.

3. Newton-Raphson Method for $\Delta\varepsilon_{eq}$

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij}^{e} + \dot{\varepsilon}_{ij}^{p}$$

Solve for

$$\Delta\varepsilon_{eq} = \frac{\Delta t \dot{\varepsilon}_{eq}^{l} \dot{\varepsilon}_0 \exp[\frac{1}{C}(\frac{\sigma_e}{A + B\varepsilon_{eq}^n} - 1)]}{\dot{\varepsilon}_{eq}^{l} + \dot{\varepsilon}_0 \exp[\frac{1}{C}(\frac{\sigma_e}{A + B\varepsilon_{eq}^n} - 1)]} = \frac{\Delta t \dot{\varepsilon}_{eq}^{l}}{1 + \frac{\dot{\varepsilon}_{eq}^{l}}{\dot{\varepsilon}_0} \exp[\frac{1}{C}(\frac{\sigma_e}{A + B\varepsilon_{eq}^n} - 1)]}$$

$$F = 1 - \frac{\dot{\varepsilon}_{eq}^{l} \Delta t}{\Delta\varepsilon_{eq}} + \frac{\dot{\varepsilon}_{eq}^{l}}{\dot{\varepsilon}_0} \exp[\frac{1}{C}(\frac{\sigma_e}{A + B\varepsilon_{eq}^n} - 1)]$$

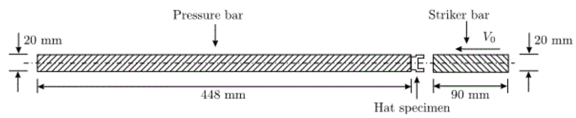
Then we can calculate $\frac{dF}{d\Delta\varepsilon_{eq}}$,

$$\frac{dF}{d\Delta\varepsilon_{eq}} = \frac{\dot{\varepsilon}_{eq}^{l} \Delta t}{{\Delta\varepsilon_{eq}}^2} + \frac{\dot{\varepsilon}_{eq}^{l}}{\dot{\varepsilon}_0} e^{\left[\frac{1}{C}\left(\frac{\sigma_e}{A + B\varepsilon_{eq}^n} - 1\right)\right]} [\frac{1}{C}(\frac{3E}{2(1+v)\left(A + B\left({\varepsilon_{eq}}^{(n)} + \varepsilon_{eq}\right)^{(n)}\right)} + \frac{1 - \frac{3E}{2(1+v)} {\sigma_e}^{(n+1)} Bn\left({\varepsilon_{eq}}^{(n)} + \varepsilon_{eq}\right)^{(n-1)}}{\left(A + B\left({\varepsilon_{eq}}^{(n)} + \varepsilon_{eq}\right)^{(n)}\right)^2})]$$

Function of F,

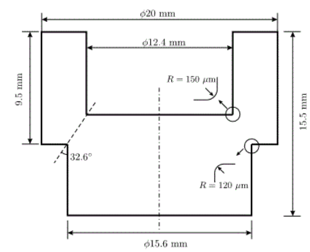
$$F = {\sigma_e}^{(n+1)} - [A + B{\varepsilon_{eq}}^{n(n+1)}][1 + Cln\dot{\varepsilon}_{eq}^*]$$

4. The Boundary/Initial value



Just like the image showed, the sample is fixed at one edge, and for the other edge, there will be an impact with initial velocity V_0.

And for simplicity, I take the sample as a 2D-sample with shape and size like below,



And the parameter of the sample is as below,

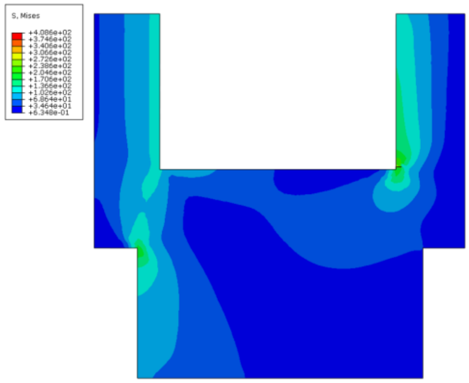
Table 1
Material constants for 91W–61Ni–3Co tungsten alloy

E (GPa)	v	ρ (kg/m^3)	$\dot{\varepsilon}_0$	C
370	0.3	17,650	1	0.03
c_p (J/kg K)	α	T_m (K)	T_0 (K)	q
150	0.9	1485	300	0.835
A (MPa)	B (MPa)	n	σ_y (MPa)	
1948	1875	0.95	1500	

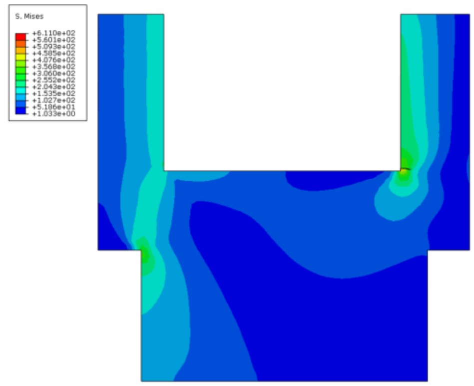
5. Results

The Velocity with the impact at sample:

(1). Velocity = **20**m/s

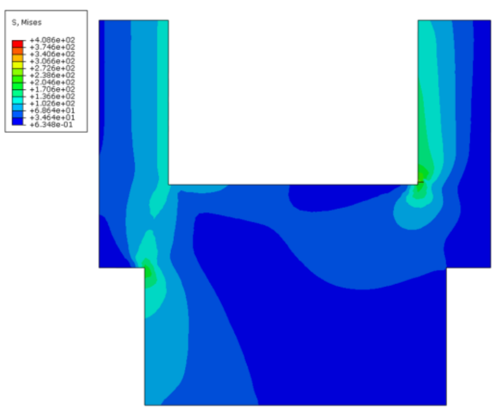


(Initial)

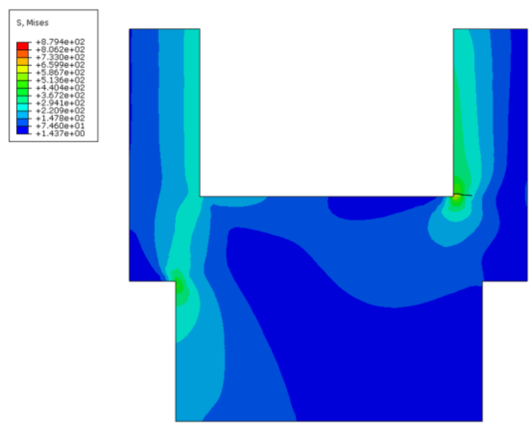


(Final)

(2). Velocity = **25**m/s

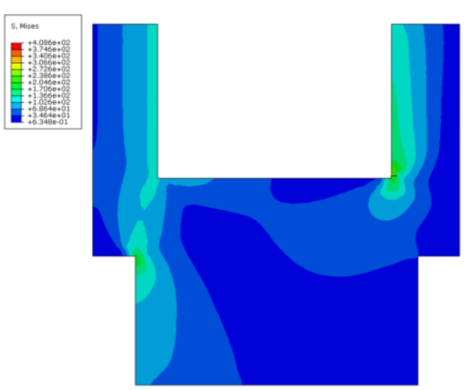


(Initial)

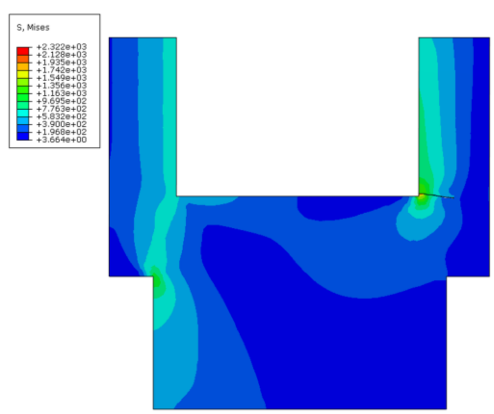


(Final)

(3). Velocity = **30**m/s

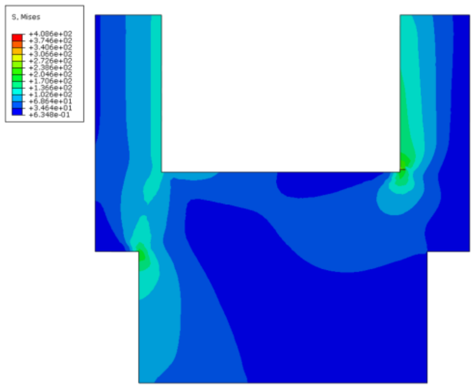


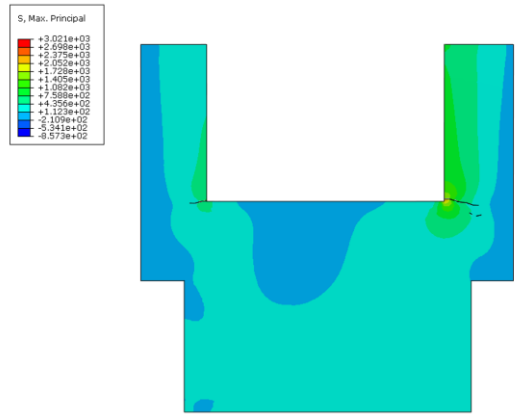
(Initial)



(Final)

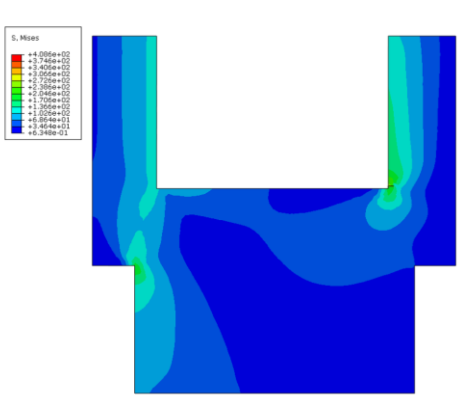
(4). Velocity = **35**m/s

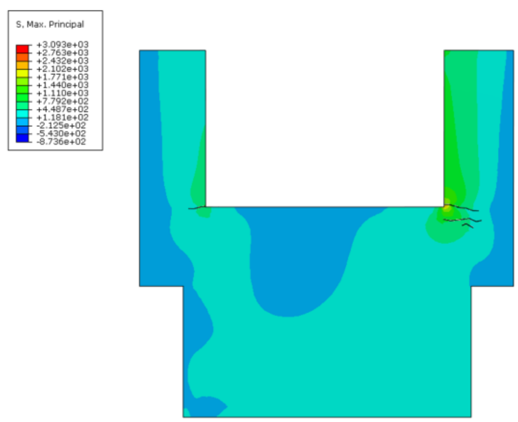




(Initial) (Final)

(5). Velocity = **40**m/s



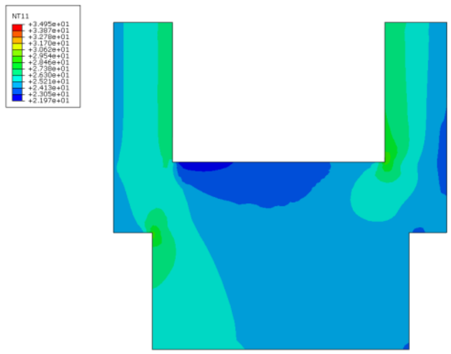


(Initial) (Final)

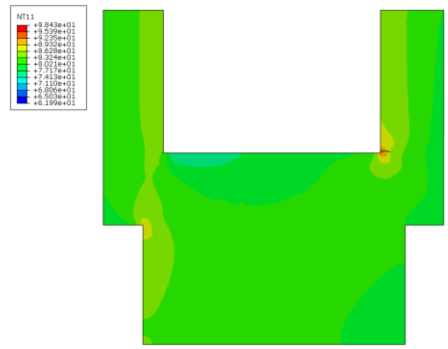
Hence, we can conclude that when the velocity increases, the shear band will increase, and it will happen in the sharp corner of the sample like the images showed above.

The Temperature:

(1). Velocity = **20**m/s

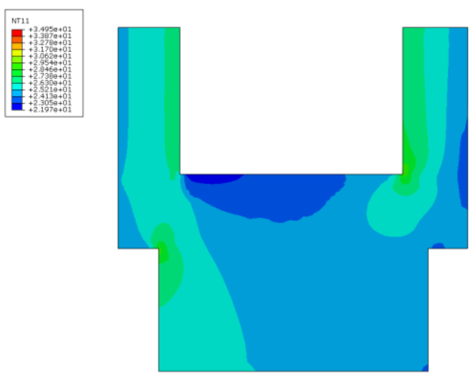


(Initial Temperature)

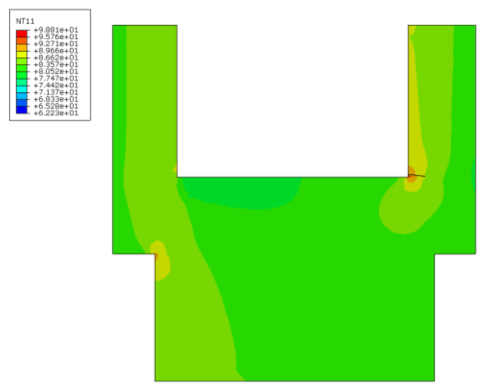


(Final Temperature)

(2). Velocity = **25**m/s

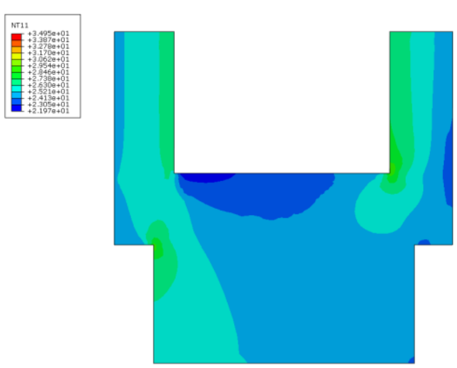


(Initial Temperature)

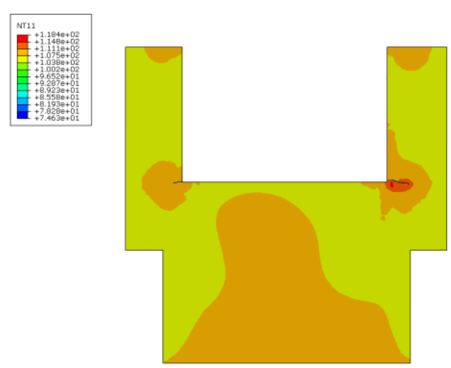


(Final Temperature)

(3). Velocity = **30**m/s

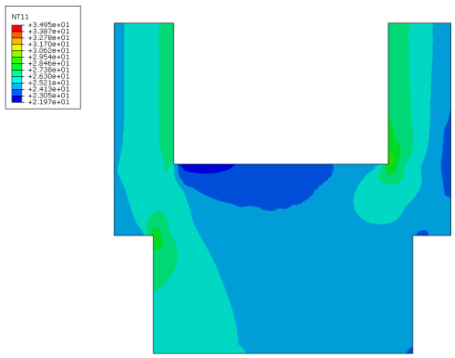


(Initial Temperature)

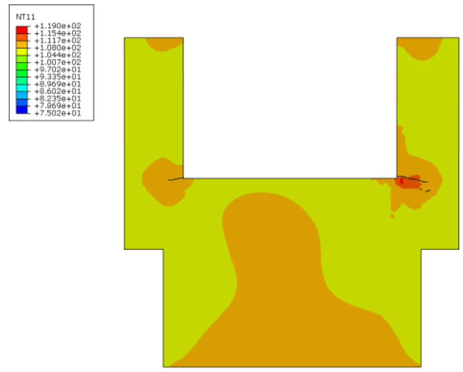


(Final Temperature)

(4). Velocity = **35**m/s

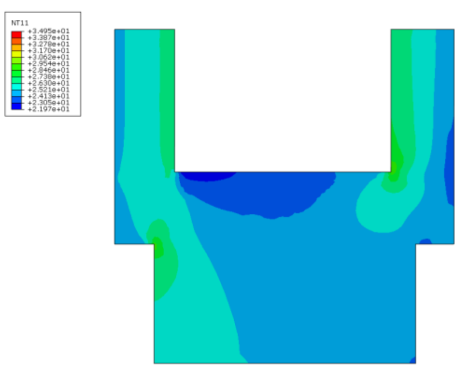


(Initial Temperature)

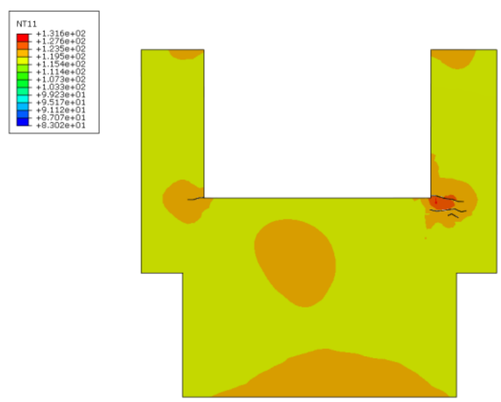


(Final Temperature)

(5). Velocity = **40**m/s



(Initial Temperature)



(Final Temperature)

We can find that at the shear band area, there exists higher temperature because of the high-strain rate. The high temperature makes the corner in the material softer then creates the crack.

6. References

'On the transition from adiabatic shear banding to fracture', January, 2006.