



Division of Engineering
Brown University

EN40: Dynamics and Vibrations

Final Examination
Friday May 15 2009

NAME: _____

General Instructions

- No collaboration of any kind is permitted on this examination.
- You may use 2 double sided pages of reference notes. No other material may be consulted
- Write all your solutions in the space provided. No sheets should be added to the exam.
- Make diagrams and sketches as clear as possible, and show all your derivations clearly. Incomplete solutions will receive only partial credit, even if the answer is correct.
- *If you find you are unable to complete part of a question, proceed to the next part.*

Please initial the statement below to show that you have read it

By affixing my name to this paper, I affirm that I have executed the examination in accordance with the Academic Honor Code of Brown University. **PLEASE WRITE YOUR NAME ABOVE ALSO!**

1-10: (30 PTS) _____

11: (15 PTS) _____

12: (15 PTS) _____

13: (20 PTS) _____

14: (20 PTS) _____

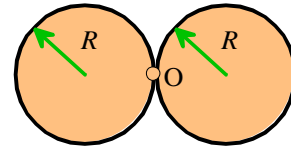
TOTAL (100 PTS) _____

FOR PROBLEMS 1-10 WRITE YOUR ANSWER IN THE SPACE PROVIDED.

ONLY THE ANSWER APPEARING IN THE SPACE PROVIDED WILL BE GRADED. ILLEGIBLE ANSWERS WILL NOT RECEIVE CREDIT.

1. What is the moment of inertia around axis O two cylinders of mass m , radius R shown?

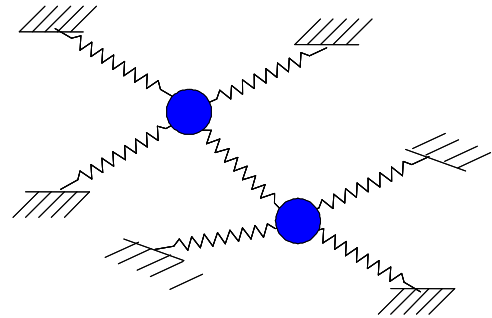
- a) $\frac{1}{2}mR^2$ b) mR^2 c) $2mR^2$ d) $3mR^2$ e) $4mR^2$



ANSWER _____ (3 POINTS)

2. How many natural frequencies of vibration are there for the system of masses and springs shown, assuming that the motion is confined to the plane of the page?

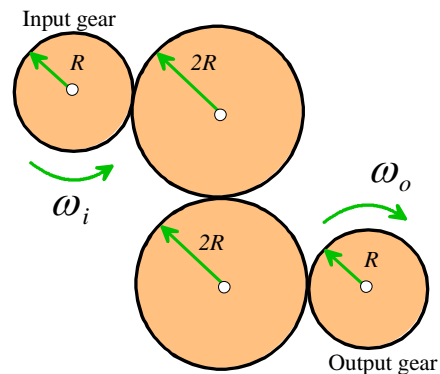
- a) 2
b) 3
c) 4
d) 6
e) Need more information



ANSWER _____ (3 POINTS)

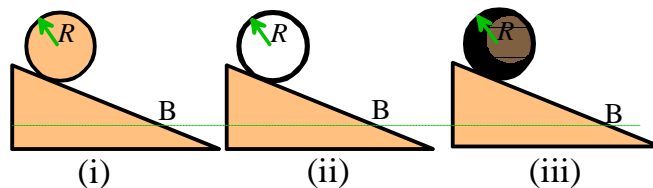
3. The figure shows a chain of 4 gears. The input gear rotates with angular speed ω_i . What is the angular velocity of the output gear?

- a) $\omega_o = \frac{1}{3}\omega_i$
b) $\omega_o = \frac{1}{2}\omega_i$
c) $\omega_o = \frac{2}{3}\omega_i$
d) $\omega_o = \omega_i$
e) $\omega_o = \frac{3}{2}\omega_i$
f) $\omega_o = 2\omega_i$



ANSWER _____ (3 POINTS)

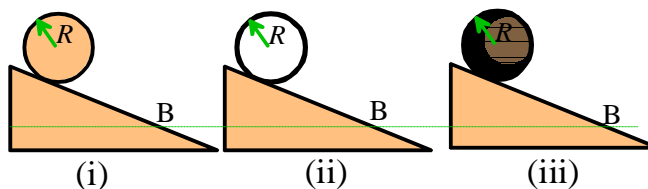
4. The figure shows (i) a solid disk; (ii) a ring and (iii) a sphere, with identical radii and mass, on a slope. Each object starts from rest from the same height, and rolls without slip. When the objects reach point B, which one is rotating more quickly?



- (a) The ring
- (b) The disk
- (c) The sphere
- (d) All three rotate, and rotate at the same rate
- (e) None of the objects rotate

ANSWER _____ (3 POINTS)

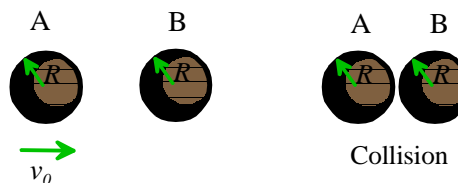
5. The figure shows (i) a solid disk; (ii) a ring and (iii) a sphere, with identical radii and mass on a *frictionless* slope. Each object starts from rest at the same height. When the objects reach point B, which one is rotating more quickly?



- (a) The ring
- (b) The disk
- (c) The sphere
- (d) All three rotate, and rotate at the same rate
- (e) None of the objects rotate

ANSWER _____ (3 POINTS)

6. The figure shows a collision between two identical spheres. The restitution coefficient for the collision $e=1$. Before the collision, A moves with speed v_0 and B is stationary. After the collision

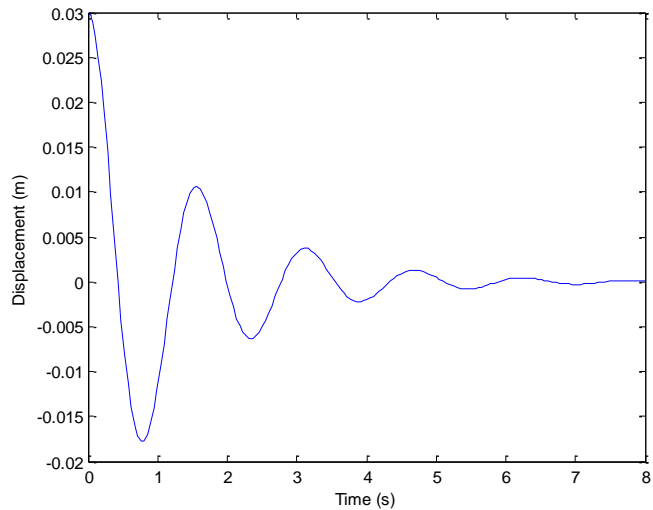


- (a) Sphere A is stationary and sphere B moves to the right
- (b) Sphere B is stationary and sphere A moves to the left
- (c) Both spheres move to the right at the same speed
- (d) Both spheres A and B move to the right with different speeds
- (e) Sphere B moves to the right and sphere A moves to the left.
- (f) Both spheres spontaneously combust

ANSWER _____ (3 POINTS)

7. A vibratory system of mass 15kg exhibits the time response shown in the figure. The effective spring constant k and viscous damping coefficient c for the system are approximately

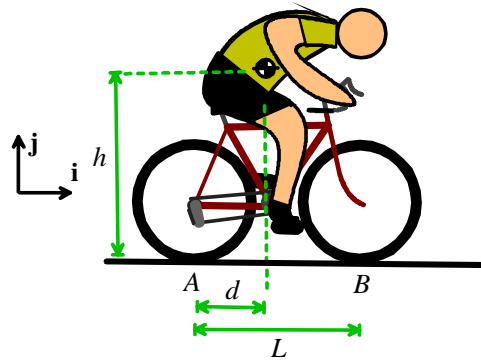
- (a) $k= 250 \text{ N/m}, c= 20 \text{ N-m/s}$
- (b) $k= 500 \text{ N/m}, c= 10 \text{ N-m/s}$
- (c) $k= 250 \text{ N/m}, c= 5 \text{ N-m/s}$
- (d) $k= 500 \text{ N/m}, c= 40 \text{ N-m/s}$
- (e) Need more information



ANSWER _____ (3 POINTS)

8 A bicycle rider traveling at speed V applies the brakes, and the bicycle skids over the ground to a full stop. If air resistance can be neglected, then the total work done by the forces exerted by the ground on the wheels during the skid is

- (a) greatest if the rider applies the front brakes only
- (b) greatest if the rider applies the rear brakes only
- (c) greatest if the rider applies both brakes together (causing both wheels to skid)
- (d) the same for each of (a), (b) and (c)
- (e) zero



ANSWER _____ (3 POINTS)

9 A system of particles or rigid bodies is said to be *conservative* if and only if

- (a) Its total energy is constant
- (b) No external forces act on the system
- (c) A potential energy can be defined for all internal forces acting in the system
- (d) A potential energy can be defined for all internal and external forces acting on the system
- (e) The system has a large factor of safety
- (f) The engineer who designed the system favors small government and fiscal responsibility.

ANSWER _____ (3 POINTS)

10. A tall mast with mass m and length L is supported by elastic cables with stiffness k and unstretched length L_0 as shown in the figure. The equation of motion for the system is

$$\frac{mL^2}{3} \frac{d^2\theta}{dt^2} + \sqrt{2}kLL_0 \sin \frac{\theta}{2} - \frac{mgL}{2} \sin \theta = 0$$

The natural (angular) frequency for *small amplitude oscillations* of θ is

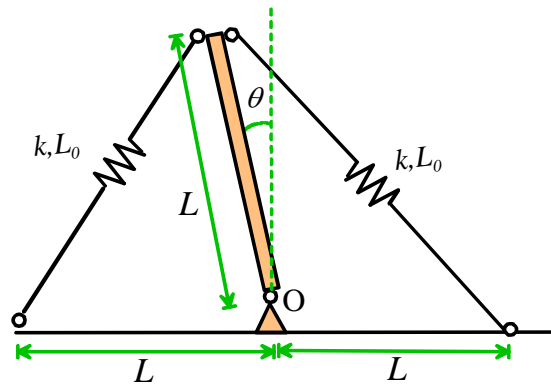
(a) $\omega_n = \sqrt{\frac{3}{2} \left(\sqrt{2} \frac{kL_0}{mL} - \frac{g}{L} \right)}$

(b) $\omega_n = \sqrt{\frac{3}{2} \left(\frac{g}{L} - \sqrt{2} \frac{kL_0}{mL} \right)}$

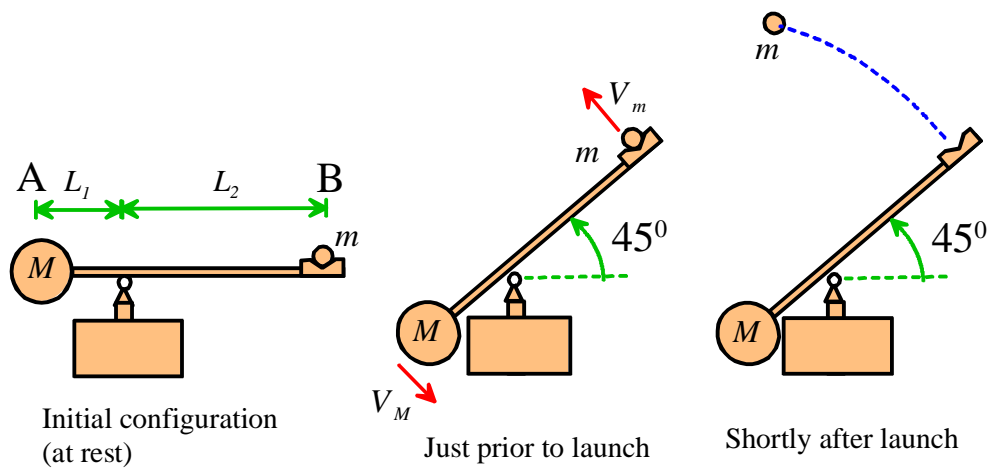
(c) $\omega_n = \sqrt{3 \left(\sqrt{2} \frac{kL_0}{mL} - \frac{g}{2L} \right)}$

(d) $\omega_n = \sqrt{3 \left(\frac{g}{2L} - \sqrt{2} \frac{kL_0}{mL} \right)}$

(e) None of the above



ANSWER _____ (3 POINTS)



11. (15 pts) A proposed design for a catapult that launches a projectile with mass m is shown in the figure. The mass of the arm AB can be neglected, and the masses m and M can be idealized as particles.

11.1 Take the potential energy of the initial configuration to be zero. Find an expression for the potential energy of the system at the instant of launch, in terms of m , M , L_1 , L_2 and the gravitational acceleration g .

(2 POINTS)

11.2 Write down the total kinetic energy of the system at the instant just prior to launch, in terms of m , M , V_m and V_M

(2 POINTS)

11.3 Write down equations that relate the angular velocity of the arm AB to the magnitude of the velocities V_m and V_M at the instant just prior to launch.

(2 POINTS)

11.4 Show that the projectile velocity at the instant of launch satisfies the equation $V_m^2 = \frac{\sqrt{2}g(ML_1 - mL_2)}{m + M \frac{L_1^2}{L_2}}$

(4 POINTS)

11.5 The solution to 11.4 can be written $V_m^2 = \frac{\sqrt{2}gL_1(1-\mu\rho)\rho^2}{1+\mu\rho^2}$ where $\mu = m/M$ $\rho = L_2/L_1$. Show that the launch velocity is maximized by the value of $\rho = L_2/L_1$ satisfying the equation

$$\mu^2\rho^3 + 3\mu\rho - 2 = 0$$

(5 POINTS)

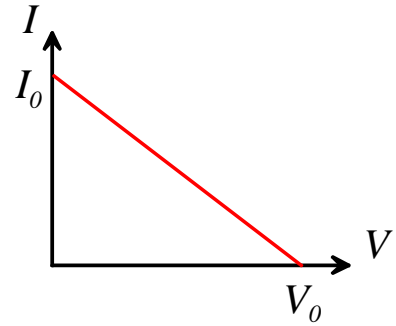
12. (15 pts) A hydrogen fuel cell has current-voltage (I - V) relationship that can be approximated by the relation

$$V = V_0 \left(1 - \frac{I}{I_0} \right)$$

where V_0 and I_0 are constants. It is used to drive an electric motor that can be characterized using the usual equations relating the current I , voltage V to motor torque T and angular speed ω

$$I = (V - \beta\omega) / R \quad T = \beta I - T_0$$

where β, T_0, R are constants (the additional constant τ_0 has been taken to be zero).



12.1 Show that the mechanical power output of the motor is given by $P = \left(\frac{\beta(V_0 - \beta\omega)}{R + V_0 / I_0} - T_0 \right) \omega$

(4 POINTS)

12.2 The cell and motor are used to power a vehicle of mass m . Show that, if friction and air resistance are neglected, and the vehicle travels along level ground, the power output of the motor is related to the acceleration a and speed v of the vehicle by $P = mav$

(4 POINTS)

12.3 Assume that the driving wheels of the vehicle have diameter D , and that the motor is connected to the axle through a transmission with gear ratio $\omega_a / \omega = r$, where ω_a is the angular speed of the axle. Find a formula for the acceleration of the vehicle in terms of $m, \omega, V_0, I_0, \beta, T_0, R, D$ and r

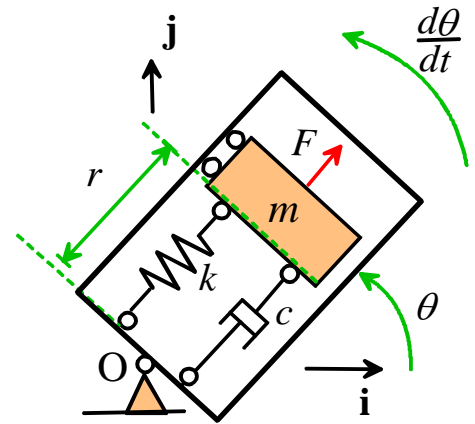
(4 POINTS)

12.4 Find a formula for the maximum possible acceleration of the vehicle in terms of $m, V_0, I_0, \beta, T_0, R, D$ and r , and determine the speed of the vehicle at the instant of maximum acceleration.

(3 POINTS)

13. (20 pts) The figure shows a simple idealization of a type of MEMS gyroscope, which is intended to be used to measure the angular velocity $d\theta/dt$ of the device.

It consists of a mass m that is supported by a spring with stiffness k and unstretched length L_0 , together with a damper with coefficient c . An electrostatic actuator subjects the mass m to a harmonic force $F = F_0 \sin \omega t$. While the device is rotating, the amplitude of vibration R_0 of the mass is measured. The driving frequency of the force is varied, and the frequency ω^* that induces the maximum vibration amplitude is determined.



The goal of this problem is to derive an equation that relates the resonant frequency ω^* to $d\theta/dt$.

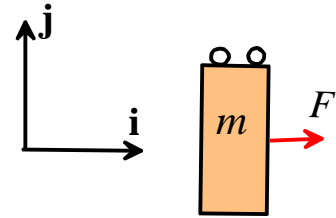
13.1 Assume the device rotates at **constant** angular velocity $d\theta/dt$ about the point O. Express the position vector of the mass m in terms of r and θ , in components in the basis shown. The dimensions of the mass may be neglected.

(2 POINTS)

13.2 Hence, find an expression for the acceleration of the mass m , in terms of $d\theta/dt$, r and time derivatives of r . There is no need to use Newton's laws to derive this result.

(2 POINTS)

13.3 The figure on the right shows the mass at an instant when $\theta = 0$. Draw the forces acting on the mass m on the figure. **NEGLECT GRAVITY**



(2 POINTS)

13.4 Express the forces exerted on the mass by the spring and damper in terms of r , dr/dt , c , k , and L_0 .

(2 POINTS)

13.5 By considering the system at the instant when $\theta = 0$, show that r is determined by the equation of motion

$$m \frac{d^2 r}{dt^2} + c \frac{dr}{dt} + \left[k - m \left(\frac{d\theta}{dt} \right)^2 \right] r = kL_0 + F_0 \sin \omega t$$

(4 POINTS)

13.6 Find an equation that relates the resonant frequency ω^* to $d\theta/dt$. Assume that the system is lightly damped.

(2 POINTS)

13.7 To avoid failure, the maximum length of the spring must not exceed $1.1L_0$. Show that the system must be designed so that

$$\frac{kL_0}{k - m(d\theta/dt)^2} + \frac{F_0\sqrt{m}}{c\sqrt{k - m(d\theta/dt)^2}} < 1.1L_0$$

for all expected values of $d\theta/dt$. Assume small damping.

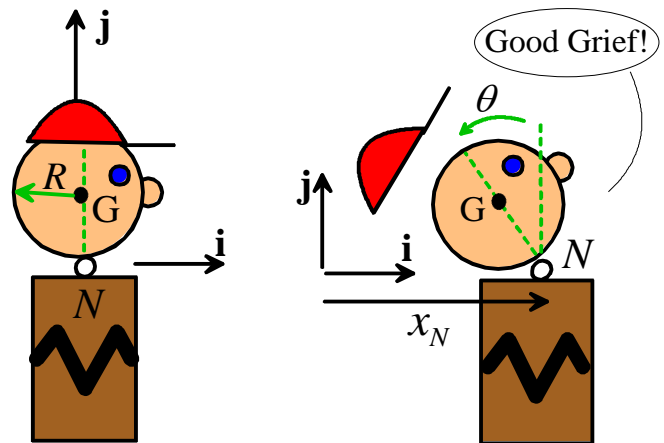
(6 POINTS)

14. (20 pts) The figure shows a passenger in a vehicle with no headrests. The car is initially at rest, and is hit from behind by another vehicle, giving the car and the passenger's torso a forward acceleration of $\mathbf{a} = a_N \mathbf{i}$.

The acceleration bends the passenger's neck through an angle θ as shown in the figure. The goal of this problem is to derive an equation of motion for θ .

Assume that:

- The head is a sphere with mass m , radius R and mass moment of inertia $2mR^2/5$.
- The neck exerts horizontal and vertical reaction forces on the head, as well as a *resisting moment against rotation* $\mathbf{M} = -k\theta \mathbf{k}$ where θ is the angle shown.



14.1 Write down an expression for the position vector of the center of mass G of the passenger's head in terms of the position of the neck x_N and the angle θ , expressing your answer as components in the $\{\mathbf{i}, \mathbf{j}, \mathbf{k}\}$ basis shown in the figure.

(2 POINTS)

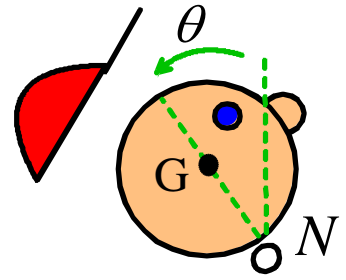
14.2 Hence, find an expression for the velocity of point G in terms of $R, \theta, dx_N/dt$, and $d\theta/dt$. Newton's laws need not be used to answer this part.

(2 POINTS)

14.3 Hence, find an expression for the acceleration \mathbf{a}_G of the center of mass G in terms of $a_N, R, d\theta/dt$ and $d^2\theta/dt^2$. Newton's laws need not be used to answer this part.

(3 POINTS)

14.4 Draw the forces and moments acting on the passenger's head, using the figure provided.



(3 POINTS)

14.5 Hence, write down the equations for linear and rotational motion of the head. **Note that N is not a fixed point.**

(3 POINTS)

14.6 Show that the equation of motion for θ is

$$\frac{7mR^2}{5} \frac{d^2\theta}{dt^2} + k\theta - mgR \sin \theta - mRa_N \cos \theta = 0$$

(4 POINTS)

14.7 Arrange the equation of motion into a form that could be solved using MATLAB.

(3 POINTS)