Division of Engineering
Brown University

EN40: Dynamics and Vibrations

Final Examination
Friday May 15 2009

NAME: ____

General Instructions

- No collaboration of any kind is permitted on this examination.
- You may use 2 double sided pages of reference notes. No other material may be consulted
- Write all your solutions in the space provided. No sheets should be added to the exam.
- Make diagrams and sketches as clear as possible, and show all your derivations clearly. Incomplete solutions will receive only partial credit, even if the answer is correct.
- *If you find you are unable to complete part of a question, proceed to the next part.*

Please initial the statement below to show that you have read it

`By affixing my name to this paper, I affirm that I have executed the examination in accordance with the Academic Honor Code of Brown University. **PLEASE WRITE YOUR NAME ABOVE ALSO!**

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1-10: (30 PTS) _____________

11: (15 PTS) _____________

12: (15 PTS) _____________

13: (20 PTS) _____________

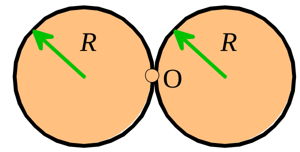
14: (20 PTS) _____________

TOTAL (100 PTS) ___________

FOR PROBLEMS 1-10 WRITE YOUR ANSWER IN THE SPACE PROVIDED.

ONLY THE ANSWER APPEARING IN THE SPACE PROVIDED WILL BE GRADED.
ILLEGIBLE ANSWERS WILL NOT RECEIVE CREDIT.

1. What is the moment of inertia around axis O two cylinders of mass m, radius R shown?

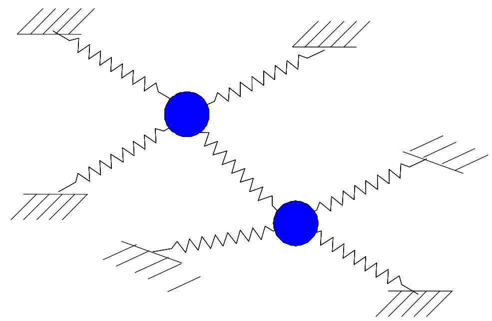


a) $\frac{1}{2}mR^2$ b) mR^2 c) $2mR^2$ d) $3mR^2$ e) $4mR^2$

Using parallel axis theorem, moment of inertia of one disk is $mR^2/2 + mR^2$. The total is therefore $3mR^2$

ANSWER______D________ **(3 POINTS)**

2. How many natural frequencies of vibration are there for the system of masses and springs shown, assuming that the motion is confined to the plane of the page?

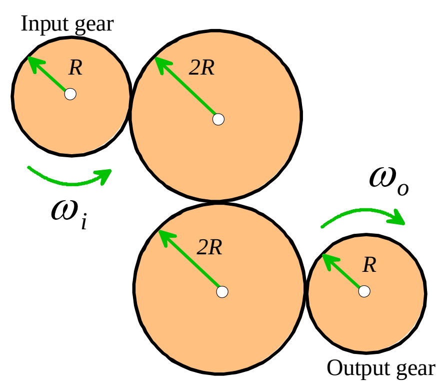


a) 2
b) 3
c) 4
d) 6
e) Need more information

Each mass can move vertically and horizontally. There are therefore 4 degrees of freedom, so we expect 4 vibration modes.

ANSWER_____C_________**(3 POINTS)**

3. The figure shows a chain of 4 gears. The input gear rotates with angular speed ω_i. What is the angular velocity of the output gear?

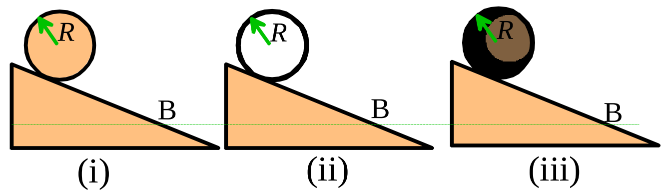


a) $\omega_o = \frac{1}{3}\omega_i$
b) $\omega_o = \frac{1}{2}\omega_i$
c) $\omega_o = \frac{2}{3}\omega_i$
d) $\omega_o = \omega_i$
e) $\omega_o = \frac{3}{2}\omega_i$
f) $\omega_o = 2\omega_i$

$$\omega_0 = \frac{R}{2R}\frac{R}{R}\frac{2R}{R}\omega_i = \omega_i$$

ANSWER________D______**(3 POINTS)**

4. The figure shows (i) a solid disk; (ii) a ring and (iii) a sphere, with identical radii and mass, on a slope. Each object starts from rest from the same height, and rolls without slip. When the objects reach point B, which one is rotating more quickly?

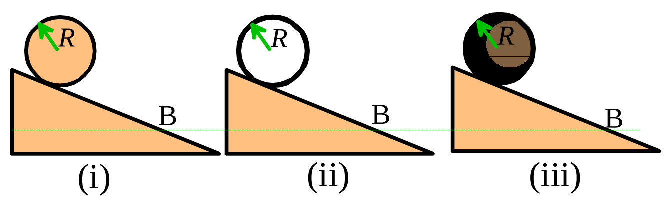


(a) The ring
(b) The disk
(c) The sphere
(d) All three rotate, and rotate at the same rate
(e) None of the objects rotate

Conservative system, so T+V=constant. Therefore $-mgh + mv^2/2 + I\omega^2/2 = 0$. Recall that $v = R\omega$ for a wheel rolling without slip, so $\omega^2 = 2mgh/\left(mR^2 + I\right)$. The object with the smallest I will rotate fastest – i.e. the sphere.

ANSWER_______c_______**(3 POINTS)**

5. The figure shows (i) a solid disk; (ii) a ring and (iii) a sphere, with identical radii and mass on a *frictionless* slope. Each object starts from rest at the same height. When the objects reach point B, which one is rotating more quickly?

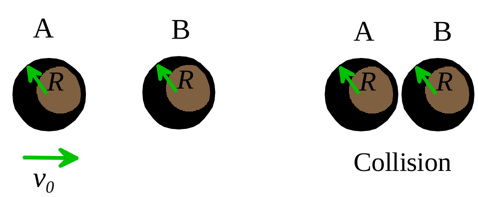


(a) The ring
(b) The disk
(c) The sphere
(d) All three rotate, and rotate at the same rate
(e) None of the objects rotate

No friction implies no moment about COM, so no angular acceleration, and no angular velocity

ANSWER_______E_______**(3 POINTS)**

6. The figure shows a collision between two identical spheres. The restitution coefficient for the collision *e=1*. Before the collision, A moves with speed v_0 and B is stationary. After the collision



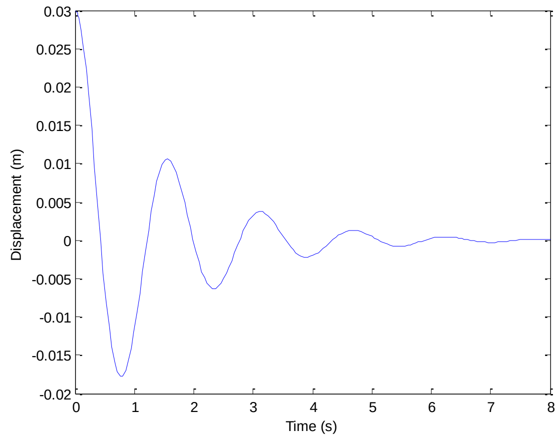
(a) Sphere A is stationary and sphere B moves to the right
(b) Sphere B is stationary and sphere A moves to the left
(c) Both spheres move to the right at the same speed
(d) Both spheres A and B move to the right with different speeds
(e) Sphere B moves to the right and sphere A moves to the left.
(f) Both spheres spontaneously combust

Momentum conservation gives $mv_0 = m(v_A + v_B)$; restitution coefficient gives $v_0 = v_B - v_A$. Solving these two gives $v_A = 0, v_B = v_0$

ANSWER________A______**(3 POINTS)**

7. A vibratory system of mass 15kg exhibits the time response shown in the figure. The effective spring constant k and viscous damping coefficient c for the system are approximately

(a) k= 250 N/m, c= 20 N-m/s
(b) k= 500 N/m, c= 10 N-m/s
(c) k= 250 N/m, c= 5 N-m/s
(d) k= 500 N/m, c= 40 N-m/s
(e) Need more information

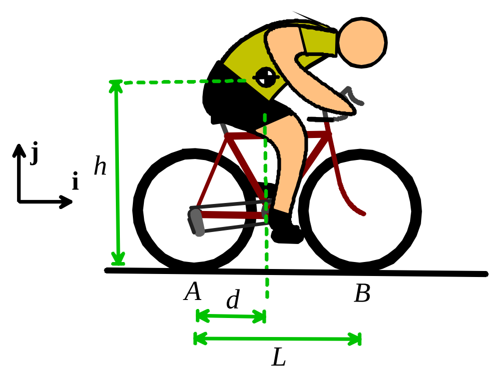


The log decrement is $\delta = \log(0.03/0.01) \Rightarrow \zeta = \delta / \sqrt{\delta^2 + 4\pi^2} = 0.17$ Period is about 1.5sec, so $\omega_n \approx 2\pi / (1.5\sqrt{1-\zeta^2}) = \sqrt{k/m} \Rightarrow k \approx 250 N/m$. Finally $c = 2m\zeta\omega_n \approx 20$

ANSWER_____A_________**(3 POINTS)**

8 A bicycle rider traveling at speed V applies the brakes, and the bicycle skids over the ground to a full stop. If air resistance can be neglected, then the total work done by the forces exerted by the ground on the wheels during the skid is

(a) greatest if the rider applies the front brakes only
(b) greatest if the rider applies the rear brakes only
(c) greatest if the rider applies both brakes together (causing both wheels to skid)
(d) the same for each of (a), (b) and (c)
(e) zero



The bicycle can be idealized as an isolated particle (a conservative system) subjected to external forces from gravity and the contact forces at the ground. The total work done by external forces must be equal to the change in kinetic energy. It doesn't matter which forces do work – the total work done is the same. Gravity does no work, so the total work done by the reaction forces is the same for (a), (b) and (c).

ANSWER_____D_________**(3 POINTS)**

9 A system of particles or rigid bodies is said to be *conservative* if and only if

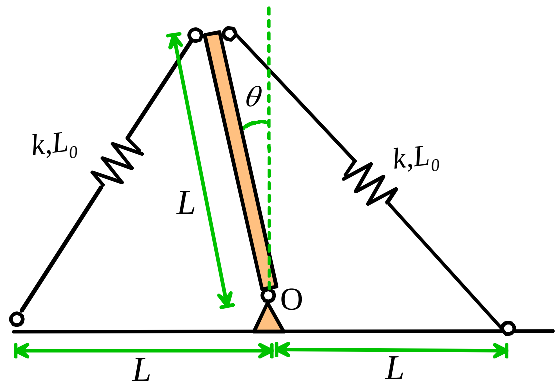
(a) Its total energy is constant
(b) No external forces act on the system
(c) A potential energy can be defined for all internal forces acting in the system
(d) A potential energy can be defined for all internal and external forces acting on the system
(e) The system has a large factor of safety
(f) The engineer who designed the system favors small government and fiscal responsibility.

ANSWER_______C_______**(3 POINTS)**

10. A tall mast with mass m and length L is supported by elastic cables with stiffness k and unstretched length L_0 as shown in the figure. The equation of motion for the system is

$$\frac{mL^2}{3}\frac{d^2\theta}{dt^2}+\sqrt{2}kLL_0\sin\frac{\theta}{2}-\frac{mgL}{2}\sin\theta=0$$

The natural (angular) frequency for *small amplitude oscillations of* θ is



(a) $\omega_n=\sqrt{\frac{3}{2}\left(\sqrt{2}\frac{kL_0}{mL}-\frac{g}{L}\right)}$

(b) $\omega_n=\sqrt{\frac{3}{2}\left(\frac{g}{L}-\sqrt{2}\frac{kL_0}{mL}\right)}$

(c) $\omega_n=\sqrt{3\left(\sqrt{2}\frac{kL_0}{mL}-\frac{g}{2L}\right)}$

(d) $\omega_n=\sqrt{3\left(\frac{g}{2L}-\sqrt{2}\frac{kL_0}{mL}\right)}$

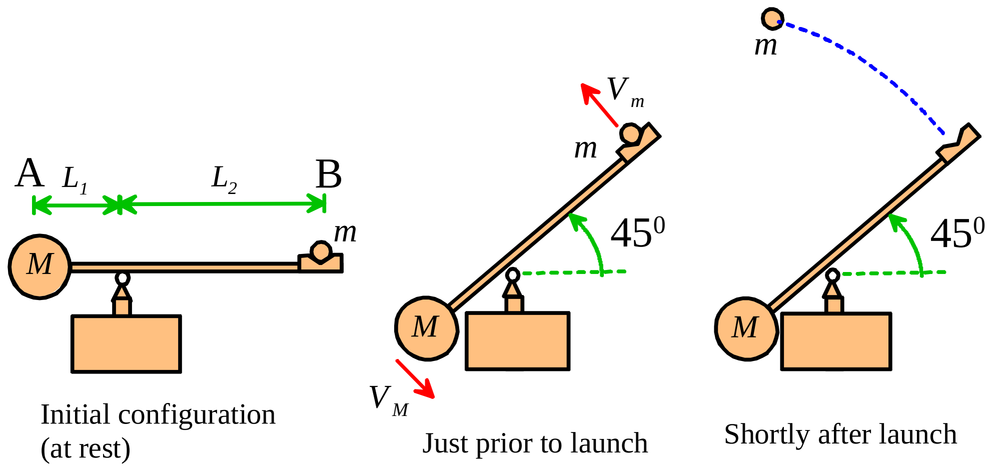
(e) None of the above

For small θ $\sin\theta/2\approx\theta/2$ and $\sin\theta\approx\theta$, so the equation of motion is

$$\frac{mL^2}{3}\frac{d^2\theta}{dt^2}+\frac{1}{2}\left(\sqrt{2}kLL_0-mgL\right)\theta=0\Rightarrow\frac{d^2\theta}{dt^2}+\frac{3}{2}\left(\sqrt{2}\frac{kL_0}{L}-\frac{g}{L}\right)\theta=0$$

The coefficient of the last term gives the square of the natural frequency.

ANSWER______A________**(3 POINTS)**



11. (15 pts) A proposed design for a catapult that launches a projectile with mass *m* is shown in the figure. The mass of the arm AB can be neglected, and the masses *m* and *M* can be idealized as particles.

11.1 Take the potential energy of the initial configuration to be zero. Find an expression for the potential energy of the system at the instant of launch, in terms of *m*, *M* , L_1, L_2 and the gravitational acceleration *g*.

$$V = \sum m_i g h_i = (mL_2 - ML_1)\frac{g}{\sqrt{2}}$$

(2 POINTS)

11.2 Write down the total kinetic energy of the system at the instant just prior to launch, in terms of *m*, *M*, V_m and V_M

$$T = \left(mV_m^2 + MV_M^2\right)/2$$

(2 POINTS)

11.3 Write down equations that relate the angular velocity of the arm AB to the magnitude of the velocities V_m and V_M at the instant just prior to launch.

$$\text{Using } \mathbf{v} = \boldsymbol{\omega} \times \mathbf{r} \text{ we get } V_m = L_2\omega \quad V_M = L_1\omega$$

(2 POINTS)

11.4 Show that the projectile velocity at the instant of launch satisfies the equation $V_m^2 = \dfrac{\sqrt{2}g(ML_1 - mL_2)}{m + M\dfrac{L_1^2}{L_2^2}}$

Energy is conserved so $T+V$=0 (initial PE and KE are zero). Thus

$$T = \left(mV_m^2 + MV_M^2\right)/2 + \left(mL_2 - ML_1\right)\frac{g}{\sqrt{2}} = 0$$

From 11.3 we see that $V_M = \dfrac{L_1^2}{L_2^2}V_m$ and so

$$T = V_m^2\left(m + M\frac{L_1^2}{L_2^2}\right) = -\left(mL_2 - ML_1\right)\frac{2g}{\sqrt{2}}$$

Which clearly yields the solution given.

(4 POINTS)

11.5 The solution to 11.4 can be written $V_m^2 = \dfrac{\sqrt{2}gL_1(1-\mu\rho)\rho^2}{1+\mu\rho^2}$ where $\mu = m/M \quad \rho = L_2/L_1$. Show that the launch velocity is maximized by the value of $\rho = L_2/L_1$ satisfying the equation

$$\mu^2\rho^3 + 3\mu\rho - 2 = 0$$

We need to maximize V_m with respect to ρ. Differentiate:

$$\frac{d}{d\rho}\frac{(1-\mu\rho)\rho^2}{1+\mu\rho^2} = \frac{2\rho - 3\mu\rho^2}{(1+\mu\rho^2)} - \frac{(1-\mu\rho)\rho^2}{(1+\mu\rho^2)^2}2\mu\rho = 0$$
$$\Rightarrow \rho\left(2 - 3\mu\rho\right)(1+\mu\rho^2) - (1-\mu\rho)\rho^2 2\mu\rho = 0$$
$$\Rightarrow \rho\left(2 - 3\mu\rho - \mu^2\rho^3\right) = 0$$

$\rho = 0$ is not a useful solution, so the max of interest follows from the equation given.

(5 POINTS)

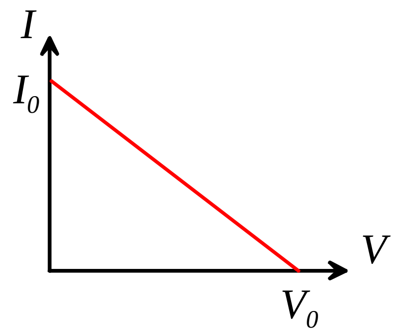
12. (15 pts) A hydrogen fuel cell has current-voltage (*I*-*V*) relationship that can be approximated by the relation

$$V = V_0\left(1 - \frac{I}{I_0}\right)$$

where V_0 and I_0 are constants. It is used to drive an electric motor that can be characterized using the usual equations relating the current *I*, voltage *V* to motor torque *T* and angular speed ω

$$I = \left(V - \beta\omega\right)/R \qquad T = \beta I - T_0$$

where β, T_0, R are constants (the additional constant τ_0 has been taken to be zero).



12.1 Show that the mechanical power output of the motor is given by $P = \left(\frac{\beta\left(V_0 - \beta\omega\right)}{R + V_0 / I_0} - T_0\right)\omega$

The power is $T\omega = (\beta I - T_0)\omega$ so we need to find the current somehow. We can do this by eliminating voltage from the fuel cell characteristic and the motor equation

$$I = \left(V - \beta\omega\right)/R \Rightarrow IR = V_0\left(1 - \frac{I}{I_0}\right) - \beta\omega \Rightarrow I = \frac{V_0 - \beta\omega}{R + \frac{V_0}{I_0}}$$

Substituting this into $T\omega = (\beta I - T_0)\omega$ yields the answer given.

(4 POINTS)

12.2 The cell and motor are used to power a vehicle of mass *m*. Show that, if friction and air resistance are neglected, and the vehicle travels along level ground, the power output of the motor is related to the acceleration *a* and speed *v* of the vehicle by $P = mav$

We can regard the car and transmission (but without the motor) as a system of rigid bodies. If friction in the transmission is neglected, this is a conservative system. It is subjected to external forces (and moments) from (a) gravity; (b) reaction forces at the wheels; and (c) the motor. The total rate of work done by the external forces must equal the rate of change of PE plus KE in the system. Gravity does no work on the system since the car is traveling along a level road; the reaction forces act on a stationary point and so do no work; so only the motor does work on the system. The PE is zero. The KE can be approximated as $T = mv^2/2$ (neglecting additional KE due to rotation of the transmission and wheels). Therefore

$$P = \frac{dT}{dt} = \frac{d}{dt}\left(\frac{mv^2}{2}\right) = mav$$

There are other, more complicated ways to show this using **F**=m**a**, but then you need to relate the force **F** to the motor power, which is very difficult to do correctly.

(4 POINTS)

12.3 Assume that the driving wheels of the vehicle have diameter D, and that the motor is connected to the axle through a transmission with gear ratio $\omega_a / \omega = r$, where ω_a is the angular speed of the axle. Find a formula for the acceleration of the vehicle in terms of m, ω, V_0, I_0, β, T_0, R, D and r

The car speed is $v = \omega_a D/2 = r\omega D/2$, and so using P from 12.1 and the P-a relation from 12.2 gives

$$P = \left(\frac{\beta (V_0 - \beta\omega)}{R + V_0 / I_0} - T_0 \right) \omega = mav = mar\omega D/2$$

$$\Rightarrow a = \frac{2}{Drm} \left(\frac{\beta (V_0 - \beta\omega)}{R + V_0 / I_0} - T_0 \right)$$

(4 POINTS)

12.4 Find a formula for the maximum possible acceleration of the vehicle in terms of of m, V_0, I_0, β, T_0, R, D and r, and determine the speed of the vehicle at the instant of maximum acceleration.

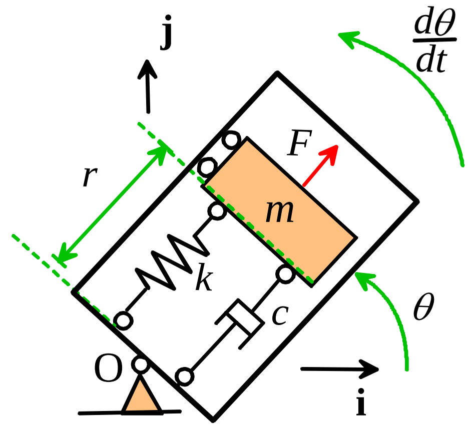
We need to find the motor speed that maximizes the acceleration from 12.3. The acceleration is clearly maximized for $\omega = 0$; so the car speed is zero, and

$$a_{\max} = \frac{2}{Drm} \left(\frac{\beta V_0}{R + V_0 / I_0} - T_0 \right)$$

(3 POINTS)

13. (20 pts) The figure shows a simple idealization of a type of MEMS gyroscope, which is intended to be used to measure the angular velocity $d\theta / dt$ of the device.

It consists of a mass *m* that is supported by a spring with stiffness *k* and unstretched length L_0, together with a damper with coefficient *c*. An electrostatic actuator subjects the mass *m* to a harmonic force $F = F_0 \sin \omega t$. While the device is rotating, the amplitude of vibration R_0 of the mass is measured. The driving frequency of the force is varied, and the frequency ω^* that induces the maximum vibration amplitude is determined.



The goal of this problem is to derive an equation that relates the resonant frequency ω^* to $d\theta / dt$.

13.1 Assume the device rotates at **constant** angular velocity $d\theta / dt$ about the point O. Express the position vector of the mass *m* in terms of *r* and θ , in components in the basis shown. The dimensions of the mass may be neglected.

$$\mathbf{r} = r\left(\cos\theta\mathbf{i} + \sin\theta\mathbf{j}\right)$$

(2 POINTS)

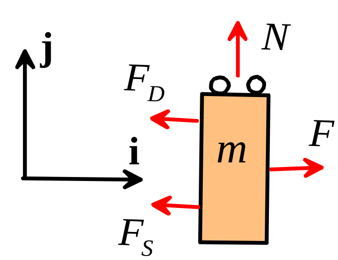
13.2 Hence, find an expression for the acceleration of the mass *m*, in terms of $d\theta / dt$, *r* and time derivatives of *r*. There is no need to use Newton's laws to derive this result.

$$\mathbf{v} = \frac{dr}{dt}\left(\cos\theta\mathbf{i} + \sin\theta\mathbf{j}\right) + r\frac{d\theta}{dt}\left(-\sin\theta\mathbf{i} + \cos\theta\mathbf{j}\right)$$

$$\mathbf{a} = \frac{d^2 r}{dt^2}\left(\cos\theta\mathbf{i} + \sin\theta\mathbf{j}\right) + 2\frac{dr}{dt}\frac{d\theta}{dt}\left(-\sin\theta\mathbf{i} + \cos\theta\mathbf{j}\right) - r\left(\frac{d\theta}{dt}\right)^2\left(\cos\theta\mathbf{i} + \sin\theta\mathbf{j}\right)$$

(2 POINTS)

13.3 The figure on the right shows the mass at an instant when $\theta = 0$. Draw the forces acting on the mass *m* on the figure. **NEGLECT GRAVITY**



(2 POINTS)

13.4 Express the forces exerted on the mass by the spring and damper in terms of *r*, *dr/dt*, *c*, *k*, and L_0.

$$\mathbf{F}_D = -c\frac{dr}{dt}\mathbf{i} \qquad \mathbf{F}_S = -k(r - L_0)\mathbf{i}$$

(2 POINTS)

13.5 By considering the system at the instant when $\theta = 0$, show that *r* is determined by the equation of motion

$$m\frac{d^2r}{dt^2} + c\frac{dr}{dt} + \left[k - m\left(\frac{d\theta}{dt}\right)^2\right]r = kL_0 + F_0 \sin\omega t$$

$\mathbf{F} = m\mathbf{a}$ with acceleration from 13.2, and forces from 13.3 and 13.4, with $\theta = 0$ yields

$$-k(r - L_0)\mathbf{i} - c\frac{dr}{dt}\mathbf{i} + F_0 \sin\omega t\mathbf{i} + N\mathbf{j} = m\left(\frac{d^2r}{dt^2}\mathbf{i} + 2\frac{dr}{dt}\frac{d\theta}{dt}\mathbf{j} - r\left(\frac{d\theta}{dt}\right)^2\mathbf{i}\right)$$

Rearranging the **i** component of this equation yields the required answer.

(4 POINTS)

13.6 Find an equation that relates the resonant frequency ω^* to $d\theta/dt$. Assume that the system is lightly damped.

Arrange the EOM in standard form

$$\frac{d^2r}{dt^2} + \frac{c}{m}\frac{dr}{dt} + \frac{1}{m}\left[k - m\left(\frac{d\theta}{dt}\right)^2\right]r = \frac{kL_0}{m} + \frac{F_0}{m}\sin\omega t$$

The max amplitude occurs when the system is forced at the natural frequency (since damping is small), so

$$\omega^* = \sqrt{\frac{1}{m}\left[k - m\left(\frac{d\theta}{dt}\right)^2\right]}$$

(2 POINTS)

13.7 To avoid failure, the maximum length of the spring must not exceed $1.1L_0$ Show that the system must be designed so that

$$\frac{kL_0}{k-m(d\theta/dt)^2}+\frac{F_0\sqrt{m}}{c\sqrt{k-m(d\theta/dt)^2}}<1.1L_0$$

for all expected values of $d\theta/dt$. Assume small damping.

We need to solve

$$\frac{d^2r}{dt^2}+\frac{c}{m}\frac{dr}{dt}+\frac{1}{m}\left[k-m\left(\frac{d\theta}{dt}\right)^2\right]r=\frac{kL_0}{m}+\frac{F_0}{m}\sin\omega t$$

For *r(t)*. Note that this is not quite a standard vibration EOM, because the RHS contains both a constant term, as well as a harmonic term.

This indicates that solution for r consists of (a) a time-independent part r_0 and (b) a vibrating part $\Delta r(t)$. We can calculate the time independent part from the EOM by setting all time varying terms to zero

$$\frac{1}{m}\left[k-m\left(\frac{d\theta}{dt}\right)^2\right]r_0=\frac{kL_0}{m}\Rightarrow r_0=\frac{kL_0}{k-m\left(\frac{d\theta}{dt}\right)^2}$$

Setting $r=r_0+\Delta r$ in the EOM yields an equation for $\Delta r(t)$

$$\frac{d^2\Delta r}{dt^2}+\frac{c}{m}\frac{d\Delta r}{dt}+\frac{1}{m}\left[k-m\left(\frac{d\theta}{dt}\right)^2\right]\Delta r=\frac{F_0}{m}\sin\omega t$$

This is now a standard vibration EOM and we can write down the solution as

$$\Delta r(t)=\frac{F_0}{m\omega_n^2}\frac{\sin(\omega t+\phi)}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2+\left(2\zeta\frac{\omega}{\omega_n}\right)^2}}$$

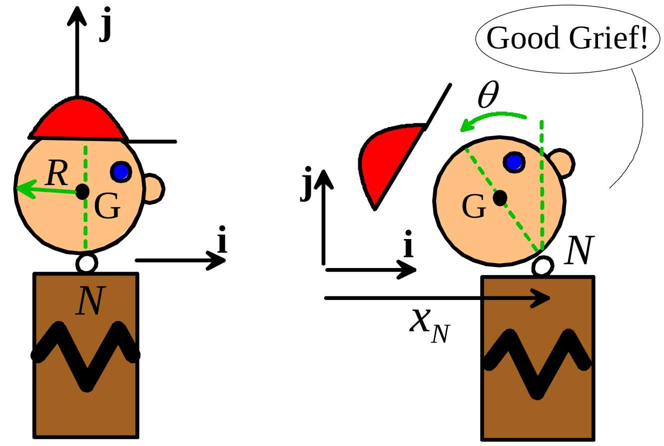
Where $\zeta=c/2m\omega_n$. The amplitude of vibration is maximized if $\omega=\omega_n$, and the max value of Δr occurs when $\sin(\omega t+\phi)=1$, so

$$\Delta r_{\max}=\frac{F_0}{m\omega_n^2}\frac{1}{2\zeta}=\frac{F_0}{c\omega_n}=\frac{F_0\sqrt{m}}{c\sqrt{k-m(d\theta/dt)^2}}$$

Finally, the condition that $r_0+\Delta r_{\max}<1.1L_0$ gives the required answer.

(6 POINTS)

14. (20 pts) The figure shows a passenger in a vehicle with no headrests. The car is initially at rest, and is hit from behind by another vehicle, giving the car and the passenger's torso a forward acceleration of $\mathbf{a} = a_N \mathbf{i}$.



The acceleration bends the passenger's neck through an angle θ as shown in the figure. The goal of this problem is to derive an equation of motion for θ.

Assume that:

- The head is a sphere with mass m, radius R and mass moment of inertia $2mR^2/5$.
- The neck exerts horizontal and vertical reaction forces on the head, as well as a *resisting moment against rotation* $\mathbf{M} = -k\theta\mathbf{k}$ where θ is the angle shown.

14.1 Write down an expression for the position vector of the center of mass G of the passenger's head in terms of the position of the neck x_N and the angle θ, expressing your answer as components in the $\{\mathbf{i,j,k}\}$ basis shown in the figure.

$$\mathbf{r} = x_N \mathbf{i} + R(-\sin\theta \mathbf{i} + \cos\theta \mathbf{j})$$

(2 POINTS)

14.2 Hence, find an expression for the velocity of point G in terms of $R, \theta, dx_N/dt,$ and $d\theta/dt$. Newton's laws need not be used to answer this part.

$$\mathbf{v} = \frac{dx_N}{dt}\mathbf{i} + R\frac{d\theta}{dt}(-\cos\theta \mathbf{i} - \sin\theta \mathbf{j})$$

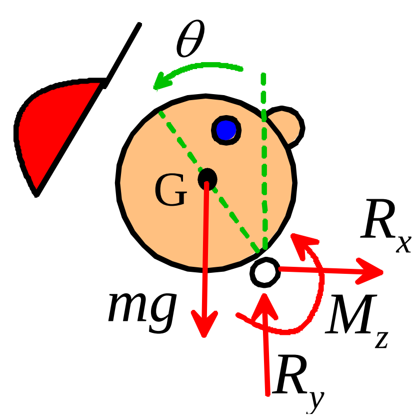
(2 POINTS)

14.3 Hence, find an expression for the acceleration $\mathbf{a}_G$ of the center of mass G in terms of a_N, R, $d\theta/dt$ and $d^2\theta/dt^2$ Newton's laws need not be used to answer this part.

$$\mathbf{a} = a_N \mathbf{i} - R\frac{d^2\theta}{dt^2}(\cos\theta \mathbf{i} + \sin\theta \mathbf{j}) - R\left(\frac{d\theta}{dt}\right)^2 (-\sin\theta \mathbf{i} + \cos\theta \mathbf{j})$$

(3 POINTS)

14.4 Draw the forces and moments acting on the passenger's head, using the figure provided.



(The moment exerted by the neck is $M_z = -k\theta$)

(3 POINTS)

14.5 Hence, write down the equations for linear and rotational motion of the head. **Note that N is not a fixed point.**

$$\mathbf{F} = m\mathbf{a} \Rightarrow R_x\mathbf{i} + (R_y - mg)\mathbf{j} = ma_N\mathbf{i} - mR\left(\frac{d\theta}{dt}\right)^2 (-\sin\theta\mathbf{i} + \cos\theta\mathbf{j})$$

$$\mathbf{M}_G = I\boldsymbol{\alpha} \Rightarrow \left(R_x R\cos\theta + R_y R\sin\theta - k\theta\right)\mathbf{k} = \frac{2}{5}mR^2\frac{d^2\theta}{dt^2}\mathbf{k}$$

NOTE THAT YOU MUST APPPLY THE ANGULAR ACCELERATION EQUATION ABOUT THE COM!!!

(3 POINTS)

14.6 Show that the equation of motion for θ is

$$\frac{7mR^2}{5}\frac{d^2\theta}{dt^2} + k\theta - mgR\sin\theta - mRa_N\cos\theta = 0$$

The preceding problem gives three equations that can be used to eliminate the reaction forces

$$R_x = ma_N - mR\frac{d^2\theta}{dt^2}\cos\theta + mR\left(\frac{d\theta}{dt}\right)^2\sin\theta$$

$$R_y = mg - mR\frac{d^2\theta}{dt^2}\sin\theta - mR\left(\frac{d\theta}{dt}\right)^2\cos\theta$$

$$\Rightarrow \left(R_x R\cos\theta + R_y R\sin\theta - k\theta\right) =$$

$$\left(ma_N - mR\frac{d^2\theta}{dt^2}\cos\theta + mR\left(\frac{d\theta}{dt}\right)^2\sin\theta\right)R\cos\theta + \left(mg - mR\frac{d^2\theta}{dt^2}\sin\theta - mR\left(\frac{d\theta}{dt}\right)^2\cos\theta\right)R\sin\theta - k\theta$$

$$= \frac{2}{5}mR^2\frac{d^2\theta}{dt^2}$$

Recalling that $\sin^2\theta + \cos^2\theta = 1$, the result reduces to the expression given.

(4 POINTS)

14.7 Arrange the equation of motion into a form that could be solved using MATLAB.

We need to turn the 2nd order differential equation into two first-order equations, by introducing $\omega = d\theta / dt$ as an additional variable. This gives

$$\frac{d}{dt}\begin{bmatrix}\theta \\ \omega\end{bmatrix} = \begin{bmatrix}\omega \\ -\frac{5k}{7mR^2}\theta + \frac{5g}{7R}\sin\theta + \frac{5a_N}{7R}\cos\theta\end{bmatrix}$$

(3 POINTS)