



Division of Engineering
Brown University

EN40: Dynamics and Vibrations

Midterm Examination
Thursday March 5 2009

NAME: Isaac Newton

General Instructions

- No collaboration of any kind is permitted on this examination.
- You may bring 3 double sided pages of reference notes. No other material may be consulted
- Write all your solutions in the space provided. No sheets should be added to the exam.
- Make diagrams and sketches as clear as possible, and show all your derivations clearly. Incomplete solutions will receive only partial credit, even if the answer is correct.
- If you find you are unable to complete part of a question, proceed to the next part.

Please initial the statement below to show that you have read it

By affixing my name to this paper, I affirm that I have executed the examination in accordance with the Academic Honor Code of Brown University. **PLEASE WRITE YOUR NAME ABOVE ALSO!**

IN

1 (3 points)

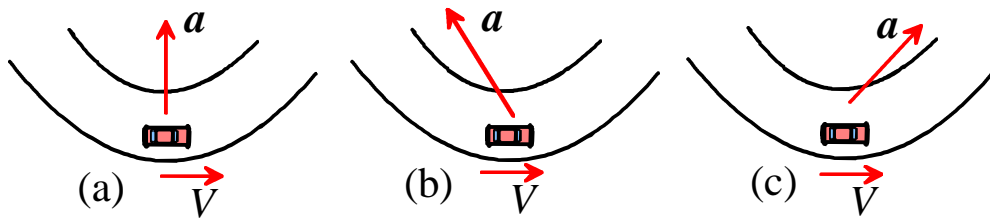
2 (4 points)

3. (8 points)

4. (7 points)

5. (8 points)

TOTAL (30 points)



1. The figure shows a car that travels along a circular road.

In Fig 1(a), the car travels at constant speed

In Fig 1(b), the driver is braking, and the car's speed is decreasing

In Fig 1(c), the driver has her foot on the gas and the car's speed is increasing.

Draw an arrow on each of figures (a), (b), (c) to show the approximate direction of the car's acceleration vector.

[1 POINT EACH]

2. The figure shows a vibration measurement from a velocity transducer. The vibration may be assumed to be harmonic. Estimate

(a) The period of oscillation

4 cycles in 0.1sec gives $T=0.1/4=0.025s$

(b) The angular frequency of oscillation

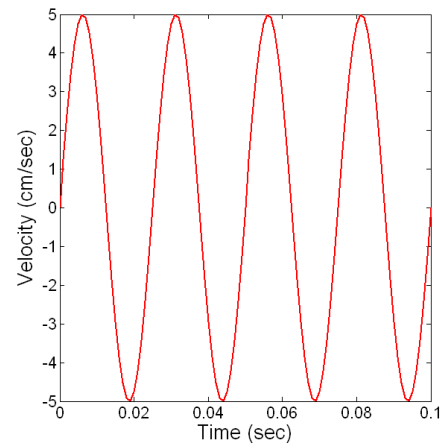
$$\text{Angular frequency is } \omega = \frac{2\pi}{T} = 80\pi \text{ rad / s}$$

(c) The amplitude of the velocity

5 cm/s

(d) The amplitude of the displacement.

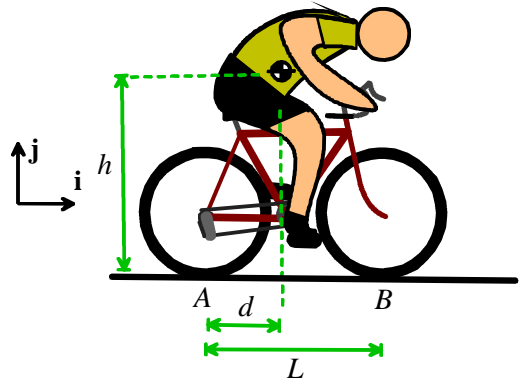
$$X_0 = V_0 / \omega = 5 / (80\pi) = 1 / (16\pi) \text{ cm}$$



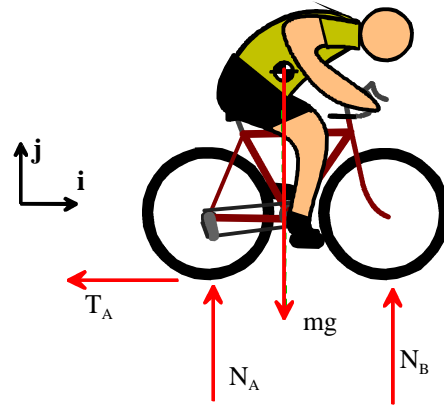
[1 POINT EACH]

3. The goal of this problem is to estimate the shortest stopping distance for a bicycle during rear-wheel braking. Assume that

- At time $t=0$ the bicycle has velocity \mathbf{Vi} . At this instant, the rider brakes hard enough to lock the rear wheel, causing it to skid.
- The coefficient of friction between the rear wheel and the ground is denoted by μ
- The front wheel rolls freely.
- Air resistance may be neglected.



3.1 Draw the forces acting on the bicycle and rider, using the figure shown. The bicycle and rider together may be idealized as a particle on a massless frame.



[2 POINTS]

3.2 Write down Newton's law of motion $\mathbf{F}=\mathbf{ma}$ and the moment balance equation $\mathbf{M}=\mathbf{0}$ about the center of mass, expressing your answer as components in the basis shown.

$$-T_A \mathbf{i} + (N_A + N_B - mg) \mathbf{j} = m \mathbf{a} \mathbf{i}$$

$$[N_B(L-d) - N_A d - T_A h] \mathbf{k} = \mathbf{0}$$

[2 POINTS]

3.3 Hence, calculate an expression for the acceleration of the bicycle in terms of μ , g , d and h .

The friction law gives $T_A = \mu N_A$. The moment equation and vertical component of Newton's law are

$$N_A + N_B - mg = 0$$

$$N_B(L-d) - N_A(d + \mu h) = 0$$

Eliminate N_B :

$$(L-d)N_A + N_A(d + \mu h) - mg(L-d) = 0$$

$$\Rightarrow N_A = mg(L-d) / (L + \mu h)$$

Finally the \mathbf{i} component of $\mathbf{F}=\mathbf{ma}$ shows that

$$a = -T_A / m = -\mu N_A / m = -\mu mg(L-d) / (L + \mu h)$$

[2 POINTS]

3.4 Deduce a formula for the stopping distance in terms of V , μ , g , d and h .

The constant acceleration formulas give

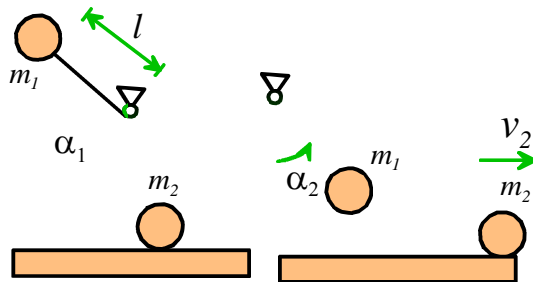
$$0 = V + at$$

$$d = Vt + \frac{1}{2}at^2 = -\frac{V^2}{2a} = V^2(L + \mu h) / 2\mu mg(L - d)$$

[2 POINTS]

4. The figure shows an experimental apparatus for measuring the restitution coefficient of, e.g. a golf-ball or a bowling ball. It uses the following procedure

- A pendulum (a golf-club head, e.g.) is swung to a known initial angle α_1 and then dropped from rest so as to strike the ball
- The angle of follow-through α_2 of the pendulum is recorded



Your goal is to derive a formula that can be used to determine the restitution coefficient e from the measured data.

4.1 Using energy conservation, derive an expression for the speed V of the mass on the end of the pendulum just before it strikes the ball, in terms of α_1 , l and the gravitational acceleration.

$$\text{Energy conservation gives } mgl(1 - \cos \alpha_1) = \frac{1}{2}mV^2 \Rightarrow V = \sqrt{2gl(1 - \cos \alpha_1)}$$

[2 POINTS]

4.2 Similarly, derive an expression for the speed v_1 of the mass on the end of the pendulum just after it strikes the ball, in terms of α_2 , l and the gravitational acceleration.

$$v_1 = \sqrt{2gl(1 - \cos \alpha_2)}$$

[1 POINT]

4.3 Use momentum conservation to calculate an expression for the velocity v_2 of the ball just after it is struck, in terms of V and v_1 , and any other necessary parameters.

Momentum conservation during the collision requires that

$$m_1V = m_1v_1 + m_2v_2 \Rightarrow v_2 = m_1(V - v_1) / m_2$$

[2 POINTS]

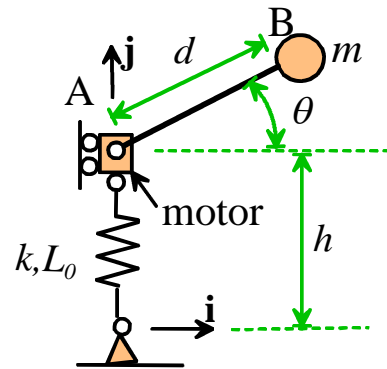
4.4 Hence, deduce a formula for the coefficient of restitution, in terms of α_1 , α_2 , l , and g , and any other necessary parameters.

The restitution coefficient formula gives

$$\begin{aligned}
 v_2 - v_1 &= -e(0 - V) \Rightarrow e = (v_2 - v_1) / V \\
 e &= m_1(V - v_1) / (Vm_2) - v_1 / V \\
 e &= \frac{m_1}{m_2} - \frac{(m_1 + m_2)}{m_2} \frac{v_1}{V} = \frac{m_1}{m_2} - \frac{(m_1 + m_2)}{m_2} \frac{\sqrt{2gl(1 - \cos \alpha_2)}}{\sqrt{2gl(1 - \cos \alpha_1)}} \\
 &= \frac{m_1}{m_2} - \frac{(m_1 + m_2)}{m_2} \frac{\sqrt{(1 - \cos \alpha_2)}}{\sqrt{(1 - \cos \alpha_1)}}
 \end{aligned}$$

[2 POINTS]

5. The figure shows an idealization of a motor mounted on a flexible vibration-isolation support. The motor turns the shaft AB at constant angular speed ω , so that $\theta = \omega t$. The motor is free to move vertically, but is prevented from moving horizontally or rotating. The vibration isolator can be idealized as a spring with stiffness k and unstretched length L_0 . The mass of the motor and the shaft may be neglected.



The goal of this problem is to derive an equation of motion for the vertical position of the motor $h(t)$.

5.1 Write down an expression for the position vector of the mass m in terms of h , d , and θ

$$\mathbf{r} = d \cos \theta \mathbf{i} + (h + d \sin \theta) \mathbf{j}$$

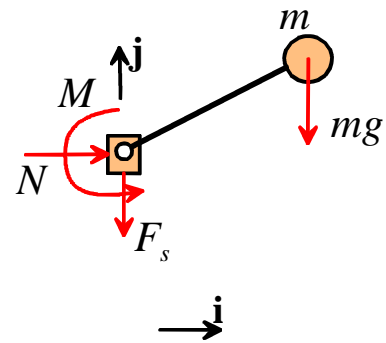
[1 POINT]

5.2 Hence, calculate a formula for the acceleration of the mass m in terms of d , θ and ω and the time derivatives of h .

$$\mathbf{a} = -d\omega^2 \cos \theta \mathbf{i} + \left(\frac{d^2 h}{dt^2} - d\omega^2 \sin \theta \right) \mathbf{j}$$

[2 POINTS]

5.3 Draw a free body diagram for the isolated portion of the system shown in the figure (you can draw forces and moments directly on the figure). Idealize the motor and shaft as a massless frame. Note that the motor is prevented from moving horizontally and is also prevented from rotating.



[3 POINTS]

5.4 Hence, show that the equation of motion for h is given by

$$\frac{d^2h}{dt^2} + \frac{k}{m}h = \frac{k}{m}L_0 - g + d\omega^2 \sin \omega t$$

Newton's law gives

$$N\mathbf{i} - (F_s + mg)\mathbf{j} = -md\omega^2 \cos \theta \mathbf{i} + m \left(\frac{d^2h}{dt^2} - d\omega^2 \sin \theta \right) \mathbf{j}$$

The spring force law is $F_s = k(h - L_0)$. Take the \mathbf{j} component and re-arrange to get the answer stated.

[2 POINTS]